Chain homotopy maps of Khovanov homology

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supported by JSPS.

International Workshop on Mathematical Fluid Dynamics
Waseda University, March 15, 2010.

History. L : link,

$$J(L) = \sum_j q^j \sum_i (-1)^i \operatorname{rk} \mathcal{H}^{i,j}(L).$$

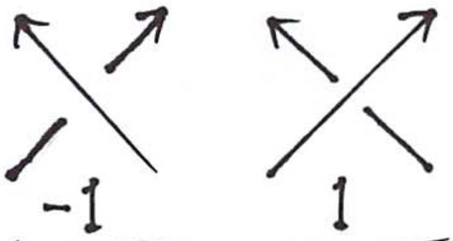
(Jones, 1984)

(Khovanov, 2000)

Redefined (Viro, 2004)

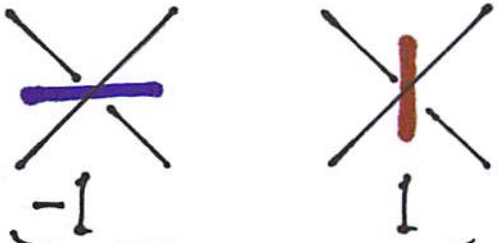
Simple Proof (I, Today's talk)
of Isotopy Invariance of $\mathcal{H}^{i,j}(L)$.

Generator S of Complex



ϵ_1

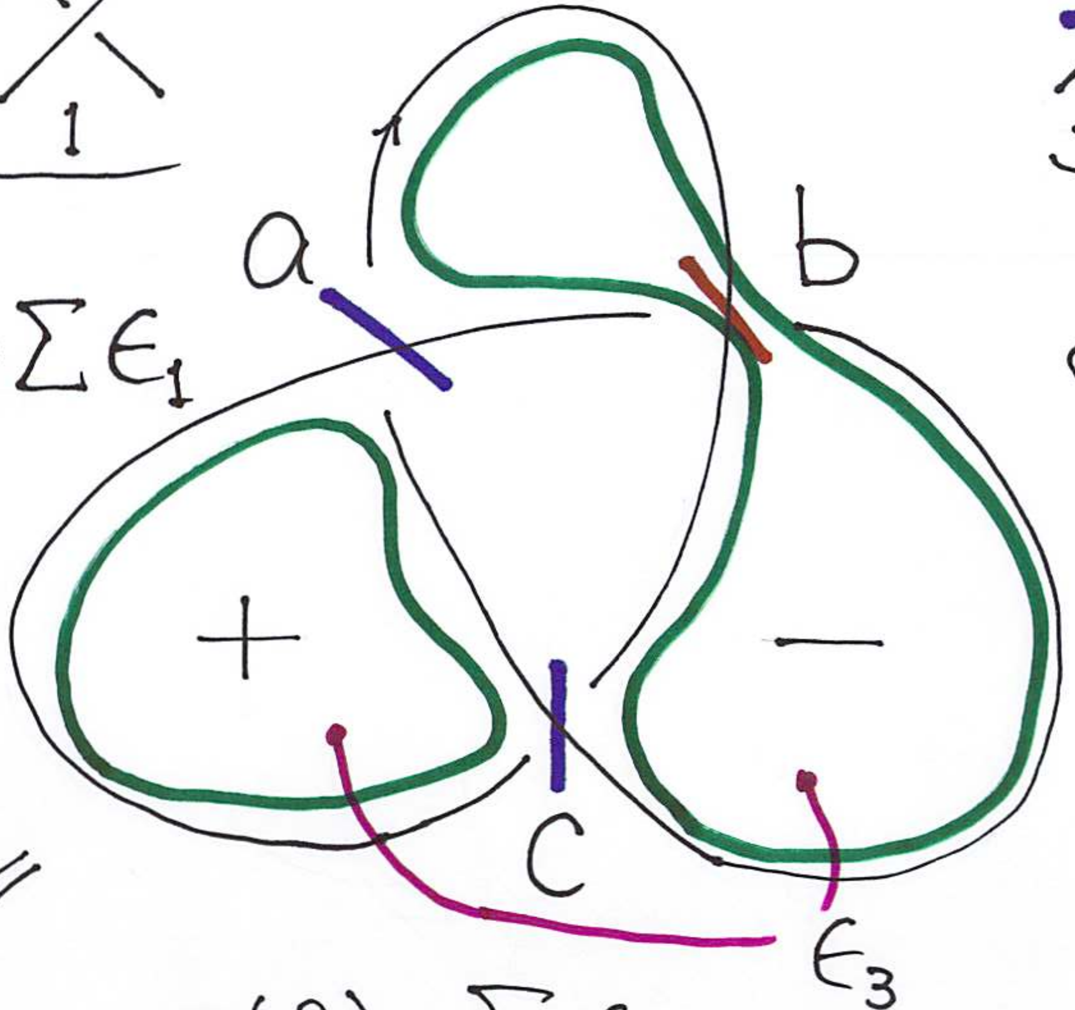
$w(S) = \sum \epsilon_1$



ϵ_2

$\sigma(S) = \sum \epsilon_2$

$S =$



$\otimes [ac]$

Word of minus markers

$\tau(S) = \sum \epsilon_3$

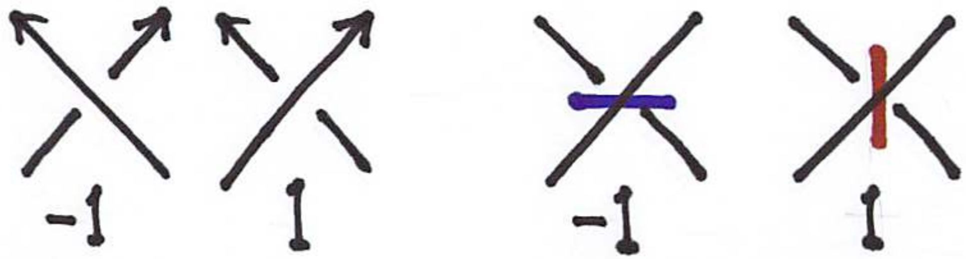
$$C^{i,j}(D)=\langle S\otimes[x] \mid i(S)=i,\ j(S)=j\rangle$$

where $i(S)=\frac{1}{2}(w(S)-\sigma(S))$,

$j(S)=w(S)+i(S)-\tau(S)$.

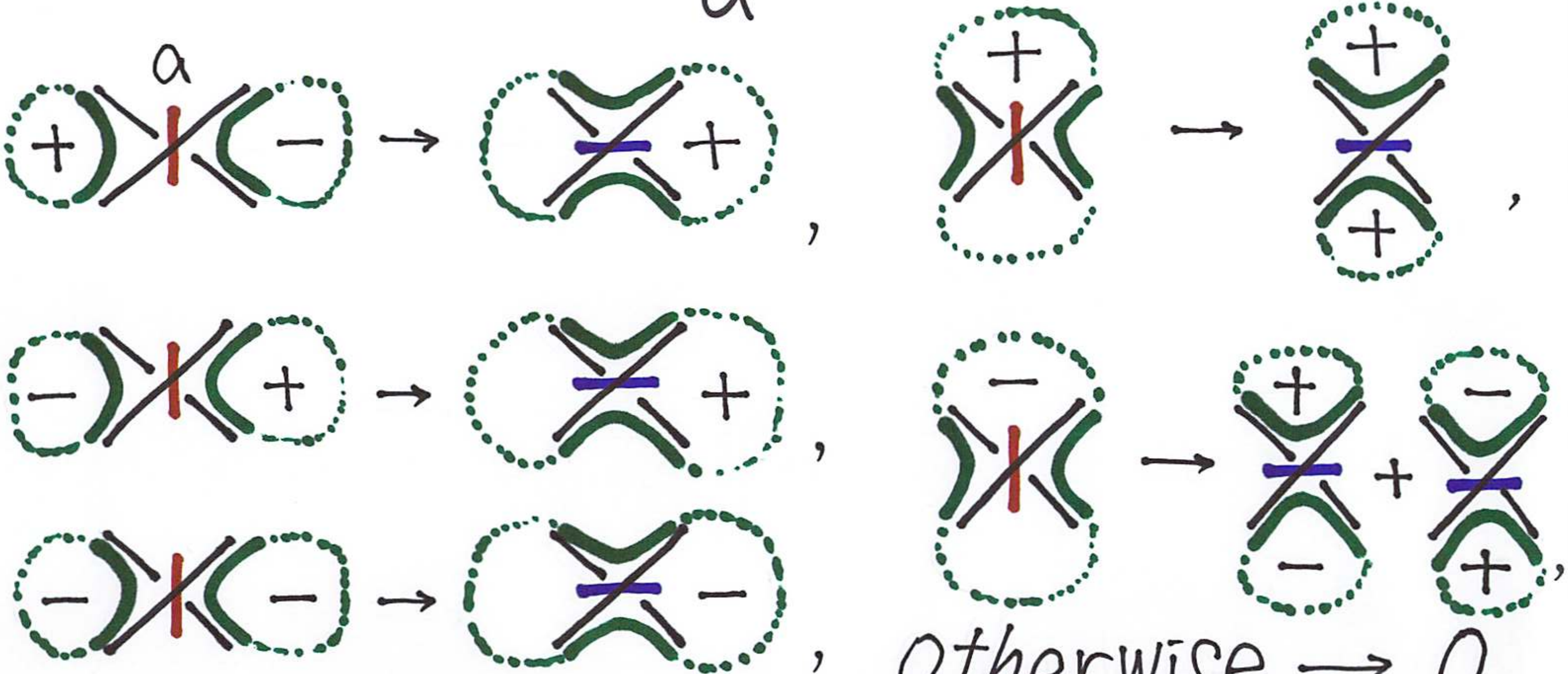
Ex.

$D=$ 



$C^{-1,-2}\Big($  $\Big)=\Big\langle$  $\otimes[a]$,  $\otimes[a]\Big\rangle$

$$d(S \otimes [x]) = \sum_{a} T \otimes [xa]$$

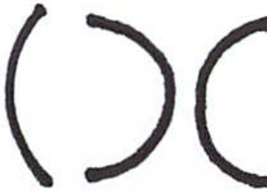


otherwise $\to 0$.

$$\rightsquigarrow H^{i,j}(D) := H^{i}(C^{*,j}(D), d).$$

$H^{i,j}(D)$ is a link invariant

$\Leftrightarrow$ (*) $\begin{cases} H^{i,j}(\end{cases}$  $) \overset{\mathrm{I}}{\cong} H^{i,j}($  $)$,

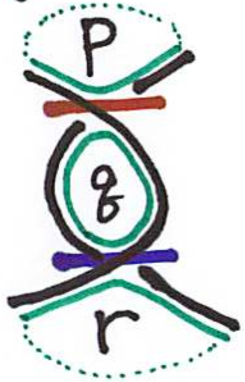
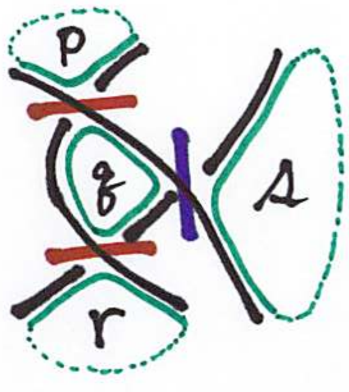
$H^{i,j}($  $) \overset{\mathrm{II}}{\cong} H^{i,j}($  $)$,

$H^{i,j}($  $) \overset{\mathrm{III}}{\cong} H^{i,j}($  $)$.

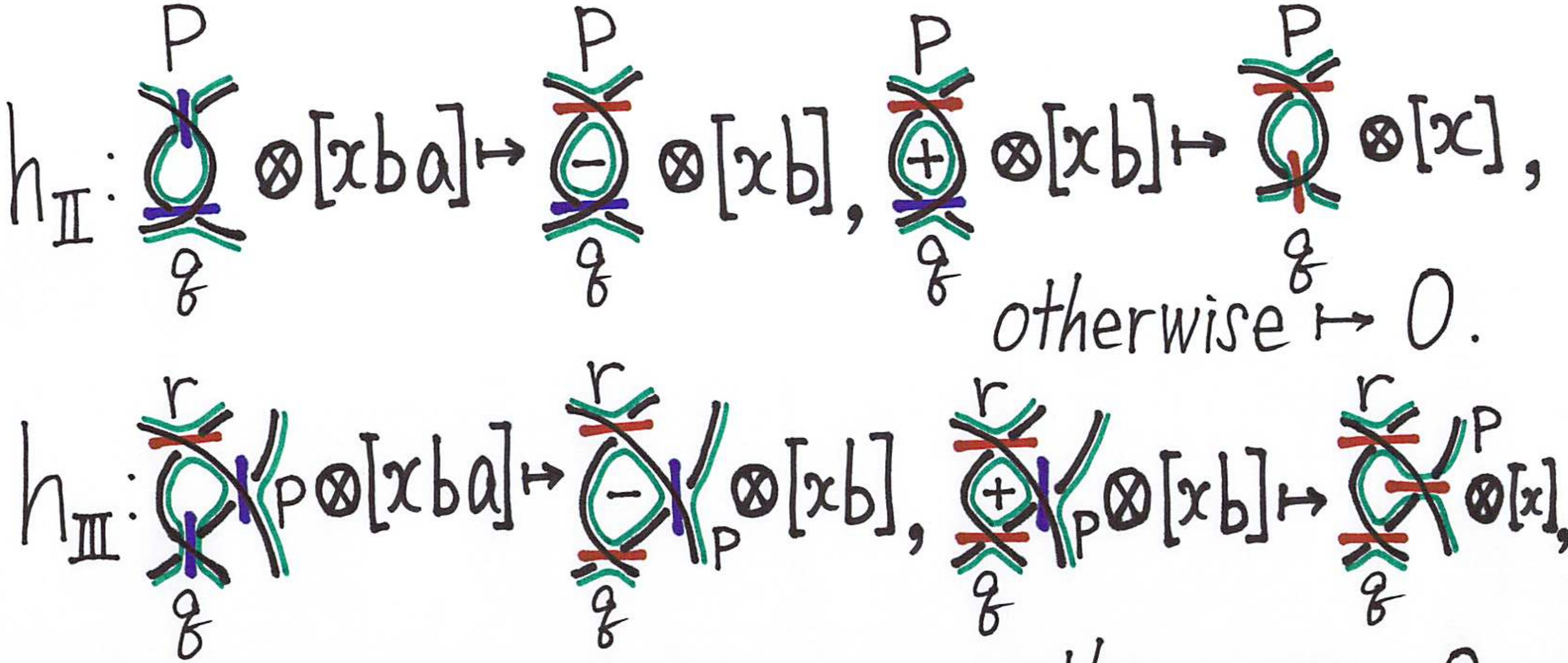
Point. $\exists \rho, h$ s.t. $d \circ h + h \circ d = \mathrm{id} - \mathrm{in} \circ \rho$

for I, II, III $\rightsquigarrow$ (*).

To prove it, find ρ, h.

	Base	Matrix Size
I :		6×6 (Viro)
II :	 etc.	12×12 or 18×18 (I.)
III :	 etc.	30×30, 36×36, 54×54 (3types)

Main Result (Explicit formula h)



(··· cf. math.GT/0901.3952.)