On the L^p-theory of the Navier-Stokes Equations in the exterior of a rotating obstacle with Oseen condition

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Outline

Problem description

- The Navier-Stokes Equations in the rotation setting
- The transformed Navier-Stokes Equations

Problem approach

- The semigroup on $L^2_\sigma(\Omega)$
- The semigroup on $L^p_\sigma(\Omega)$
- The Kato iteration

Physical setting of the problem

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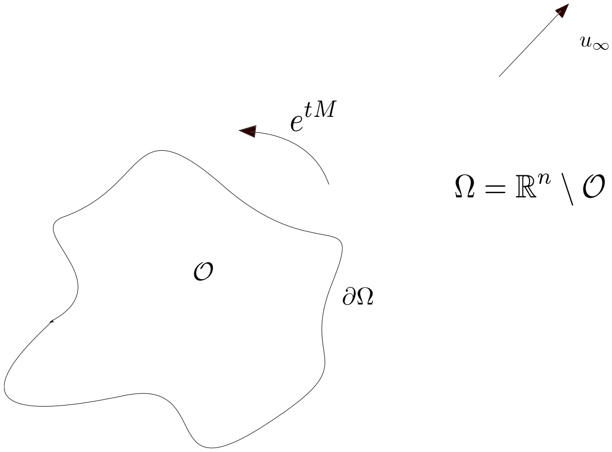

$$\Omega = \mathbb{R}^n \setminus \mathcal{O}$$

The Equations

The usual Navier-Stokes Equations:

$$\begin{aligned}
\partial_t v - \Delta v + (v \cdot \nabla) v + \nabla q &= 0, && y \in \Omega(t),\ t > 0; \\
\operatorname{div} v &= 0, && y \in \Omega(t),\ t > 0; \\
v &= My, && y \in \partial\Omega(t),\ t > 0; \\
v(y,0) &= v_0(y), && y \in \Omega;
\end{aligned}$$

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Oseen's condition:

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\lim_{|y| \to \infty} v(y,t) = u_\infty \qquad t > 0.
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The Transformation

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Coordinate transformation:

$$y(t) = e^{tM}x \qquad u(x,t) = e^{-tM}(v(y,t) - u_\infty) \qquad \mathrm{p}(x,t) = \mathrm{q}(y,t).$$

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Known results closely related to the problem

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Rotation

- Borchers (1992): weak solutions
- Hishida (1999/2001): existence of a local mild solution in L^2 space
- Geissert/Heck/Hieber (2006): extension to L^p space , $1 < p < \infty$
- Hishida/Shibata (2009): global solutions for small initial data

Rotation and Oseen

- Shibata (2008): $u_\infty = ae_3$, global L^p-L^q-estimates
- Farwig/Neustupa (2008): $u_\infty = ae_3$, calculation of the essential spectrum
- Hansel (to appear): arbitrary u_∞, existence of a local mild solution in $\mathbb{R}^d$

Goal of this talk

What we want to prove:

Theorem

Let $d \geq 2$, $d \leq p \leq q < \infty$ *and* $u_0 - b \in L^p_\sigma(\Omega)$. *There exists* $T_0 > 0$ *and a unique mild solution to system* (1) *which satisfies*

$$t \mapsto t^{\frac{d}{2}(\frac{1}{p}-\frac{1}{q})}u(t) \in C([0,T_0); L^q_\sigma(\Omega)),$$

$$t \mapsto t^{\frac{d}{2}(\frac{1}{p}-\frac{1}{q})+\frac{1}{2}}\nabla u(t) \in C([0,T_0); L^q(\Omega)^{d^2}).$$

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- linearized problem: semigroup existence on $L^2_\sigma(\Omega)$
- semigroup extension to $L^p_\sigma(\Omega)$, $1 < p < \infty$
- nonlinearity: Kato iteration

Recall: mild solution

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Definition (mild solution)

Let $0 < T < \infty$. We call a function $u \in C([0,T); L^p_\sigma(\Omega))$ a *mild solution* of system (1), if it satisfies the integral equation

$$u(t) = T^p_{\Omega,b}(t)(u_0 - b) - \int_0^t T^p_{\Omega,b}(t-s)P_\Omega(F_1(u,u)(s))\mathrm{d}s$$

$$+ \int_0^t T^p_{\Omega,b}(t-s)P_\Omega(F_2)\mathrm{d}s$$

where $T^p_{\Omega,b}$ denotes the semigroup generated by $A_{\Omega,b}$ on $L^p_\sigma(\Omega)$.

$$F_1(u,v) = (u\cdot\nabla)v \qquad F_2 = -\Delta b + (b\cdot\nabla)b - ((Mx - u_\infty)\cdot\nabla)b + Mb$$

Solving the linearized problem

Define:

$$A_{\Omega,b}u := P_\Omega\big(\Delta u + ((Mx - u_\infty)\cdot\nabla)u - Mu + (u\cdot\nabla)b + (b\cdot\nabla)u\big)$$

$$D(A_{\Omega,b}) := \left\{f \in W^{2,p}(\Omega)^d \cap W_0^{1,p}(\Omega)^d \cap L^p_\sigma(\Omega) : ((Mx - u_\infty)\cdot\nabla)f \in L^p(\Omega)^d\right\}$$

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- $\Omega = D, \mathbb{R}^d$ with D a bounded (smooth) domain: $A_{\Omega,b}$ generates a strongly continious semigroup.
- In both cases the semigroup satisfies L^p-L^q-*smoothing estimates*.
- Idea for exterior domain: reduction to these cases.

Existence proof via the Lumer-Phillips Theorem - a sketch

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Step 1: The existence of the exterior domain semigroup on $L^2_\sigma(\Omega)$.

- dissipativity of the operator

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  - show: $\exists C \in \mathbb{R} \quad \forall \omega > C \quad \forall u \in D(A_{\Omega,b}) \qquad \mathrm{Re}\langle (A_{\Omega,b} - \omega)u, u\rangle \leq 0$

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  - proof: localization shows

$$R(\lambda, A_{\Omega,b})f = U_\lambda(\mathrm{Id} - P_\Omega T_\lambda)^{-1} f = U_\lambda \Sigma_{n=0}^{\infty}(P_\Omega T_\lambda)^n f.$$

    where
    - $P_\Omega T_\lambda$ are the pertubating terms
    - $U_\lambda f = \phi R(\lambda, A_{\mathbb{R}^d,b}) + (1-\phi)R(\lambda, A_{D,b}) - B_1(\nabla\phi(R(\lambda, A_{\mathbb{R}^d,b}) - R(\lambda, A_{D,b})))$

A representation formula for the semigroup

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Step 2: The extension of the semigroup to $L^p_\sigma(\Omega)$, $1 < p < \infty$.
Let

$$T_n(t) := \int_0^t T_{n-1}(t-s)H(s)\mathrm{d}s,$$

$$T_0 := \phi T_{\mathbb{R}^d} + (1-\phi)T_{D,b} - B_1(\nabla\phi(T_{\mathbb{R}^d} - T_{D,b})),$$

where $\mathcal{L}(H) = P_\Omega T_\lambda$
$\Rightarrow T(t)f = \sum_{n=0}^{\infty} T_n(t)f$ is the semigroup on $L^2_\sigma(\Omega)$

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- Recall: $R(\lambda, A_{\Omega,b})f = U_\lambda \Sigma_{n=0}^{\infty}(P_\Omega T_\lambda)^n f$
- $\mathcal{L}(T_0 f) = U_\lambda f$
- $\mathcal{L}(T_1 f) = \mathcal{L}(T_0 * Hf) = U_\lambda P_\Omega T_\lambda f$

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- $\mathcal{L}(T_0 f) = U_\lambda f$
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- $\mathcal{L}(T_n f) = \mathcal{L}(T_{n-1} * Hf) = U_\lambda (P_\Omega T_\lambda)^n f$

Extension of the representation formula to L^p-space

We have:

$$T(t)f = \Sigma_{n=0}^{\infty} T_n(t)f$$

is the semigroup on $L^2_\sigma(\Omega)$.
One can show:

$$\begin{aligned} \|T(t)f\|_p &\leq \Sigma_{n=0}^{\infty} \|T_n(t)f\|_p \\ &\leq C\|f\|_p \qquad \text{for all } f \in L^2_\sigma(\Omega) \cap L^p_\sigma(\Omega) \end{aligned}$$

- $T(t)$ can be (uniquely) extended to all $L^p_\sigma(\Omega)$
- we have a representation formula for the semigroup on $L^p_\sigma(\Omega)$, $1 < p < \infty$

L^p-L^q-smoothing estimates

Key ingredient for Kato's iteration argument:
Let $1 < p < \infty$ und $d \le p \le q \le \infty$.
$\exists \quad C > 0$ such that

$$\|T_{\Omega,b}(t)u\|_q \le Ct^{-\frac{d}{2}(\frac{1}{p}-\frac{1}{q})}e^{\omega t}\|u\|_p$$
$$\|\nabla T_{\Omega,b}(t)u\|_p \le Ct^{-\frac{1}{2}}e^{\omega t}\|u\|_p \quad \text{for all } t \in \mathbb{R}^+, u \in L^p_\sigma(\Omega)$$

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- estimates hold on $\mathbb{R}^d$, bounded domain D
- representation formula allows to extend these estimates to the exterior domain semigroup $T^p_{\Omega,b}$ on $L^p_\sigma(\Omega)$

The nonlinear terms

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Step 3: Treatment of the nonlinear terms via Kato's iteration procedure.
The mild solution is exactly the fixpoint of the following sequence:

$$u_{j+1}(t) = u_1 + Gu_j(t),$$

where

$$u_1(t) = T^p_{\Omega,b}(t)(u_0 - b)$$

$$Gu_j(t) = -\int_0^t T^p_{\Omega,b}(t-s)F_1(u_j, u_j)(s)\mathrm{d}s + \int_0^t T^p_{\Omega,b}(t-s)F_2\mathrm{d}s.$$

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By successive approximation of $t^{\frac{1}{d}(\frac{1}{p}-\frac{1}{q})}u_j(t)$, $t^{\frac{1}{d}(\frac{1}{p}-\frac{1}{q})+\frac{1}{2}}\nabla u_j(t)$ in the Banach space $C([0, T), L^q_\sigma(\Omega))$ we show the existence of a local mild solution to the (nonlinear) problem.

Acknowledgment

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Thank you very much for your attention!