Numerical Modelling in Hemodynamics

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Outline

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1. Continuum mechanical modelling / Hemorheology
2. Numerical methods for blood flow simulations (FEM)
3. Fluid-Structure interaction

Motivation

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$\Longrightarrow$ importance of further detailed study e.g. using computational science

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- plasma . . . 54 %

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  - deformability
  - tendency to align with the flow field at high shear rates

$\Longrightarrow$ effect non-Newtonian behaviour of blood

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Hemodynamic factors:
flow separation, flow recirculation, and oscillatory wall shear stress:
important role in the localization and development of vascular diseases $\Longrightarrow$

- healthy patient: high shear rate, typical vessel lengths: no time to build microstructures

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- disease states: diseases in which the arterial geometry has altered (aneurysms), aggregates get more stable

non-Newtonian models more relevant

Non-Newtonian nature of blood

Blood: plasma + cells (45% volume concentration)

vessel	radius	RE
aorta	1.25	3400
arteries	0.2	500
arterioles	$1.5.10^{-3}$	0.7
veins	0.25	140
vena cava	1.5	3300

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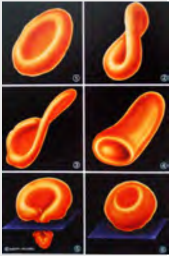

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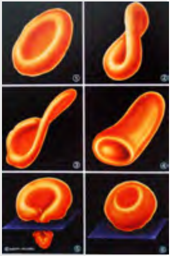

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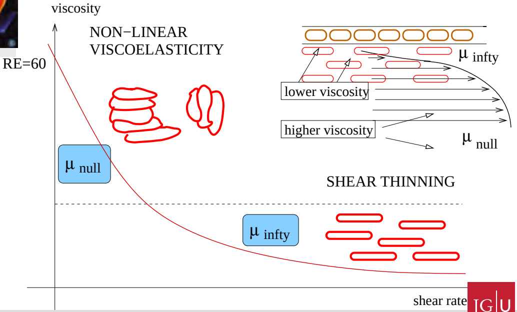

Characteristic numbers

$$Re = \frac{\rho\, U\, L}{\mu} \qquad \alpha = \frac{L}{2}\sqrt{\frac{\rho\, \omega}{\mu}}$$

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$\rho \dots$ density; 1.06×10^{-3} kg $\cdot$ m^{-3},
$U \dots$ characteristic velocity,
$\boldsymbol{u} = (u_1, \dots, u_d) \dots$ fluid velocity
$L \dots$ characteristic length
$\mu \dots$ characteristic viscosity; $3 - 5.5$ mPa $\cdot$ s
$\omega \dots$ characteristic angular frequency
$Re \in (0.0015, 6100) \qquad \alpha \in (0.003, 30)$

Pictures

Fig.1,2, 3

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$$D(\nabla \boldsymbol{u}) := \left(\frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right)_{i,j=1}^{d}$$

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- **Asymptotic limits:**

$\mu_0 = 65.7cP\ (20^0 C), \ \mu_0 = 45.1cP\ (37^0 C)$
$\mu_\infty = 4.47cP\ (20^0 C), \ \mu_\infty = 3.07cP\ (37^0 C)$

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Non-Newtonian models

Mathematical model

- Flow problem
- Structure equation
- Fluid-structure interaction

Flow problem

- shear-thinning properties

$$\begin{aligned} \rho\partial_t \boldsymbol{u} + \rho\,(\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u} - \operatorname{div}\left[2\mu(|D(\boldsymbol{u})|)D(\boldsymbol{u})\right] + \nabla p &= 0 \\ \operatorname{div}\boldsymbol{u} &= 0 \end{aligned}$$

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power-law model

$$\mu = \mu(D(\boldsymbol{u})) = \nu(1 + \gamma |D(\boldsymbol{u})|^2)^{\frac{p-2}{2}}$$

Carreau-Yasuda model

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  - challenge: developing nonlinear viscoelastic constitutive models for blood
  - the simplest model that captures the shear rate dependence and elasticity the rate-type shear thinning model (Oldroyd-B type model)

$$\tau = -pI + \tau^v + \tau^e$$

$$\lambda \left(\frac{\partial \tau^e}{\partial t} + \boldsymbol{u} \cdot \nabla \tau^e - \tau^e \cdot (\nabla \boldsymbol{u})^T - \nabla \boldsymbol{u} \cdot \tau^e \right) + \tau^e = 2 \left(\frac{\mu(D(\boldsymbol{u}))}{\mu_0} - (1 - \alpha) \right) D(\boldsymbol{u})$$

viscoelastic shear-dependent fluids

$$div(\mathbf{u}) = 0$$

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- nonlinear coupled parabolic-hyperbolic system
- singular behaviour for large Reynolds, Weissenberg

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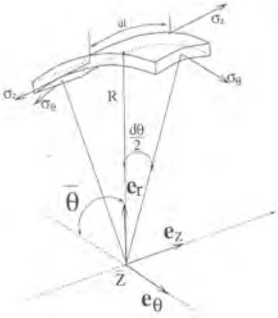
Domain deformation

Unknown $\eta(x_1, t) = R(x_1, t) - R_0(x_1) \Longrightarrow$ **generalized string model** for cylindrical geometry with a non-constant reference radius $R_0(x_1)$:

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{|\sigma_{x_1}|}{\rho_w} \frac{\left(\frac{\partial^2 \eta}{\partial x_1^2} + \frac{\partial^2 R_0}{\partial x_1^2}\right)}{\left[1 + \left(\frac{\partial R_0}{\partial x_1}\right)^2\right]^2} + \frac{E\eta}{\rho_w (R_0 + \eta) R_0} - c \frac{\partial^3 \eta}{\partial t \partial x_1^2}$$

$$= \frac{(-\mathbf{T}_f \boldsymbol{n} \cdot \boldsymbol{e}_r - P_w)}{\rho_w h}$$

- ρ_W wall density
- h wall thickness
- E the Young modulus of elasticity
- $|\sigma_{x_1}| = E/3$ shear modulus
- c viscoelastic constant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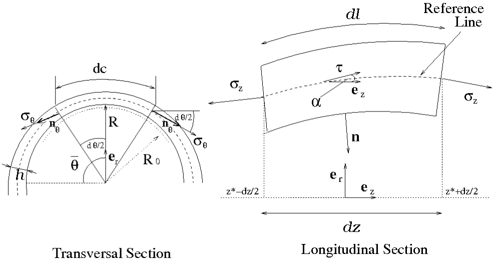
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Transversal Section
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Reference Line
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ez
z*−dz/2
z*+dz/2
dz
Longitudinal Section

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1. Dirichlet boundary condition

$$\boldsymbol{u}(x_1, R_0 + \eta, t) = \frac{\partial \eta}{\partial t}\boldsymbol{N} \qquad \text{on } \Gamma^w, \tag{1}$$

$\boldsymbol{N}$ is unit outward normal to the domain boundary

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2. $$\mathbf{T}_S \boldsymbol{N} = \mathbf{T}_f \boldsymbol{N}$$

   the equation for domain deformation (1).