# 第 4 回 日独流体数学国際研究集会

# The 4th Japanese-German International Workshop on Mathematical Fluid Dynamics

Nov. 28 - Dec.2, 2011 at Waseda University

## ■ Program ■

| | | Mon, Nov. 28<br>63 bldg.<br>04 room | Tue, Nov. 29<br>63 bldg.<br>04 room | Wed, Nov. 30<br>62 bldg.<br>Meeting room | Thu, Dec. 1<br>63 bldg.<br>03 room | Fri, Dec. 2<br>63 bldg.<br>03 room |
|---|---|---|---|---|---|---|
| ↑<br>Mini Course<br>↓ | 10:30<br>-11:30 | **Zajaczkowski ①** | **Farwig ①** | **Shibata ②** | **Kobayashi ③** | **Zajaczkowski ④** |
| | 11:30<br>-12:00 | Coffee Break | | | | |
| | 12:00<br>-13:00 | **Kobayashi ①** | **Zajaczkowski ②** | **Farwig ②** | **Shibata ③** | **Kobayashi ④** |
| | 13:00<br>-14:30 | Lunch Break | | | | |
| | 14:30<br>-15:30 | **Shibata ①** | **Kobayashi ②** | **Zajaczkowski ③** | **Farwig ③** | **Shibata ④** |
| | 15:30<br>-16:00 | Coffee Break | | | | |
| ↑<br>Scientific Talks<br>↓ | 16:00<br>-16:30 | **Okihiro Sawada**<br>(Gifu Univ.) | **Satoshi Yokoyama**<br>(Tokyo Univ.) | **Tsukasa Iwabuchi**<br>(Touhoku Univ.) | **Tomoyuki Nakatsuka**<br>(Nagoya Univ.) | **Kazuhiro Oeda**<br>(Waseda Univ.) |
| | 16:30<br>-17:00 | **Mads Kyed**<br>(TU Darmstadt) | **Hajime Koba**<br>(Tokyo Univ.) | **Ryo Takada**<br>(Touhoku Univ.) | **Kohei Soga**<br>(Waseda Univ.) | **Hidemitsu Wadade**<br>(Waseda Univ.) |
| | 17:00<br>-17:30 | Coffee Break | | | | |
| | 17:30<br>-18:00 | **Georg Schöchtel**<br>(TU Darmstadt) | **Dario Götz**<br>(TU Darmstadt) | **Christoph Albert**<br>(TU Darmstadt) | **Yusuke Yamauchi**<br>(Waseda Univ.) | **Jyunichi Harada**<br>(Waseda Univ.) |
| | 18:00<br>-18:30 | **Norikazu Yamaguchi**<br>(Toyama Univ.) | **Manuel Nesensohn**<br>(TU Darmstadt) | **Loenz v. Below**<br>(TU Darmstadt) | | **Raphael Schulz**<br>(TU Darmstadt) |

**Reinhard Farwig**
Technische Universität Darmstadt

Title:

**Weak solutions of the nonstationary Navier-Stokes equations and their regularity**

Abstract:

We consider the nonstationary Navier-Stokes system in a domain $\Omega \subset \mathbb{R}^3$. By a classical result, for any data $u(0) = u_0 \in L^2_\sigma(\Omega)$, there exists a weak solution $u \in L^\infty(0,T,L^2(\Omega)) \cap L^2(0,T,H^1_0(\Omega))$ satisfying the energy inequality

$$\frac{1}{2}\|u(t)\|_2^2 + \nu \int_0^t \|\nabla u(\tau)\|_2^2 \, d\tau \le \frac{1}{2}\|u(0)\|_2^2.$$

Then the famous open Millennium Problem of Clay Mathematics Institute 2000 concerns the global regularity, say, $u \in L^s(0,T,L^q(\Omega))$ with $\frac{2}{s} + \frac{3}{q} = 1$ (*Serrin's condition*), and also uniqueness of this weak solution. Further questions concern the validity of the physically relevant energy equality known to hold when $u \in L^4(0,T,L^4(\Omega))$ (rather than an energy inequality), optimal conditions on the initial value or local conditions weaker than Serrin's one to guarantee at least local regularity on an initial interval or locally in time, respectively.
In this lecture series we consider the following items:

- The energy equality holds when

  $$u \in L^3(0,T;W^{1/2,18/7}(\Omega)) \subset L^3(0,T;L^{9/2}(\Omega))$$

  using less regularity in space compared to recent results of Cheskidov, Friedlander, Shvydkoy and Constantin
- Optimal initial value conditions to get local strong solutions in Serrin's class, namely

  $$\int_0^\infty \|e^{-\tau A_2} u_0\|_q^s \, d\tau < \infty$$

- Local and also one-sided conditions of Serrin type or even weaker than Serrin's condition to guarantee local regularity
- Conditions on the kinetic energy or the dissipation energy to guarantee local regularity

References:

R. Farwig, C. Komo: Regularity of weak solutions to the Navier-Stokes equations in exterior domains. Nonlinear Differ. Equ. Appl. **17**, 303-321 (2010)

R. Farwig, H. Kozono, H. Sohr: Local in Time Regularity Properties of the Navier-Stokes Equations. Indiana Univ. Math. J. **56**, 2111-2131 (2007)

R. Farwig, H. Kozono, H. Sohr: Energy-Based Regularity Criteria for the Navier-Stokes Equations. J. Math. Fluid Mech. **11**, 1-14 (2008)

R. Farwig, H. Sohr: Optimal Initial Value Conditions for the Existence of Local Strong Solutions of the Navier-Stokes Equations. Math. Ann. **345**, 631-642 (2009)

R. Farwig, H. Sohr: On the existence of local strong solutions for the Navier-Stokes equations in completely general domains. Nonlinear Anal. **73**, 1459-1465 (2010)

R. Farwig, H. Sohr, W. Varnhorn: On Optimal Initial Value Conditions for Local Strong Solutions of the Navier-Stokes Equations. Ann. Univ. Ferrara **55**, 89-110 (2009)

R. Farwig, H. Sohr, W. Varnhorn: Necessary and Sufficient Conditions on Local Strong Solvability of the Navier-Stokes System. Appl. Anal. **90**, 47-58 (2011)

R. Farwig, H. Sohr, W. Varnhorn: Extension of Serrin's uniqueness and regularity conditions for the Navier-Stokes equations. J. Math. Fluid Mech. (to appear)

R. Farwig, Y. Taniuchi: On the energy equality of Navier-Stokes equations in general unbounded domains. Arch. Math. **95**, 447-456 (2010)

Date:

① Tuesday, Nov.29 10:30-11:30 ② Wednesday, Nov. 30 12:00-13:00
③ Thursday, Dec. 1 14:30-15:30

**Takayuki Kobayashi**
Saga University

Title:

**Weighted $L^p$ estimates for the Stokes semigroup**

Abstract:

We consider the decay estimates for the solutions to the Navier-Stokes equations in some unbounded domains; $\mathcal{R}^n$, $\mathcal{R}^n_+$, an exterior domain and a perturbed half space. We discuss

(i) resolvent estimates in a weighted $L^p$ space for the Stokes operator

(ii) local energy decay estimates for the Stokes semigroup

(iii) weighted $Lp - Lq$ estimates for the Stokes semigroup

(iv) weighted $L^p$ estimates for the solutions to the Navier-Stokes equations

Date:

① Monday, Nov. 28 12:00-13:00 ② Tuesday, Nov.29 14:30-15:30
③ Thursday, Dec. 1 10:30-11:30 ④ Friday, Dec. 2 12:00-13:00

**Wojciech Zajaczkowski**
Polish Academy of Sciences

Lecture ①:

**Navier-Stokes equations (NSE): global regular solutions and asymptotics**

Abstract:

We consider the NSE in a cylindrical domain with the slip boundary conditions. Assuming smallness of $L_2$ norm of derivative of initial velocity and the external force with respect to the variable along the axis of the cylinder we are able to find an a priori estimate for velocity in $W_2^{2,1}$ space for a finite time interval. Then applying the Leray-Schauder fixed point theorem we prove the existence in the space for velocity and gradient of pressure in $L_2$. Using that the existence time is sufficiently large and using the natural decay estimates in NSE we prove global existence step by step for all time. There is no restrictions on the magnitudes of velocity, gradient of pressure and the external force.

Date:

① Monday, Nov. 28 10:30-11:30

Lecture ②:

**Nonhomogeneous Navier-Stokes equations (NSE): global existence of regular solutions**

Abstract:

We consider nonhomogeneous incompressible NSE (density satisfies the equation of continuity for divergence free velocity) in a cylindrical domain with slip boundary conditions. Assuming smallness of grad of initial density in L infinity norm and derivatives of initial velocity and the external force with respect to the variable along the axis of the cylinder in $L_2$ norm we find an a priori estimate for arbitrary large finite time for velocity in $W_2^{2,1}$ and gradient of pressure in $L_2$. Next, applying the Leray-Schauder fixed point theorem we prove the existence of global regular solutions to the nonhomogeneous NSE. There is no restriction on the magnitudes of velocity, gradient of pressure, density and the external force.

Date:

② Tuesday, Nov. 29 12:00-13:00

Lecture ③:

**Inflow-outflow problem for the Navier-Stokes equations (NSE)**

Abstract:

We consider the NSE in a bounded cylinder with the slip boundary conditions and inflow-outflow flux on the parts of the boundary of the cylinder which are

perpendicular to its axis. We need smallness of derivatives of the flux with respect to variable tangent to the boundary and time in some spaces. Moreover we need smallness of $L_2$ norms of derivatives with respect to the variable along the axis of the cylinder of the initial velocity and the external force. Then we are able to prove an a priori estimate for velocity in $W_2^{2,1}$ and gradient of pressure in $L_2$ for arbitrary time. To find the estimate we need an estimate and the existence of weak solutions. A key point in the proof of the estimate for the weak solutions are some weighted estimates. Next by the Leray-Schauder fixed point theorem we prove the existence of global regular solutions in the previously mentioned regularity.
We have to underline that there are no restrictions on the magnitudes of the flux, velocity, gradient of pressure and the external force.

Date:

③ Wednesday, Nov. 30 14:30-15:30

Lecture ④:

**Navier-Stokes equations (NSE) coupled with the heat convection: regularity, global existence**

Abstract:

We consider the Navier-Stokes equations with the slip boundary conditions coupled with the heat equation with the Neumann boundary conditions in a bounded cylinder. The cases without and with inflow-outflow are examined. To prove global a priori estimate for regular solutions we need smallness of $L_2$ norms of the derivative with respect to the variable along the axis of the cylinder of the initial velocity, initial temperature, the external force and the heat source. In the case of the inflow-outflow we need also smallness of derivatives of the flux with respect to tangent directions to the boundary and time. The existence of global regular solutions is proved by the Leray-Schauder fixed point theorem.
We do not have restrictions on the magnitudes of velocity, temperature, the external force, the heat source and the gradient of pressure.

Date:

④ Friday, Dec. 2 10:30-11:30

**Yoshihiro Shibata**
Waseda University

Title:

**$\mathcal{R}$ sectoriality of the Stokes operator for the compressible viscous fluid flow and its application to the nonlinear problem**

Abstract:

We discuss the $\mathcal{R}$ sectoriality of the Stokes operator for the compressible viscous fluid flow in a general domain, which implies the generation of analytic semigroup and maximal $L_p$- $L_q$ regularity at the same time. The proof relies on the $\mathcal{R}$ sectoriality for the model problems in the whole space and half-space. Then, it is also proved in the perturbed half-space by some perturbation method and therefore, the parameter elliptic technique allows us to prove the $\mathcal{R}$ sectoriality in a general domain. From the $\mathcal{R}$ sectoriality implies the generation of analytic semigroup and maximal $L_p$-$L_q$ regularity with help of the Mihlin-Hörmander Fourier multiplier theorem and the Weis operator valued Fourier multiplier theorem. Applying these facts, we discuss a local in time unique existence theorem of the Navier-Stokes equation with non-slip condition for the compressible viscous fluid flow and also a global in time unique existence theorem in a bounded domain.

Date:

① Monday, Nov. 28 14:30-15:30 ② Wednesday, Nov. 30 10:30-11:30
③ Thursday, Dec. 1 12:00-13:00 ④ Friday, Dec. 2 14:30-15:30

## Christoph Albert

Technische Universität Darmstadt

Title:

**Absorption of a gas by a falling liquid film**

Abstract:

A falling film is a thin layer of liquid running down a vertical wall. This setup is widely used in chemical engineering, for applications in which large mass transfer rates between liquid and gas are desired. The presence of surface waves on the film is known to increase the mass transfer rates. In this talk, volume of fluid simulations of falling films at different Reynolds numbers and different wave regimes are presented. The influence of the hydrodynamics on mass transport is investigated.

Date:

Wednesday, Nov. 30 17:30-18:00

## Lorenz v. Below

Technische Universität Darmstadt

Title:

**The Kalton-Weis Theorem in Lebesgue spaces**

Abstract:

In Kalton and Weis proved that two sectorial operators $A$ and $B$ with commuting resolvents in a Banach space $X$ admit a joint bounded $H^\infty$-calculus whenever $A$ and $B$ each admit a bounded $H^\infty$-calculus and the geometry of the underlying Banach space $X$ satisfies certain assumptions. These geometric assumptions are fulfilled e. g. by the Lebesgue spaces $L^p(\Omega)$ for $1 < p < \infty$.

We present an elementary proof of the aforementioned result in the special case where $X = L^p(\Omega)$ and applications thereof.

References:

N. J. Kalton and L. Weis, *The $H^\infty$-calculus and sums of closed operators*, Math. Ann. **321** (2001), no. 2, 319–345. MR 1866491 (2003a:47038)

Date:

Wednesday, Nov. 30 18:00-18:30

## Dario Götz

Technische Universität Darmstadt

Title:

**The spin-coating process and two-phase flow for generalized Newtonian fluids**

Abstract:

In this talk, a mathematical analysis of a model describing the spin-coating process is presented. In the spin-coating process, a small droplet of a coating liquid is placed in the center of a rapidly spinning disc. Centrifugal force then drive the liquid outwards and a thin film of the coating material is formed.

The process is modeled as a one-phase free boundary problem on a perturbed layer for a generalized Newtonian fluid. Relying on linear results for the Newtonian case previously attained by Denk et al., existence and uniqueness of strong solutions for small data is shown.

Date:

Tuesday, Nov. 29 17:30-18:00

## Jyunichi Harada

Waseda University

Title:

**Blow-up solutions and its singularities of the heat equations with a nonlinear boundary condition**

Abstract:

We study blow-up phenomenons of positive solutions to the heat equation with a nonlinear boundary condition. We give sufficient conditions for a single point blow-up and study the blow-up profile and its spacial singularities of such a single point blow-up solution. To apply arguments in Herrero-Velazquez (92-93), we establish some estimates for a heat kernel of some linearized equation.

Date:

Friday, Dec. 2 17:30-18:00

## Tsukasa Iwabuchi

Touhoku University

Title:

**Global and almost global solutions for the Navier-Stokes equations in Besov spaces and Triebel-Lizorkin spaces**

Abstract:

We consider the existence of the global and almost global solutions for the Navier-Stokes equations in the spaces which have scaling invariant properties to the equations, where almost global solutions are solutions which existence time is bounded below by the exponential order of the norms of initial data. On the global solutions, we study in the space of all functions of bounded mean oscillation using

the Triebel-Lizorkin spaces. The existence of the almost global solutions are also studied in the larger function spaces than those for the global solutions.

Date:

Wednesday, Nov. 30 16:00-16:30

## Hajime Koba

Tokyo University

Title:

**Asymptotic stability of Ekman boundary layers in rotating stratified fluids**

Abstract:

We investigate the stability of Ekman boundary layers in rotating stratified fluids. We show the existence of a weak solution of an Ekman perturbed system, which satisfies the strong energy inequality. Moreover, we prove that the weak solution is smooth with respect to time when time is sufficiently large.

Date:

Tuesday, Nov. 29 16:30-17:00

## Mads Kyed

Technische Universität Darmstadt

Title:

**Time-periodic solutions to the Navier-Stokes equations**

Abstract:

I will present some new existence theorems for the time-periodic three-dimensional Navier-Stokes system in the whole-space. In addition, results on the asymptotic behavior of the corresponding solutions will be discussed.

Date:

Monday, Nov. 28 16:30-17:00

## Tomoyuki Nakatsuka

Nagoya University

Title:

**On uniqueness of stationary solutions to the Navier-Stokes equations in exterior domains**

Abstract:

The purpose of this talk is to introduce a new uniqueness criterion for solutions to the stationary Navier-Stokes equations in 3-dimensional exterior domains. Kozono-Yamazaki proved the solvability of the exterior problem by introducing the weak $L^p$ space, and we consider the solutions in their class. Using the dual equation with the aid of the bootstrap argument, we can show that if solutions $u$ and $v$ satisfy the conditions that $u$ is small in the weak $L^3$ space and $u, v \in L^p$ for some $p > 3$, then $u = v$.

Date:

Thursday, Dec. 1 16:00-16:30

## Manuel Nesensohn

Technische Universität Darmstadt

Title:

**$L_p$-theory for a generalized viscoelastic fluid model on a fix domain and with a free surface**

Abstract:

The investigated model is a generalization of the classical Oldroyd-B model. Existence and uniqueness of strong solution on fix domains is investigated in the $L_p$-setting. For bounded and unbounded domains large time existence is shown, provided the initial data is sufficiently small. For bounded domains, local in time solvability is proven for arbitrary large initial data. Further, an approach to the corresponding free boundary value problem without surface tension in Lagrangian coordinates is presented. Local in time existence of a unique strong $L_p$-solution for arbitrary large initial data is studied.

Date:

Tuesday, Nov. 29 18:00-18:30

## Kazuhiro Oeda

Waseda University

Title:

**Stationary solutions of a three species population model with a protection zone**

Abstract:

We consider a three species prey-predator model with a protection zone for the prey species. We discuss the existence and the non-existence of positive stationary solutions by using the bifurcation theory. From an ecological viewpoint, a positive stationary solution means a coexistence state of the three species.

Date:

Friday, Dec. 2 16:00-16:30

## Okihiro Sawada

Gifu University

Title:

**Norm-inflation arguments of mild solutions to the Navier-Stokes equations in the critical space**

Abstract:

A Cauchy problem of the Navier-Stokes equations in the whole space with initial data in the critical Besov space is considered via mild solutions. It is explained the results by Bourgain-Pavlovic who showed a lack of equicontinuity of solutions in the natural topology, that is, the mild solution does not continuously depend on the initial velocity. The proof is based on the norm-inflation argument; the Besov norm of a mild solution instantaneously becomes large even for small initial velocity. In this talk, the term-wise estimates of successive approximation are established. Furthermore, it is shown that there exists a subsequence of successive approximation which diverges at any positive time in the Besov norm, if the initial velocity is large.

Date:

Monday, Nov. 28 16:00-16:30

## Georg Schöchtel

Technische Universität Darmstadt

Title:

**Motion of Inertial Particles in Gaussian Fields driven by an infinite-dimensional Fractional Brownian Motion**

Abstract:

The motion of an inertial particle in a fractional Gaussian random field is studied. The motion is described by Newton's second law for a particle on a 2D torus, with force proportional to the difference between a background fluid velocity and the particle velocity itself. The fluid velocity satisfies a linear stochastic PDE driven by an infinite-dimensional fractional Brownian motion. This model captures the clustering phenomenon of preferential concentration observed in real world and numerical experiments, i.e. particles cluster in regions of low vorticity and high strain rate. We prove almost sure existence and uniqueness of particle paths and give sufficient conditions to rewrite this system as a random dynamical system with a global random pullback attractor. Further we give upper bounds of the almost sure constant Hausdorff dimension of the random attractor.

Date:

Monday, Nov. 28 17:30-18:00

## Raphael Schulz

Technische Universität Darmstadt

Title:

**Spatial Behaviour of the Navier Stokes Flow in a Rotating Frame**

Abstract:

We consider the 3-dimensional rotating Navier-Stokes Equations in weighted $L^\infty$ spaces. For large $|x|$ the solution behaves like $|x|^{-3}$. Furthermore, we work out the leading terms of the spatial asymptotic away from the rotating axis.

Date:

Friday, Dec. 2 18:00-18:30

## Kohei Soga

Waseda University

Title:

**Stochastic and variational approach to the Lax-Friedrichs scheme**

Abstract:

I will present a stochastic variational problem which is equivalent to the Lax-Friedrichs scheme applied to nonlinear PDEs of hyperbolic type. The stochastic variational problem consists of space-time inhomogeneous random walks and an action functional. Difference solutions of the PDEs obtained through the Lax-Friedrichs scheme are represented by the stochastic variational problem. Convergence of the difference solutions to exact ones is proved from a stochastic point of view.

Date:

Thursday, Dec. 1 16:30-17:00

## Ryo Takada

Touhoku University

Title:

**Local well-posedness for the Navier-Stokes equations in the rotational framework**

Abstract:

We consider the initial value problems for the Navier-Stokes equations with the

Coriolis force. We prove the local in time existence and uniqueness of the mild solution for every initial velocity field in the Sobolev spaces. Furthermore, we give an exact characterization for the time interval of its local existence in terms of the Coriolis parameter and the initial velocity. It follows from our characterization that the existence time of solution can be taken arbitrarily large provided the speed of rotation is sufficiently fast.

Date:

Wednesday, Nov. 30 16:30-17:00

## Hidemitsu Wadade

Waseda University

Title:

**Weighted Trudinger-Moser type inequality and its application**

Abstract:

We consider the weighted Trudinger-Moser type inequality with its sharp constant. In the paper by Adachi-Tanaka (Proc.Amer.Math.Soc.,1999), the sharp constant of the Trudinger-Moser type inequality of the scaling invariant form was obtained. We generalize this result to the homogeneous weighted version, and we apply this inequality to construct a time global solution of the Klein-Gordon equation including the weighted exponential nonlinear term.

Date:

Friday, Dec. 2 16:30-17:00

## Norikazu Yamaguchi

Toyama University

Title:

**A mathematical justification of the penalty method for the Stokes and Navier-Stokes equations**

Abstract:

Penalty method is widely used in numerical computation of the Stokes and Navier-Stokes equations for removing pressure from the equations. In this talk, I will report some error estimates between solution to original Stokes/Navier-Stokes equation and solution to penalized Stokes/Navier-Stokes equation in the whole space.

Date:

Monday, Nov. 28 18:00-18:30

**Yusuke Yamauchi**
Waseda University

Title:

**Life span of positive solutions for a semilinear heat equation with non-decaying initial data**

Abstract:

In this talk, we show an upper bound of the life span of positive solutions of a semilinear heat equation for non-decaying initial data. The bound is expressed by the limit inferior of the data at space infinity around a specific direction. We also show that the minimal time blow-up occurs when initial data attains its maximum at space infinity.

Date:

Thursday, Dec. 1 17:30-18:00

**Satoshi Yokoyama**
Tokyo University

Title:

**Construction of weak solutions of a certain stochastic Navier-Stokes equation**

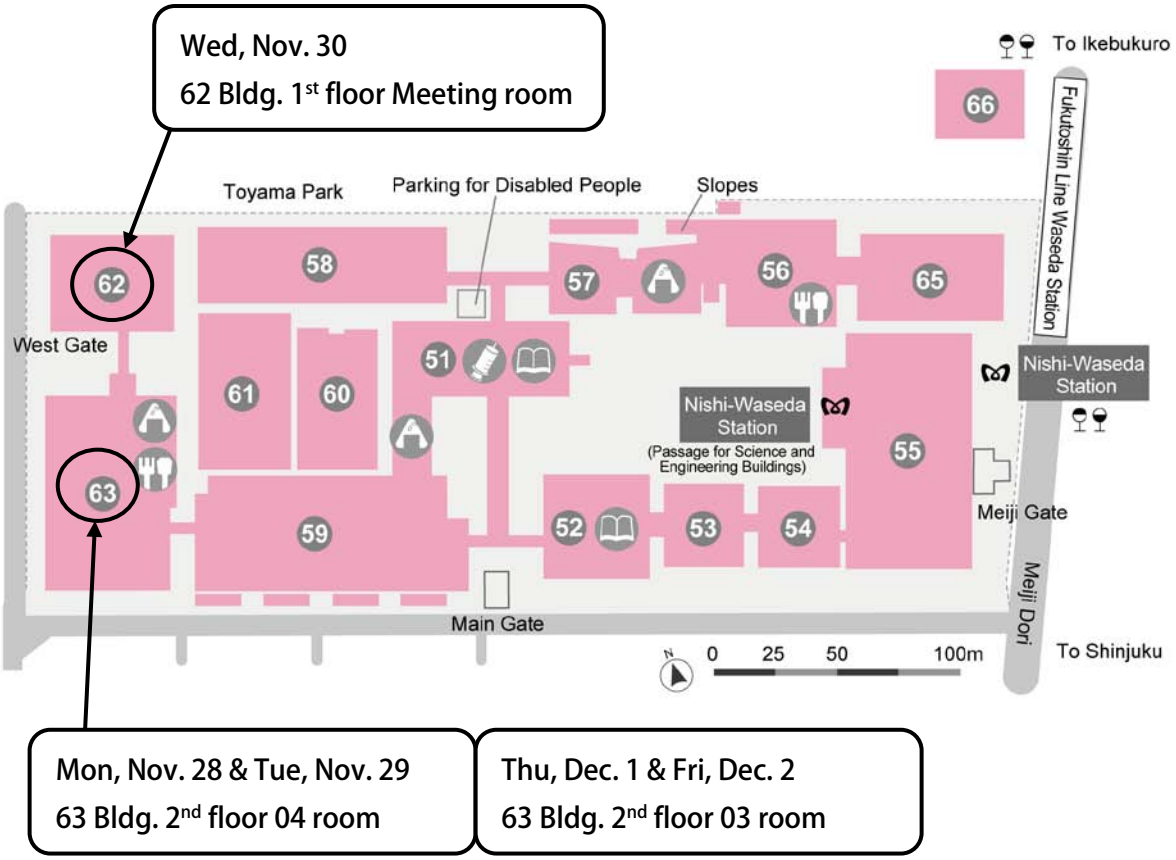
Abstract:

We prove the existence of weak solutions of stochastic Navier-Stokes equation on a two-dimensional torus, which appears in a certain variational problem. Our equation does not satisfy the coercivity condition. We construct its weak solutions due to an approximation by a sequence of solutions of equations with enlarged viscosity terms and then by showing an a priori estimate for them.

Date:

Tuesday, Nov. 29 16:00-16:30

## Campus Map





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http://www.japan-germany.sci.waseda.ac.jp/event/201111workshop.html

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