Vortex interactions and instabilities in stratified and rotating fluids

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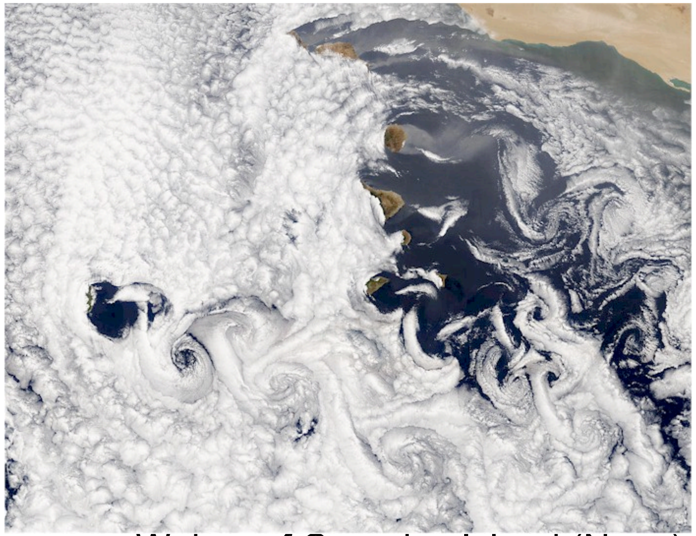
Geophysical flows



Wakes of Canaries Island (Nasa)

✓ Planetary rotation

✓ Stable stratification

$$\frac{D\boldsymbol{u}}{Dt} + 2\Omega_b \boldsymbol{e_z} \times \boldsymbol{u} = -\frac{1}{\rho_0}\boldsymbol{\nabla} p - \frac{g\rho'}{\rho_0}\boldsymbol{e_z} + \nu\Delta\boldsymbol{u}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} = 0$$

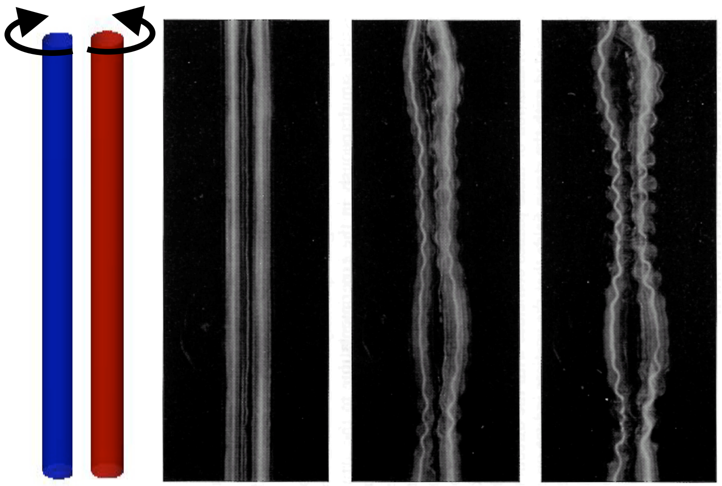
$$\frac{D\rho'}{Dt} + \frac{\partial\bar{\rho}}{\partial z}u_z = D\Delta\rho'$$

Brunt-Väisälä frequency: $N = \sqrt{-\frac{g}{\rho}\frac{\partial\bar{\rho}}{\partial z}}$

Counter-rotating vortex pair

homogeneous fluid

- ✓ Crow instability (symmetric bending)
- ✓ Elliptic instability (antisymmetric core deformation)



Leweke & Williamson (1998)

Counter-rotating vortex pair

homogeneous fluid

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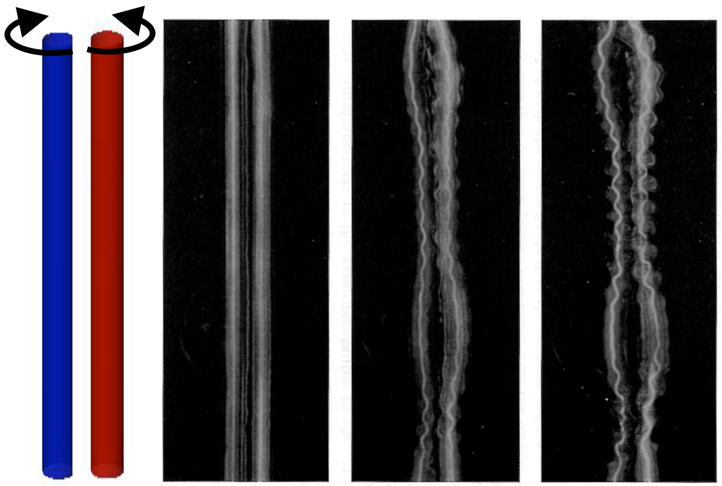
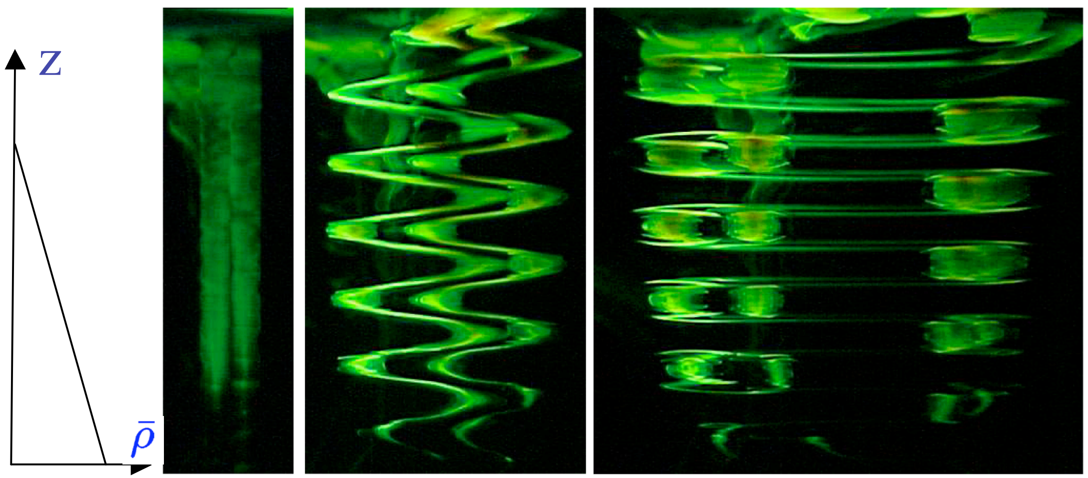


Leweke & Williamson (1998)

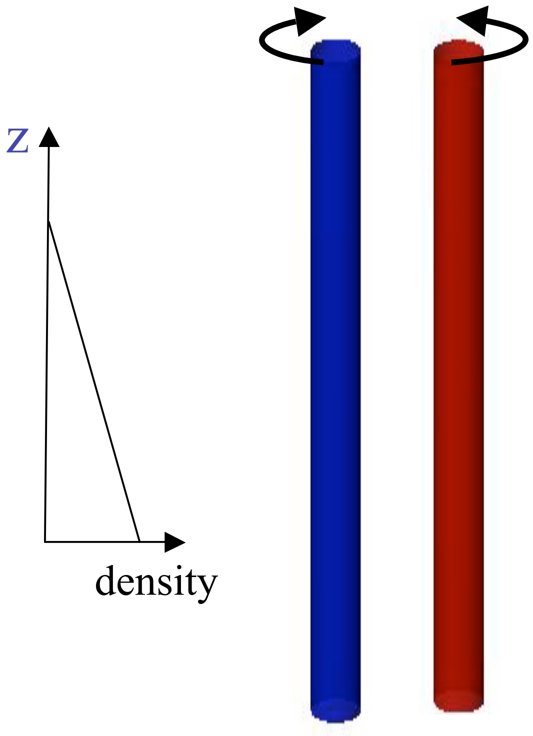
stably stratified fluid

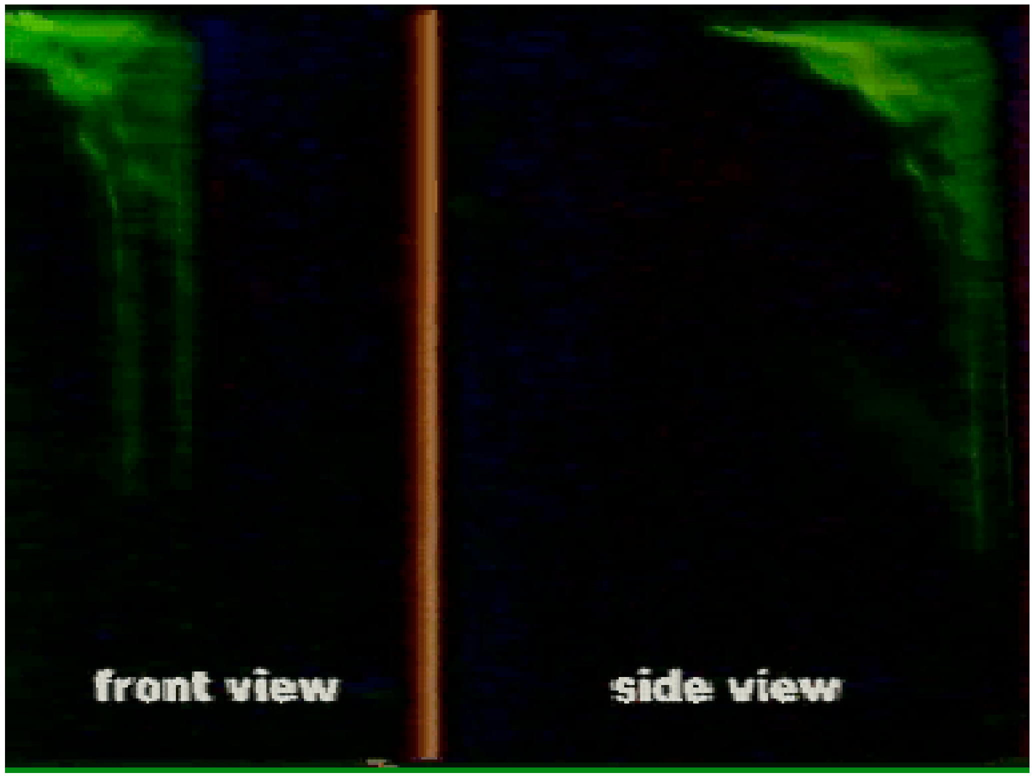
✓ Zigzag instability (antisymmetric bending)



(Billant & Chomaz, 2000)

Zigzag instability of a counter-rotating vortex pair in a strongly stratified fluid

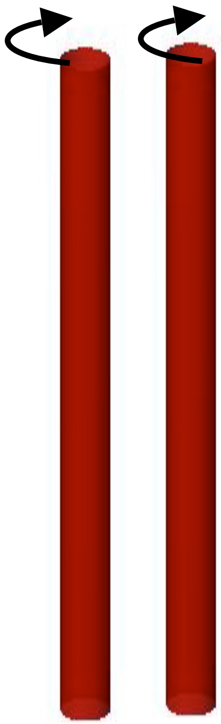


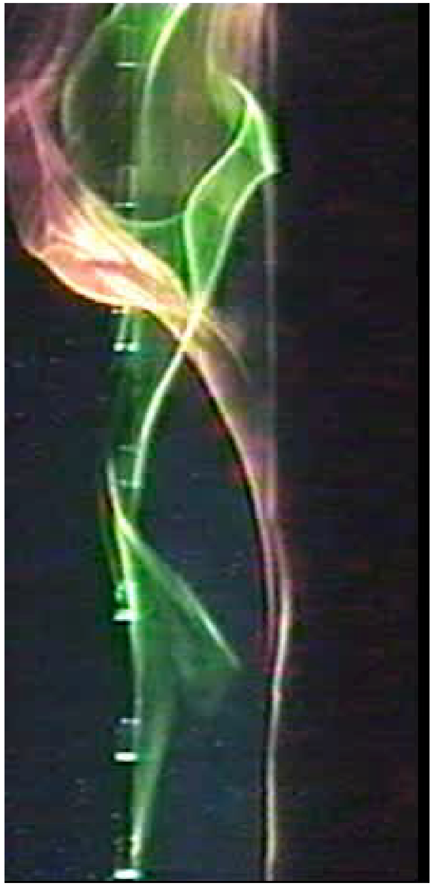


Co-rotating vortex pair

homogeneous fluid

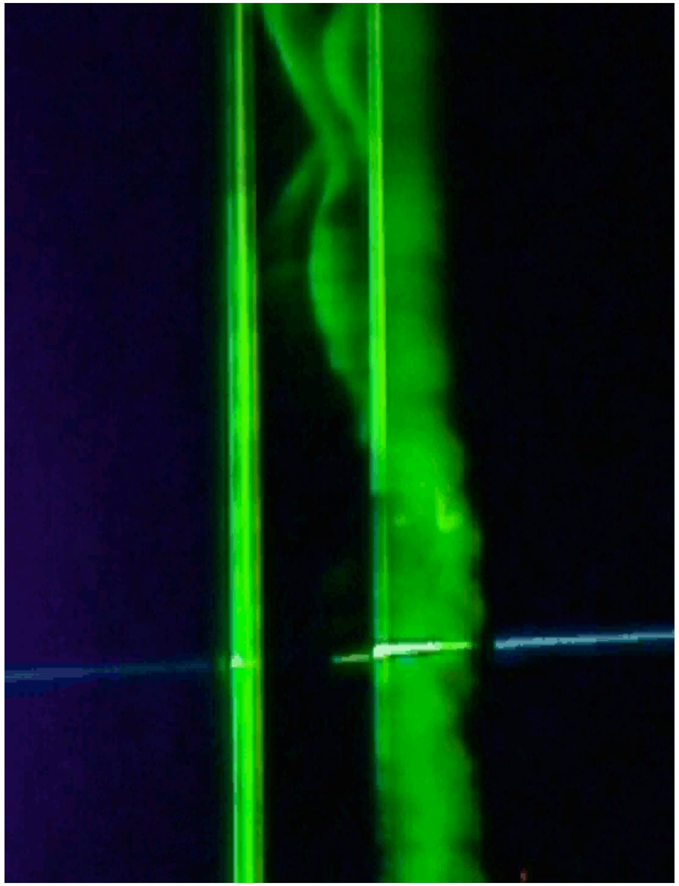
✓ Elliptic instability (antisymmetric core deformation)





stratified fluid

✓ Zigzag instability (symmetric bending of the whole vortex)



Co-rotating vortex pair

homogeneous fluid

✓ Elliptic instability
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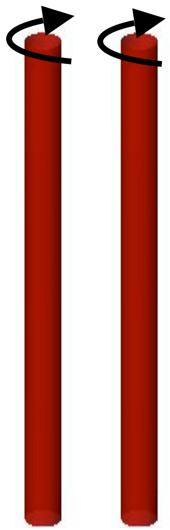
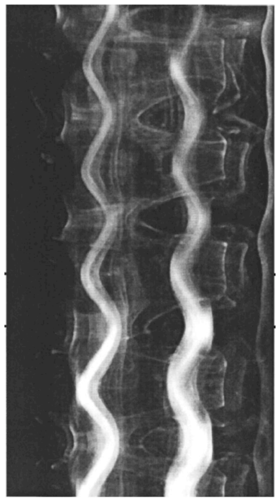
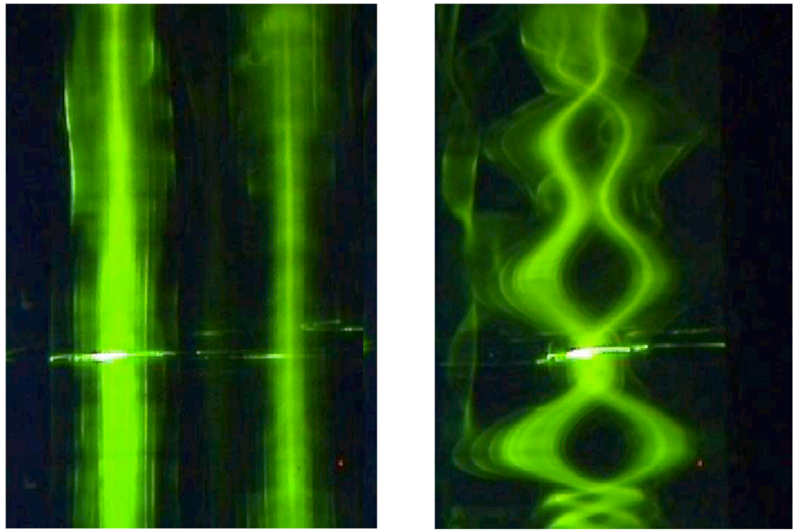




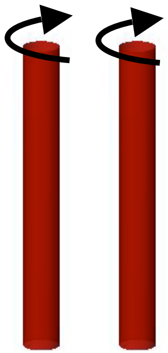
Meunier & Leweke (2001)

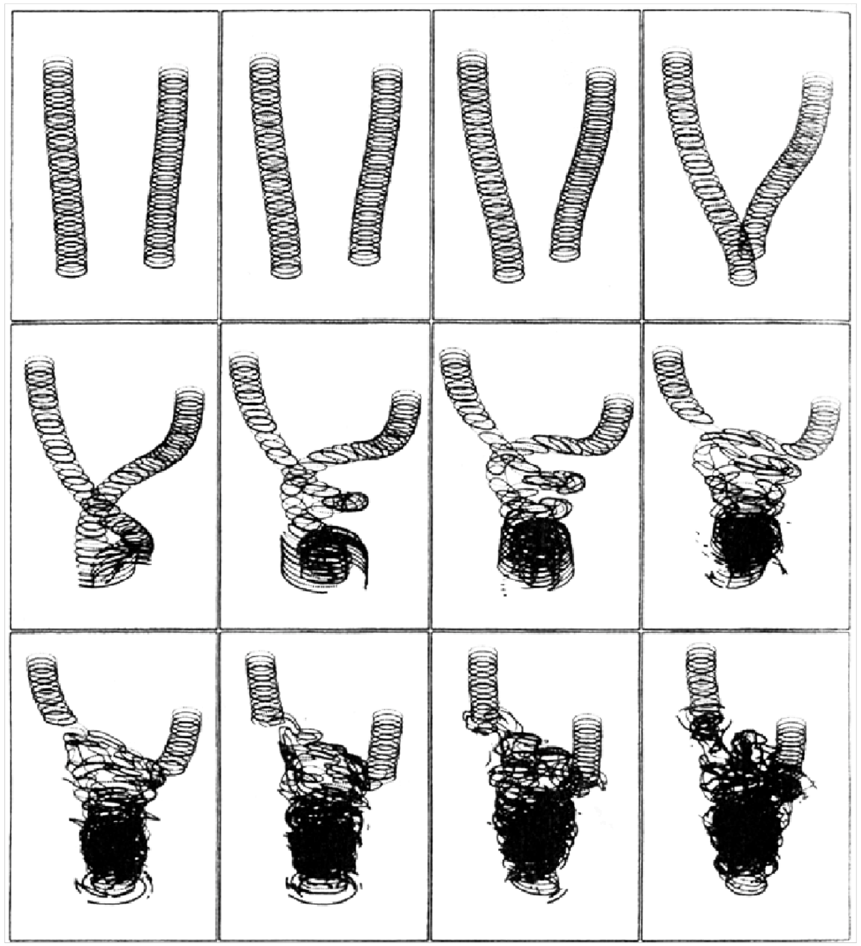
stratified fluid

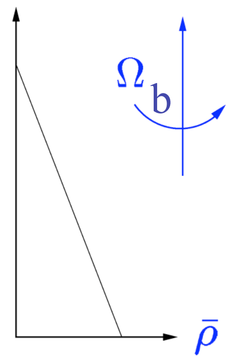
✓ Zigzag instability
(symmetric bending of the whole vortex)



Quasi-Geostrophic fluids:
(Strong stratification and rapid rotation)

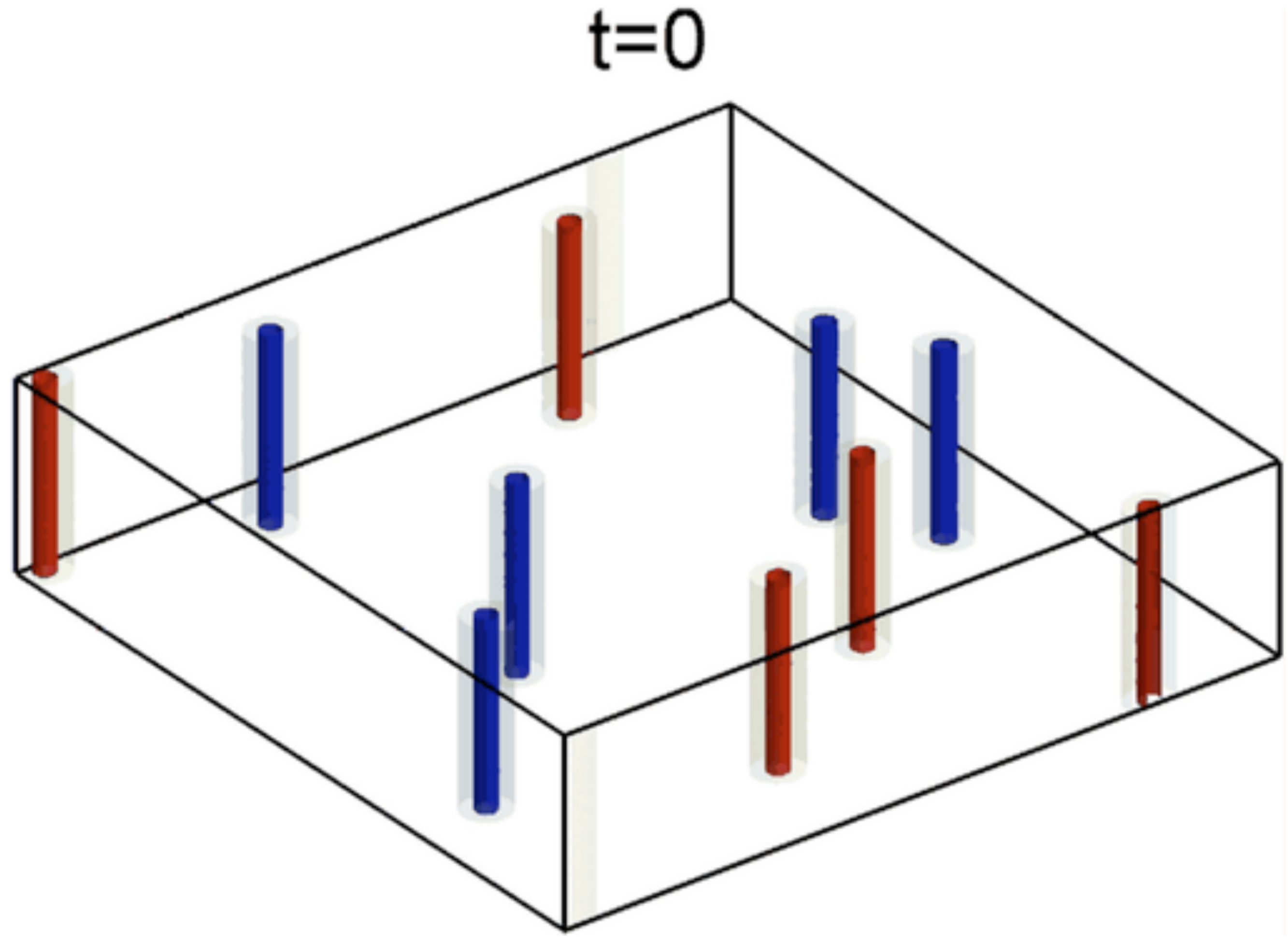






Dritschel &
de la Torre Juàrez
(1996)

Random configuration of vertical vortices in a stratified fluid (DNS, Fh=0.8, Re=1060, Ro=∞)

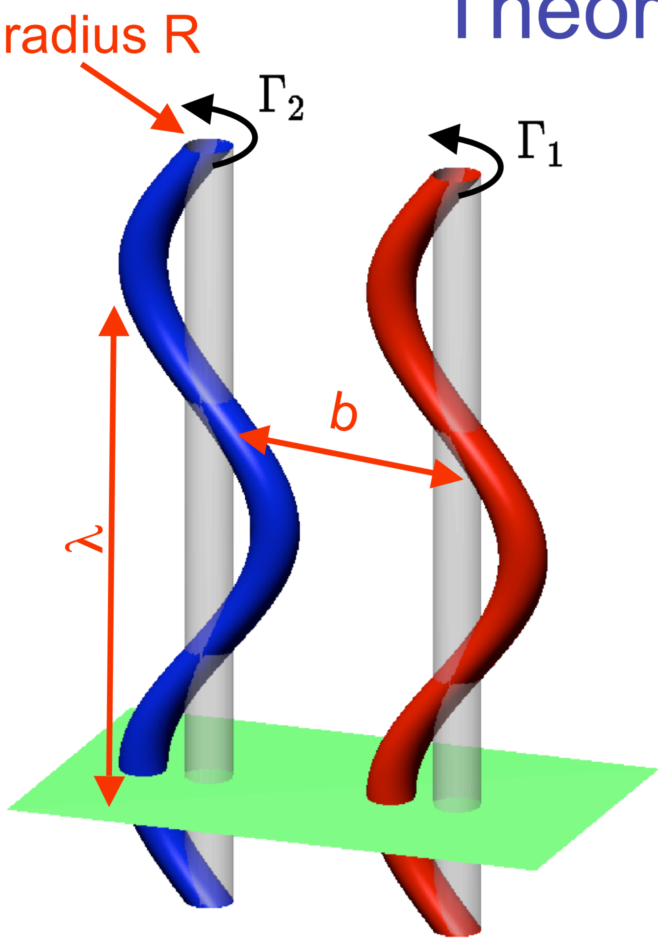


Deloncle (2007)

Question:

Why different 3D instabilities are observed in stratified-rotating fluids compared to homogeneous fluids ?

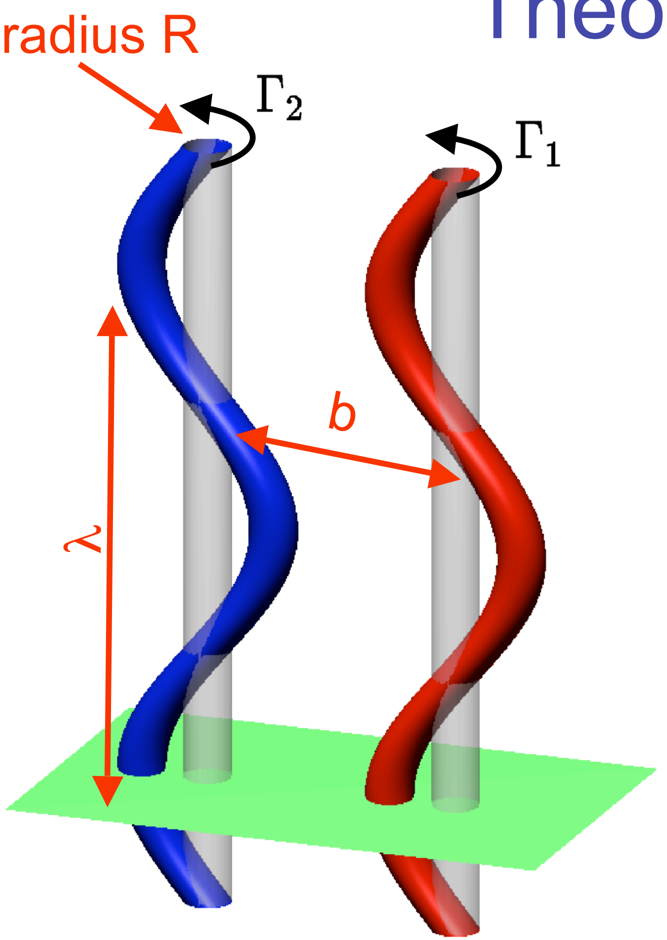
Theoretical approach



Hypotheses:

- Inviscid fluid
- well-separated vortices : b >> R
- bending deformations
- long-wavelength : λ >> R

Theoretical approach



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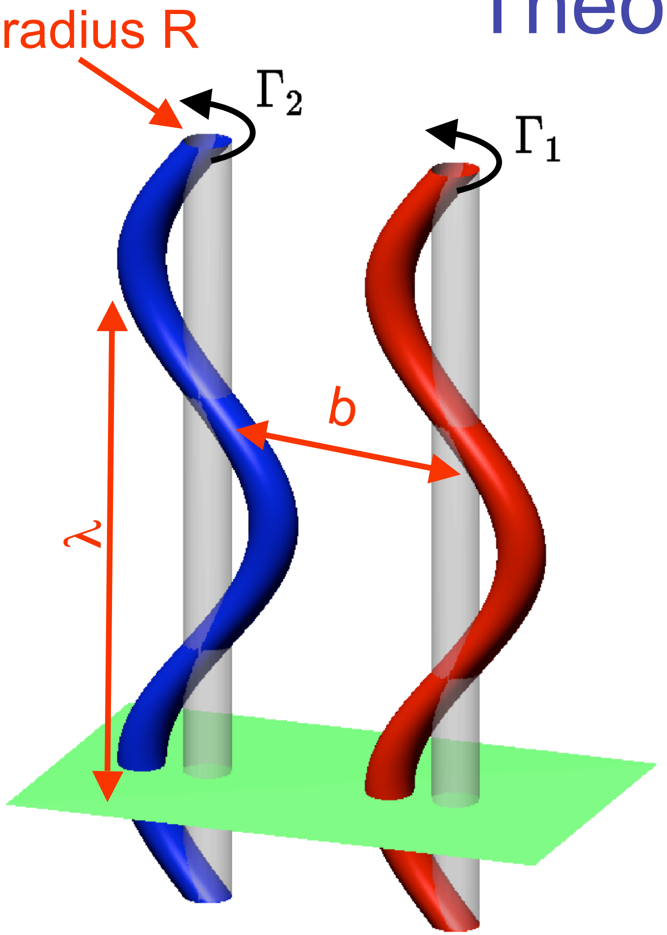
=> Vortex filaments method

Theoretical approach



Hypotheses:

- Inviscid fluid
- well-separated vortices : b >> R
- bending deformations
- long-wavelength : λ >> R

=> Vortex filaments method

- stratified and rotating fluid

=> The Kelvin theorem (conservation of the circulation) is not valid

=> The Helmholtz theorem (vortex lines=material lines) is not valid

Asymptotic stability analysis

Expansion with the small parameters: $\frac{R}{b} \ll 1 \qquad kR \ll 1$

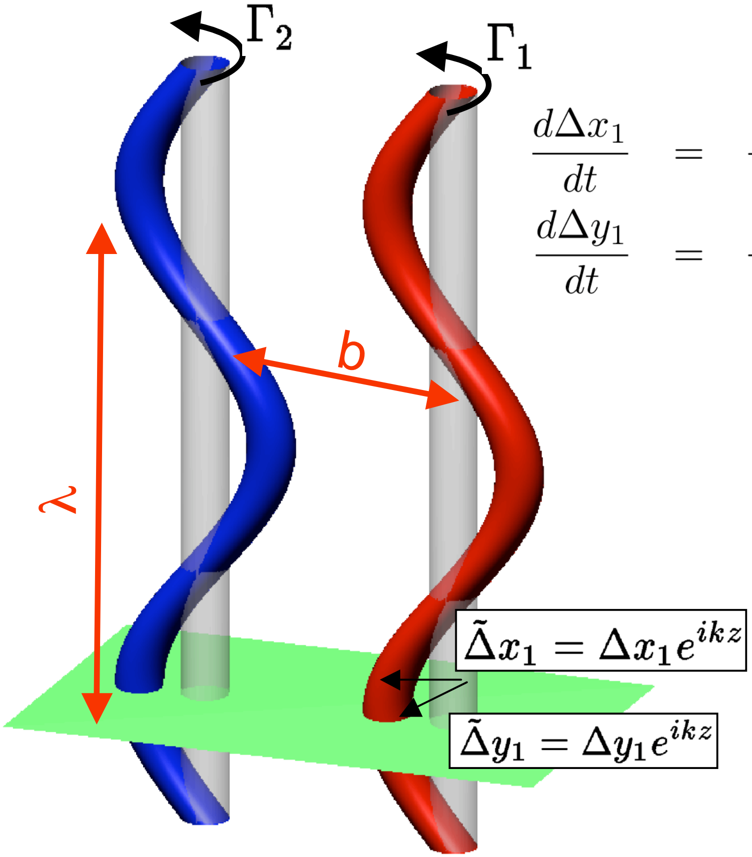
Base flow:
(near each vortex core)

$$\boldsymbol{U}_b = \underbrace{r\Omega(r)\boldsymbol{e}_\theta}_{\text{single axisymmetric vortex}} + \underbrace{\frac{R^2}{b^2}\boldsymbol{U}_{bs}}_{\text{Straining flow due to the companion vortex}} + O\left(\frac{R^3}{b^3}\right)$$

Perturbation:

$$\boldsymbol{u} = \tilde{\boldsymbol{u}}(r,\theta,t)e^{ikz} = \left[\underbrace{\tilde{\boldsymbol{u}}_0}_{\text{2D displacements}} + \underbrace{\frac{R^2}{b^2}\tilde{\boldsymbol{u}}_1}_{\text{strain effects}} + \underbrace{k^2R^2\tilde{\boldsymbol{u}}_2}_{\text{3D effects}} + \dots\right]e^{ikz}$$

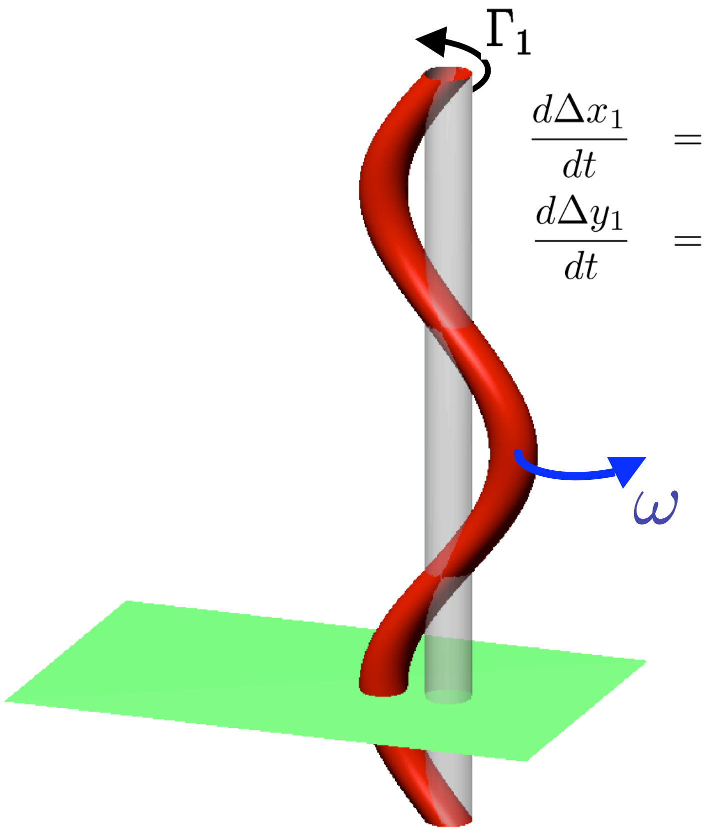
Results



$$
\begin{aligned}
\frac{d\Delta x_1}{dt} &= -\frac{\Gamma_2}{2\pi b^2}\Delta y_1 + f\Delta y_1 + \frac{\Gamma_2}{2\pi b^2}\psi\Delta y_2 - \frac{\Gamma_1}{2\pi R^2}\omega_r\Delta y_1 + \frac{|\Gamma_1|}{2\pi R^2}\omega_i\Delta x_1 \\
\frac{d\Delta y_1}{dt} &= -\frac{\Gamma_2}{2\pi b^2}\Delta x_1 - f\Delta x_1 + \frac{\Gamma_2}{2\pi b^2}\chi\Delta x_2 + \frac{\Gamma_1}{2\pi R^2}\omega_r\Delta x_1 + \frac{|\Gamma_1|}{2\pi R^2}\omega_i\Delta y_1
\end{aligned}
$$

Similar to the equations for vortex filaments in homogeneous fluid (Crow 1970; Jimenez 1975; Robinson & Saffman 1982)

Results



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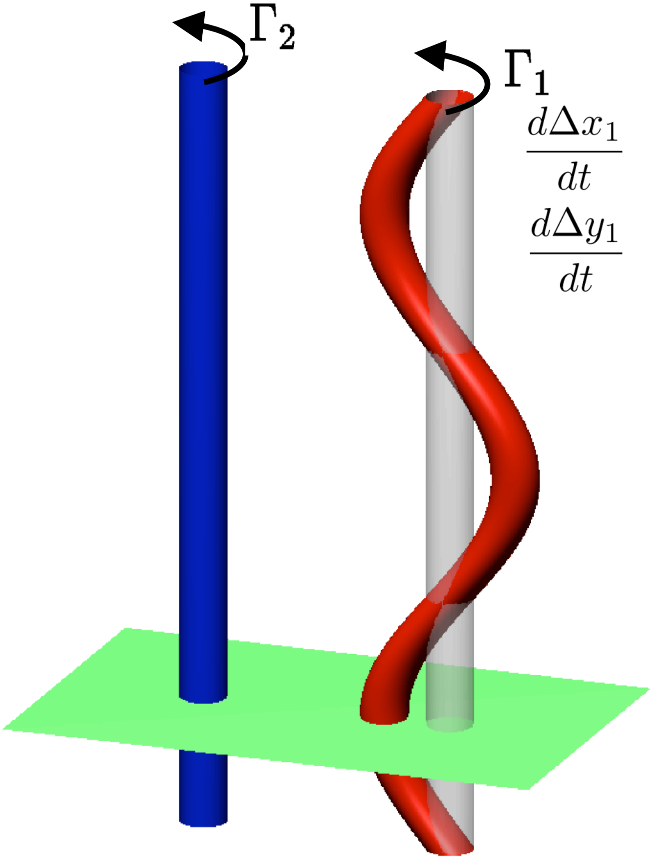
\end{aligned}
$$

Self-induction

Effect of the vortex on itself

Results



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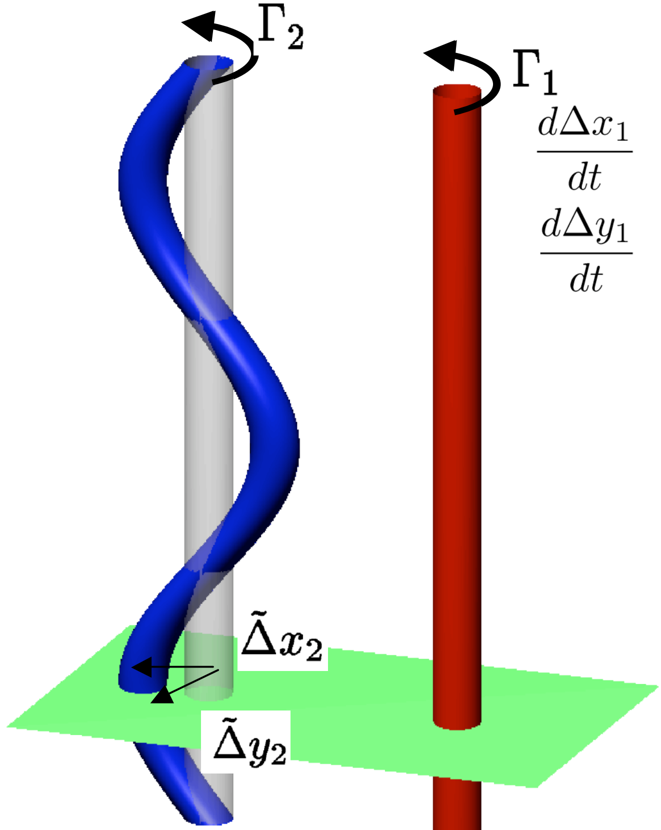
Strain

Self-induction

Advection of the perturbation of vortex 1
by the basic flow of vortex 2

Results



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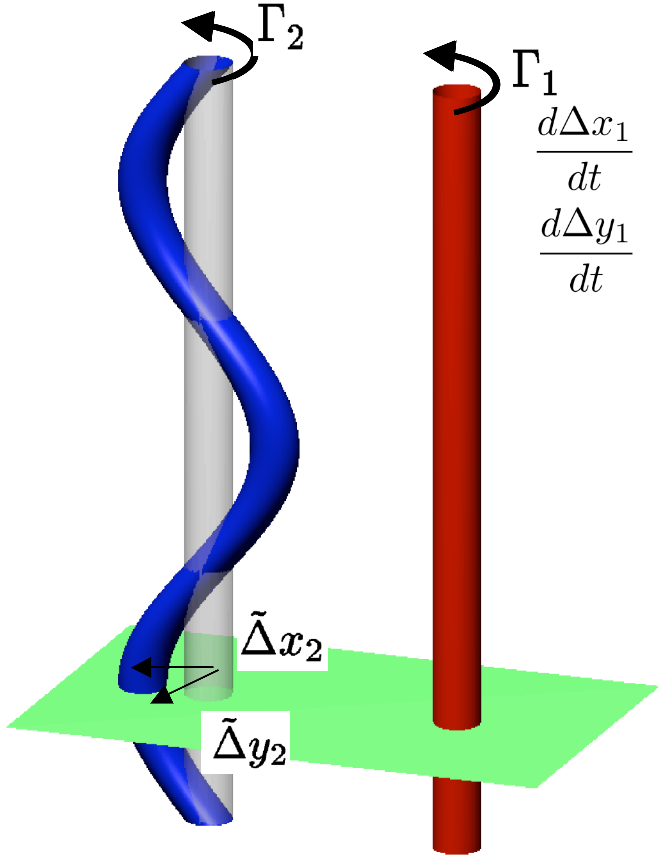
Strain

Self-induction

Mutual Induction

Advection of vortex 1 by the
perturbation of vortex 2

Results



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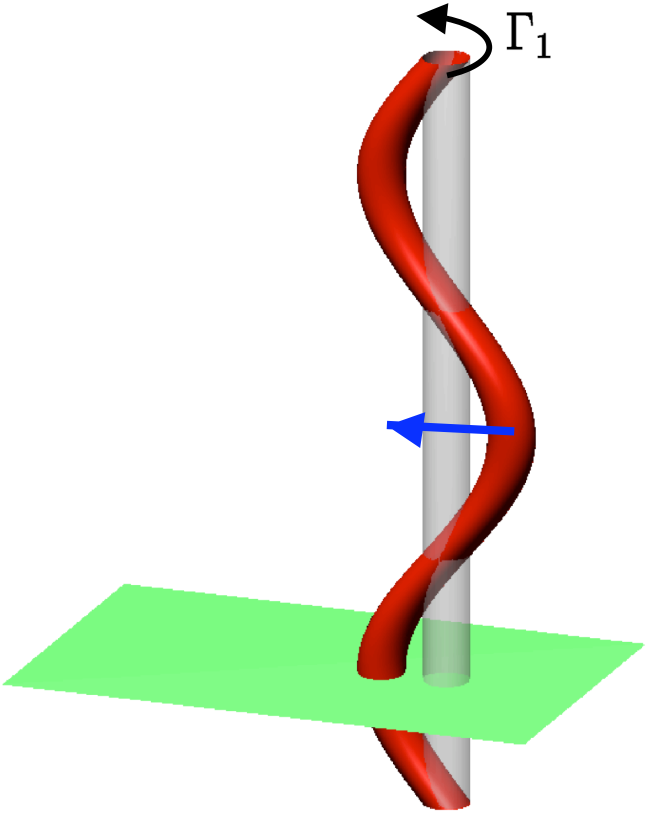
Strain

Mutual Induction

Self-induction

The nature of the fluid enters the problem only through ω, ψ, χ

Self-induction in homogeneous fluid



$$\omega = \frac{k^2R^2}{2}\left(\ln\frac{kR}{2} + \gamma_e - D\right)$$

$$D = \lim_{\eta_0\to\infty}\int_0^{\eta_0} \xi^3\Omega(\xi)^2 d\xi - \ln\eta_0$$

$$\gamma_e = 0.5772\ldots$$

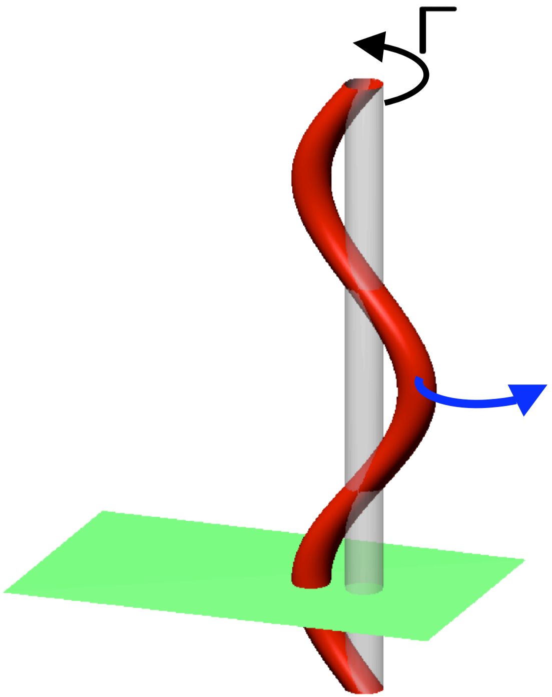
Widnall et al (1971), Moore & Saffman (1972),
Leibovich et al (1986)

$\omega<0 \Rightarrow$ Self-induced rotation opposite to the direction of rotation of the vortex

Self-induction in stratified and rotating fluid

$$\omega = -\frac{1}{2}\left(\frac{kF_hR}{Ro}\right)^2\left[\ln\left(\frac{kF_hR}{2Ro}\right) - \delta(F_h, Ro) + \gamma_e\right]$$

$$F_h = \frac{\Gamma}{2\pi R^2 N} \qquad Ro = \frac{\Gamma}{4\pi R^2 \Omega_b} \qquad \delta(F_h,Ro) = \lim_{\eta_0\to\infty}\int_0^{\eta_0} \xi^3\Omega(\xi)^2\frac{(Ro\Omega(\xi)+1)^2}{1-F_h^2\Omega(\xi)^2}d\xi - \ln\eta_0$$



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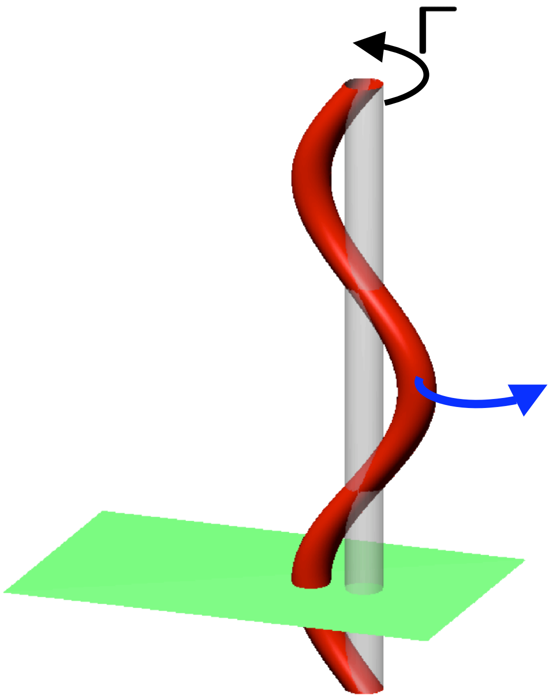

- $F_h < 1$: $\omega > 0$ whatever Ro and $\Omega(r)$

  => Self-induced rotation in the same direction of rotation as the vortex

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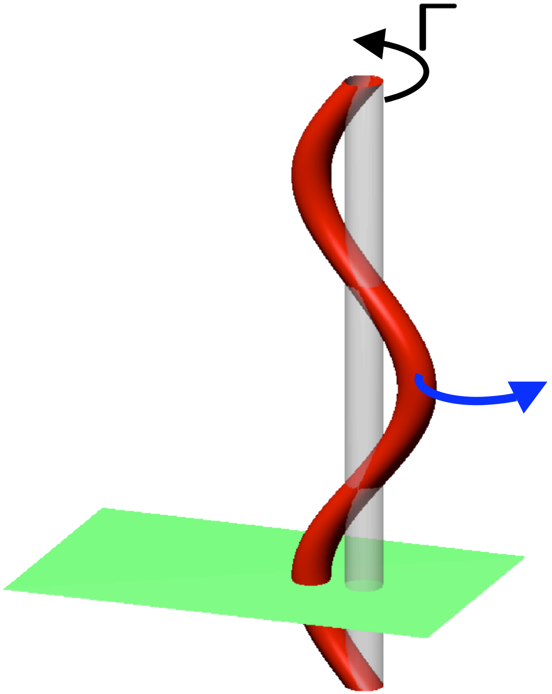
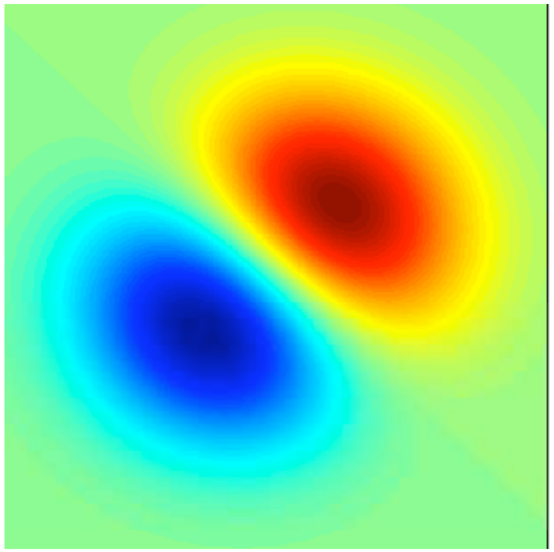


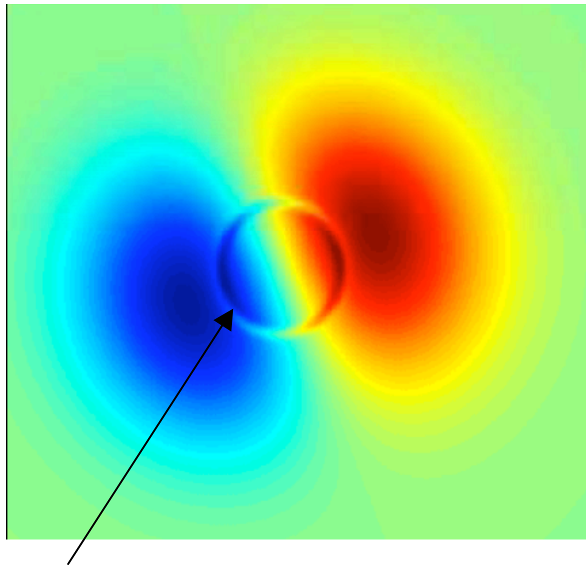
- $F_h < 1$: $\omega > 0$ whatever Ro and $\Omega(r)$
  => Self-induced rotation in the same direction of rotation as the vortex
- $F_h > 1$: $Im(\omega) < 0$
  $\Rightarrow$ damping due to a singularity where $\Omega(r_c) = N$

Vertical vorticity of the perturbation

$F_h = 0.9$



$F_h = 1.1$



Critical layer $\Omega(r_c) = N$

viscous critical layer analysis =>

$$\text{thickness} \sim \frac{1}{Re^{1/3}}$$

$$u_z \propto Re^{1/3}$$

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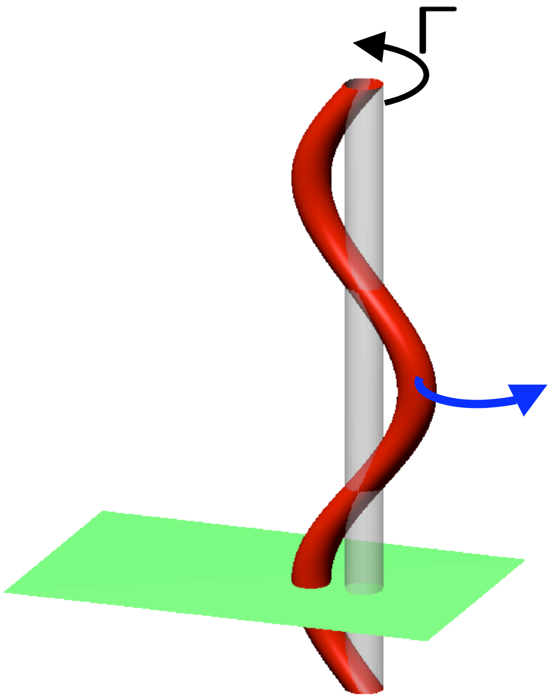

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- $F_h > 1$: $Im(\omega) < 0$

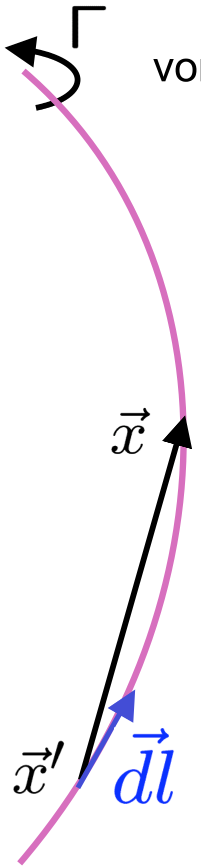
  $\Rightarrow$ damping due to a singularity where $\Omega(r_c) = N$

- $F_h > F_{hc}(Ro)$: $Re(\omega) < 0$

Simple explanation for the sign of the self-induction

Homogeneous fluid

$$\frac{D\Gamma}{Dt} = 0$$



Biot-Savart law

$$\vec{u}(\vec{x}) = -\frac{\Gamma}{4\pi}\int \frac{(\vec{x}-\vec{x}')\times\vec{dl}}{|\vec{x}-\vec{x}'|^3}$$

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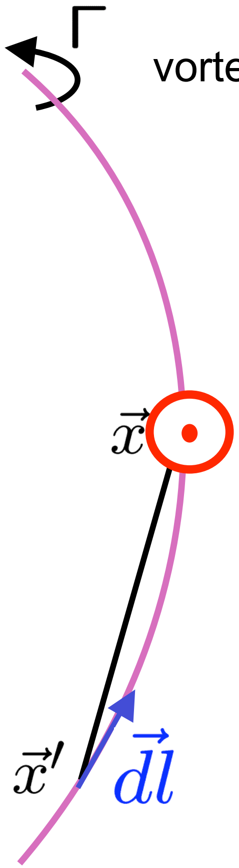

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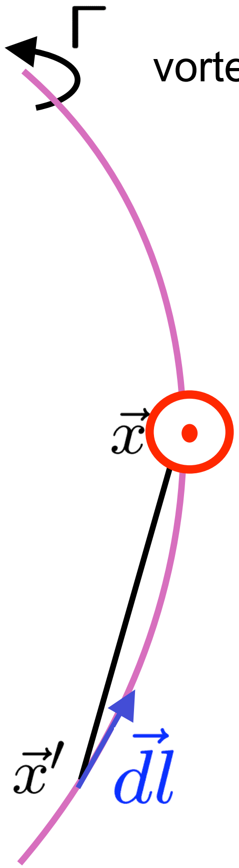

vortex filament

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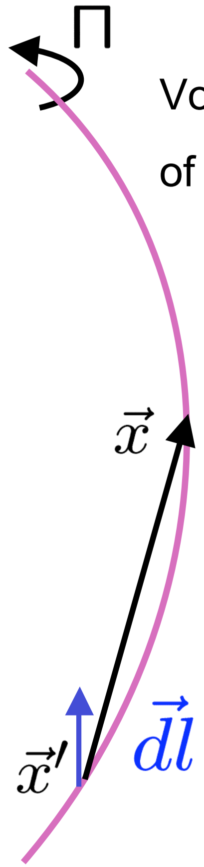
Quasigeostrophic fluid $(Ro \ll 1, F_h \ll 1)$

Potential vorticity : $\Pi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial \tilde{z}^2}$

$$\frac{D\Pi}{Dt} = 0$$



Vortex line

of potential vorticity

Biot-Savart law

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Miyazaki et al (2000)

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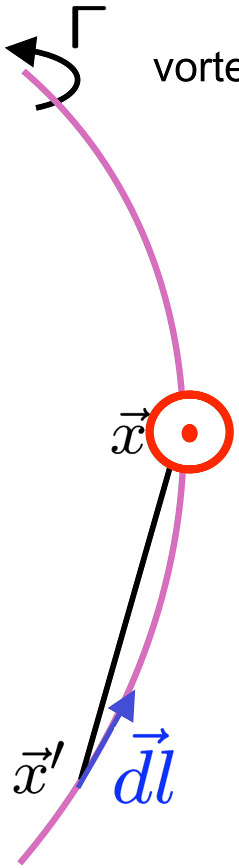

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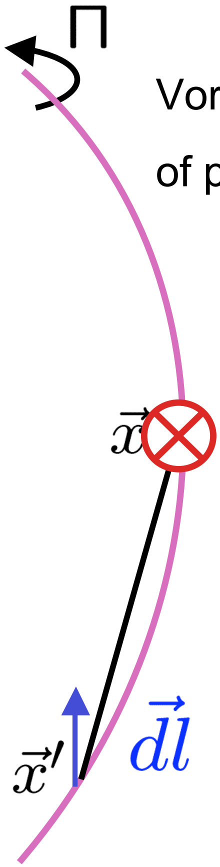

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Miyazaki et al (2000)

Mutual-induction functions

	Homogeneous Fluid (Crow 1970)	Stratified and rotating fluid
1st function	$\chi = kbK_1(kb)$	$\chi = \beta K_1(\beta) + \beta^2 K_0(\beta)$
2nd function	$\Psi = kbK_1(kb) + k^2 b^2 K_0(kb)$	$\Psi = \beta K_1(\beta)$

$$\left(\beta = \frac{kF_h b}{2|Ro|}\right)$$

Conditions of validity:

- whatever Ro
- $F_h \ll \frac{b^2}{R^2}$

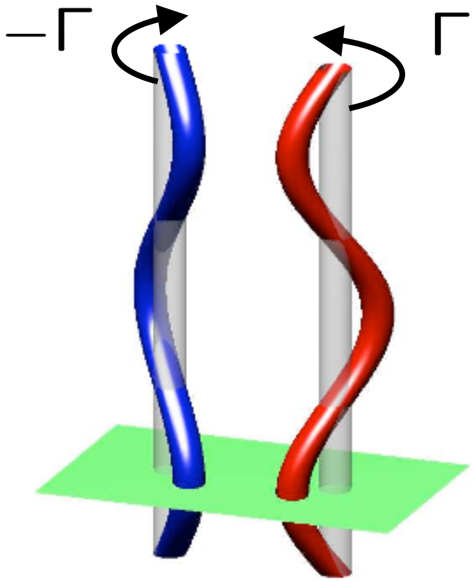
Stability of a counter-rotating vortex pair

$$\Gamma_1 = +\Gamma \qquad \Gamma_2 = -\Gamma$$

$$[\Delta x_1(t), \Delta y_1(t)] = [\bar{\Delta} x_1, \bar{\Delta} y_1] e^{\sigma t}$$

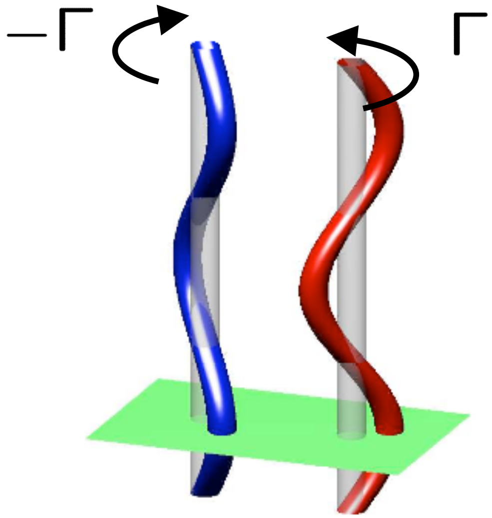
$$[\Delta x_2(t), \Delta y_2(t)] = [\bar{\Delta} x_2, \bar{\Delta} y_2] e^{\sigma t}$$

Symmetric mode



$$\bar{\Delta} x_1 = -\bar{\Delta} x_2 \qquad \bar{\Delta} y_1 = \bar{\Delta} y_2$$

Antisymmetric mode

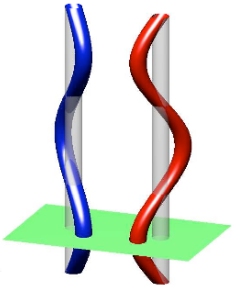


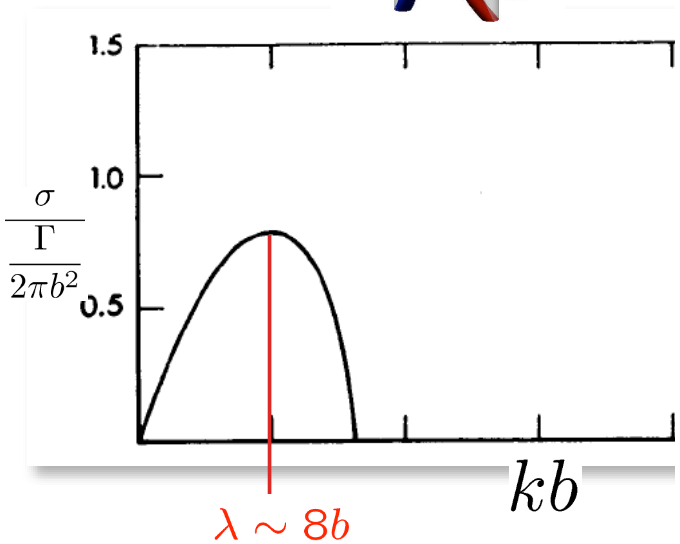
$$\bar{\Delta} x_1 = \bar{\Delta} x_2 \qquad \bar{\Delta} y_1 = -\bar{\Delta} y_2$$

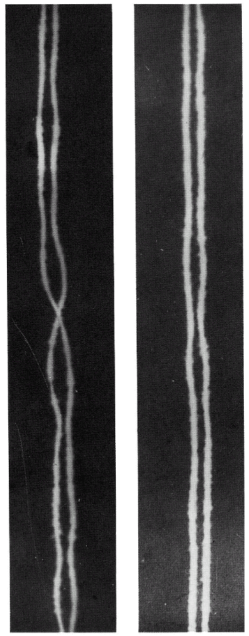
Stability of a counter-rotating vortex pair in homogeneous fluid (Crow 1970)

Symmetric mode

$$\frac{\sigma_s^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1 + \chi + \frac{b^2}{R^2}\omega\right)\left(1 - \Psi - \frac{b^2}{R^2}\omega\right)$$

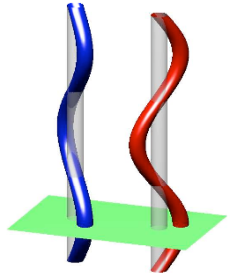






Antisymmetric mode:

$$\frac{\sigma_a^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1 - \chi + \frac{b^2}{R^2}\omega\right)\left(1 + \Psi - \frac{b^2}{R^2}\omega\right)$$

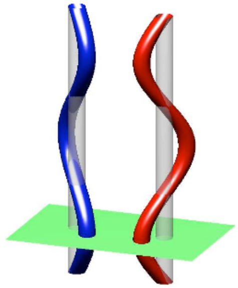


stable

Stability of a counter-rotating vortex pair in a strongly stratified fluid

Symmetric mode

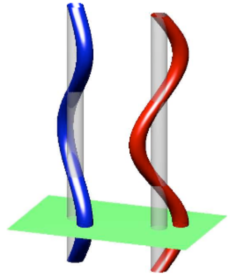
$$\frac{\sigma_s^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1 + \chi + \frac{b^2}{R^2}\omega\right)\left(1 - \Psi - \frac{b^2}{R^2}\omega\right)$$

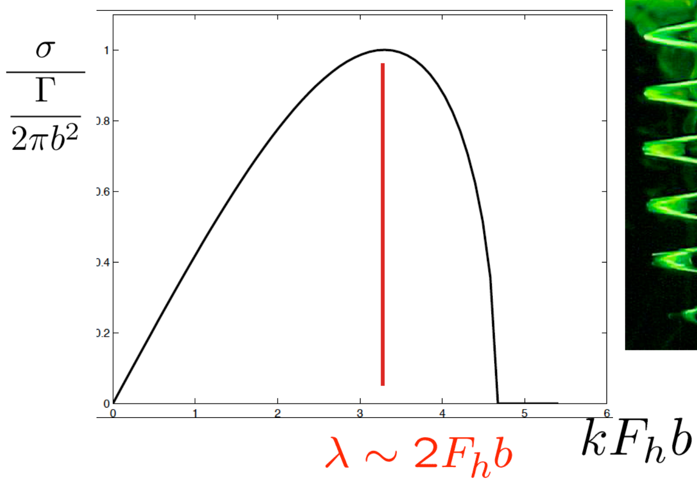


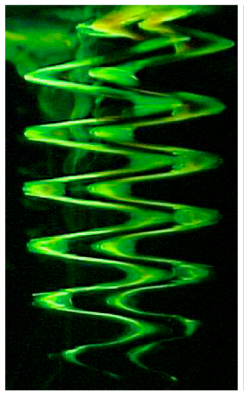
stable

Antisymmetric mode:

$$\frac{\sigma_a^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1 - \chi + \frac{b^2}{R^2}\omega\right)\left(1 + \Psi - \frac{b^2}{R^2}\omega\right)$$



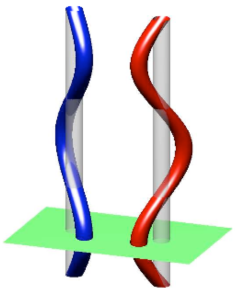




Origin of the exchange of stability ?

For very long wave: $kb \ll 1$

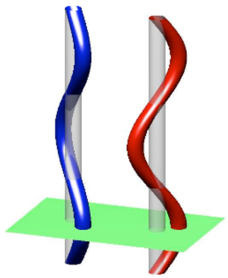
Symmetric mode



$$\frac{\sigma_s^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1+\chi+\frac{b^2}{R^2}\omega\right)\left(1-\Psi-\frac{b^2}{R^2}\omega\right) = -2\frac{b^2}{R^2}\omega+\ldots$$

Unstable if $\omega < 0$

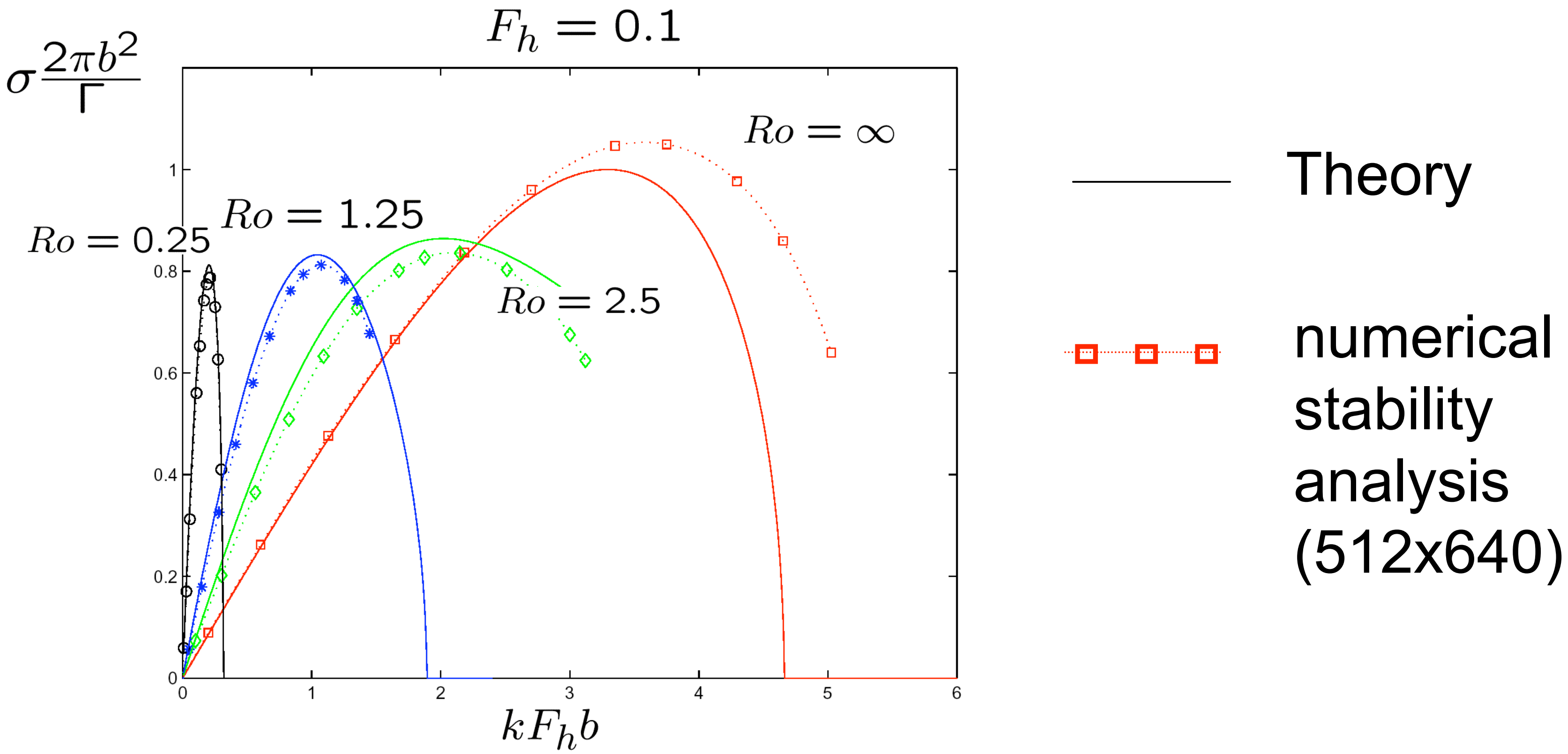
Antisymmetric mode:



$$\frac{\sigma_a^2}{\left(\frac{\Gamma}{2\pi b^2}\right)^2} = \left(1-\chi+\frac{b^2}{R^2}\omega\right)\left(1+\Psi-\frac{b^2}{R^2}\omega\right) = 2\frac{b^2}{R^2}\omega+\ldots$$

Unstable if $\omega > 0$

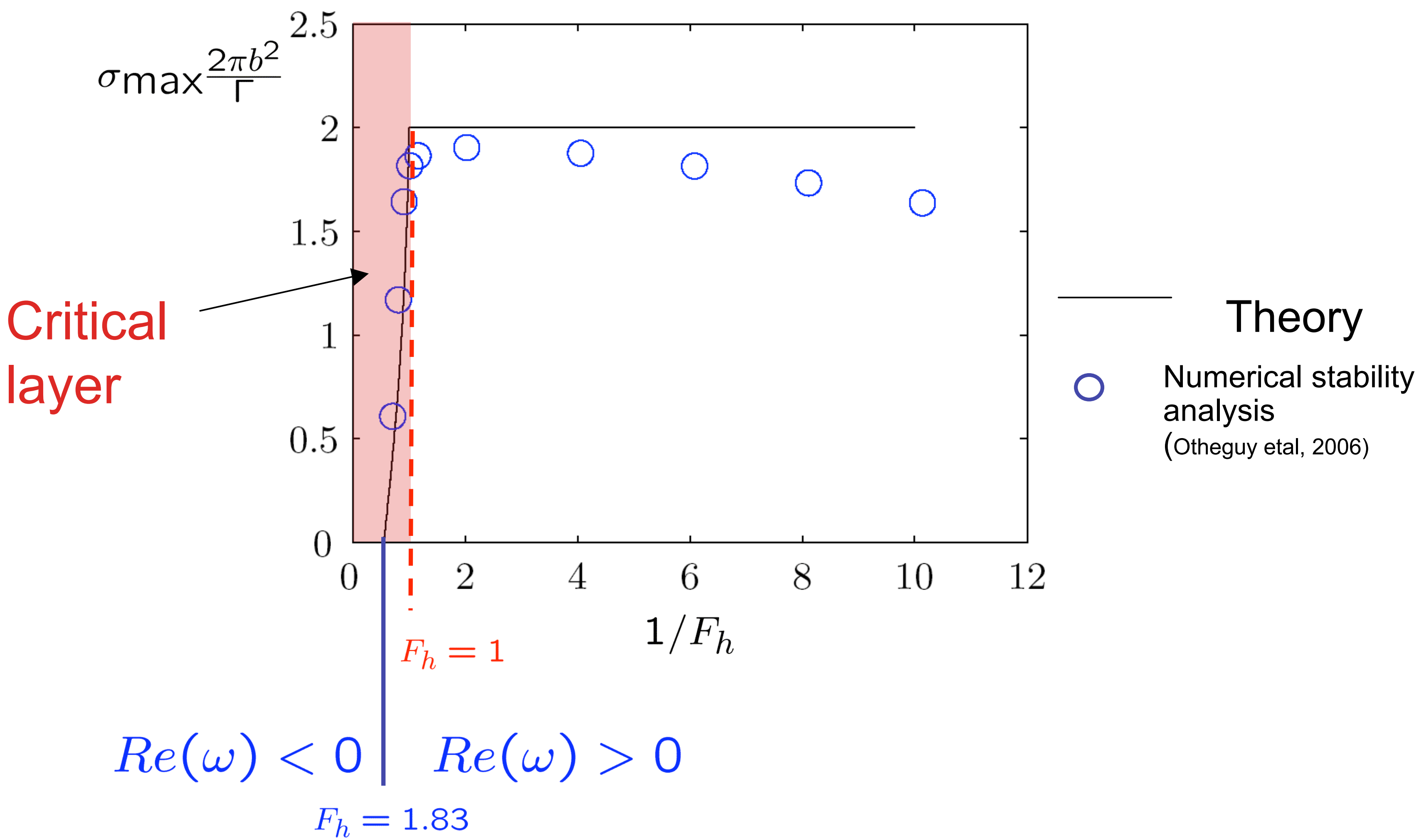
counter-rotating vortex pair: Effect of the Rossby number



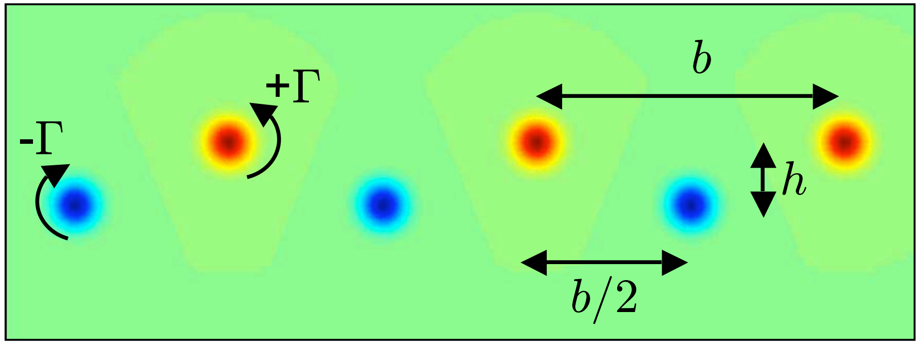
Property of the zigzag instability:

$$\sigma \max \propto \frac{\Gamma}{2\pi b^2} \qquad \lambda \max \propto \frac{F_h b}{f(Ro)} \qquad \begin{matrix} f(Ro) \propto Ro \text{ for } Ro \to 0 \\ f(Ro) = 1 \text{ for } Ro \to \infty \end{matrix}$$

Co-rotating vortex pair: Comparison theory/numeric $Ro = \infty$



Stability of the Von Karman Street in a stratified and rotating fluid

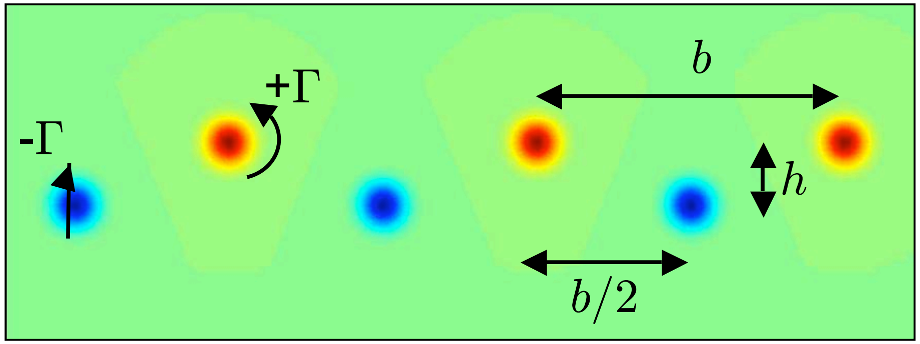


$$\kappa = \frac{h}{b}$$

Equation for the vortex m

$$\begin{aligned}
\frac{\mathrm{d}\Delta x_{1,m}}{\mathrm{d}t} &= -\frac{\Gamma}{2\pi}\omega_{\mathrm{Re}}(k_z, F_h, Ro)\Delta y_{1,m} + \frac{|\Gamma|}{2\pi}\omega_{\mathrm{Im}}(k_z, F_h, Ro)\Delta x_{1,m} \\
&\quad -\frac{\Gamma}{2\pi}\sum_{p\neq m}\frac{\Delta y_{1,m}}{b_{pm}^2} + \frac{\Gamma}{2\pi}\sum_{p\neq m}\frac{\psi_{pm}\Delta y_{1,p}}{b_{pm}^2} \\
&\quad +\frac{\Gamma}{2\pi}\sum_{q}\frac{2\tilde{b}_{qm}h\Delta x_{1,m} + (\tilde{b}_{qm}^2 - h^2)\Delta y_{1,m}}{L_{qm}^4} \\
&\quad +\frac{\Gamma}{2\pi}\sum_{q}\frac{-\tilde{b}_{qm}h(\chi_{qm}+\psi_{qm})\Delta x_{2,q} - (\tilde{b}_{qm}^2\psi_{qm} - h^2\chi_{qm})\Delta y_{2,q}}{L_{qm}^4}
\end{aligned}$$

Stability of the Von Karman Street in a stratified and rotating fluid



$$\kappa = \frac{h}{b}$$

Equation for the vortex m

$$\begin{aligned}
\frac{\mathrm{d}\Delta x_{1,m}}{\mathrm{d}t} \quad = \quad & -\frac{\Gamma}{2\pi}\omega_{\mathrm{Re}}(k_z, F_h, Ro)\Delta y_{1,m} + \frac{|\Gamma|}{2\pi}\omega_{\mathrm{Im}}(k_z, F_h, Ro)\Delta x_{1,m} \\
& -\frac{\Gamma}{2\pi}\sum_{p\neq m}\frac{\Delta y_{1,m}}{b_{pm}^2} + \frac{\Gamma}{2\pi}\sum_{p\neq m}\frac{\psi_{pm}\Delta y_{1,p}}{b_{pm}^2} \\
& +\frac{\Gamma}{2\pi}\sum_{q}\frac{2\tilde{b}_{qm}h\Delta x_{1,m} + (\tilde{b}_{qm}^2 - h^2)\Delta y_{1,m}}{L_{qm}^4} \\
& +\frac{\Gamma}{2\pi}\sum_{q}\frac{-\tilde{b}_{qm}h(\chi_{qm} + \psi_{qm})\Delta x_{2,q} - (\tilde{b}_{qm}^2\psi_{qm} - h^2\chi_{qm})\Delta y_{2,q}}{L_{qm}^4}
\end{aligned}$$

Self-induction

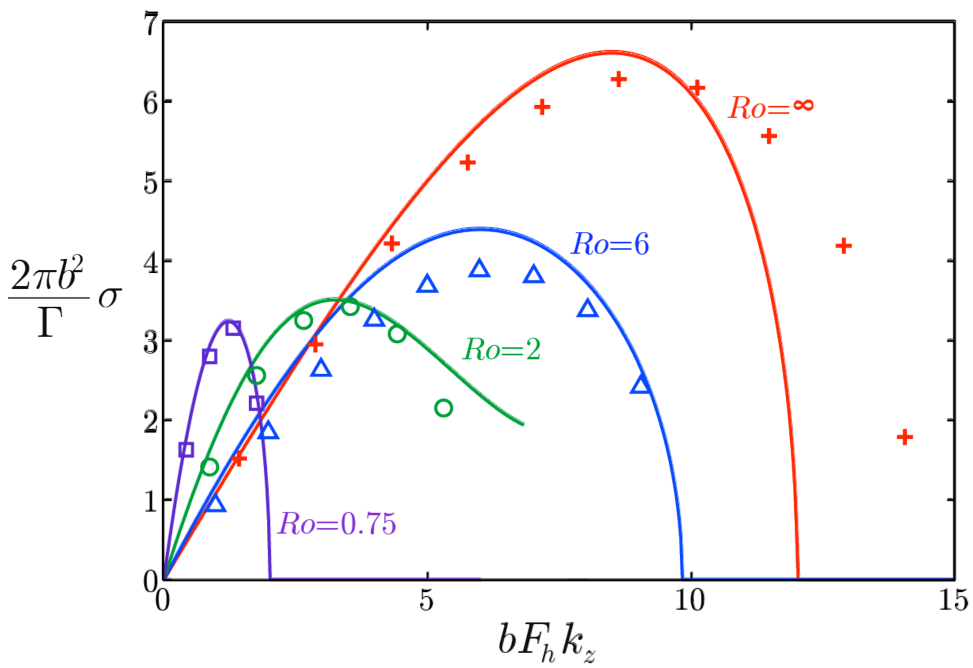
Strain (1st row) + Mutual Induction (1st row)

Strain (2nd row)

Mutual Induction (2nd row)

Is the theory valid ? : Comparison theory/numeric

$$\kappa = \frac{h}{b} = 0.2 \qquad F_h = 0.1$$



Theory

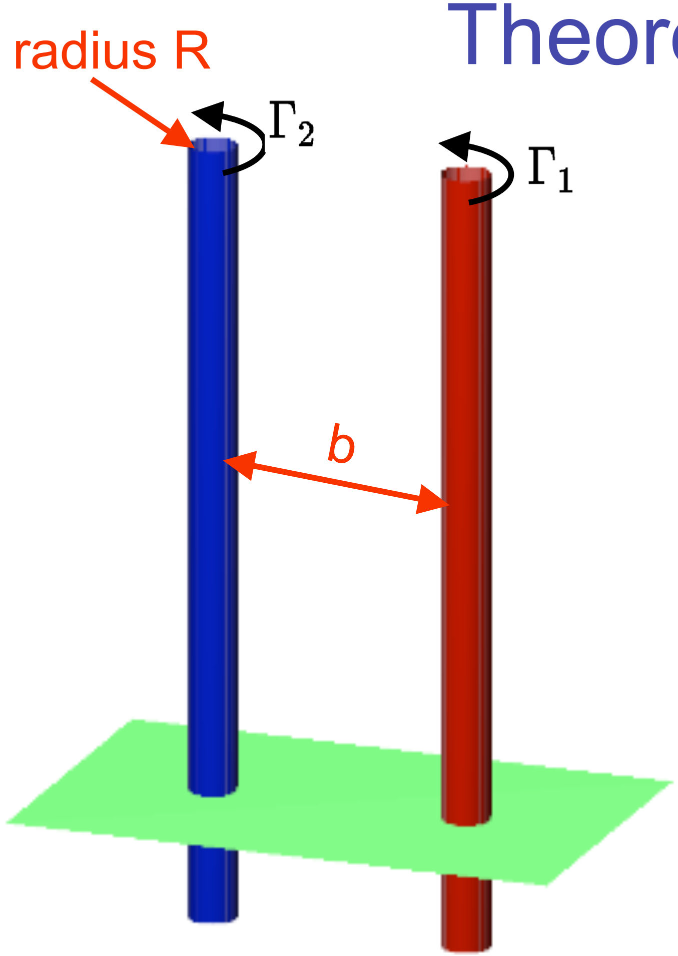
numerical stability analysis (512x640)

Conclusions

- ✓ General approach to look at the instabilities due to vortex interactions in stratified and rotating fluids (equivalent to vortex filaments approach)

- ✓ The reversal of the sign of the self-induction function explains why different bending instabilities are observed in stratified-rotating fluids/ homogenous fluids

Theoretical approach



Hypotheses:

- Inviscid fluid
- well-separated vortices : b >> R