On the compressible Navier-Stokes flow with Neumann boundary conditions

Dario Götz
IRTG 1529
TU Darmstadt

TECHNISCHE UNIVERSITÄT DARMSTADT

joint work with Prof. Yoshihiro Shibata (Waseda University)

The 5th Japanese-German International Workshop
on Mathematical Fluid Dynamics

Waseda University, June 14, 2012

Compressible Navier-Stokes flow with free boundary

TECHNISCHE UNIVERSITÄT DARMSTADT

Consider the free boundary problem

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) = 0, & \text{in } (0,T) \times \Omega(t), \end{cases}$$

Compressible Navier-Stokes flow with free boundary

TECHNISCHE UNIVERSITÄT DARMSTADT

Consider the free boundary problem

$$
\begin{cases}
\partial_t \rho + \operatorname{div}(\rho u) &= 0, \quad \text{in } (0,T) \times \Omega(t), \\
\rho(\partial_t u + u \cdot \nabla u) - \operatorname{div} S(u,\rho) &= 0, \quad \text{in } (0,T) \times \Omega(t),
\end{cases}
$$

where the stress tensor is

$$S(u,\rho) = 2\mu Du + \mu' \operatorname{div} u \, \mathrm{id} - P(\rho)\mathrm{id},$$

P is a smooth function of ρ with $P'(\rho) > 0$

Compressible Navier-Stokes flow with free boundary

Consider the free boundary problem

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) &= 0, \quad \text{in } (0,T) \times \Omega(t), \\ \rho(\partial_t u + u \cdot \nabla u) - \operatorname{div} S(u,\rho) &= 0, \quad \text{in } (0,T) \times \Omega(t), \\ -S(u,\rho)\nu &= 0, \quad \text{on } (0,T) \times \partial\Omega(t), \end{cases}$$

where the stress tensor is

$$S(u,\rho) = 2\mu Du + \mu' \operatorname{div} u \,\mathrm{id} - P(\rho)\mathrm{id},$$

P is a smooth function of ρ with $P'(\rho) > 0$

Compressible Navier-Stokes flow with free boundary

TECHNISCHE UNIVERSITÄT DARMSTADT

Consider the free boundary problem

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho u) &= 0, & \text{in } (0,T)\times\Omega(t), \\ \rho(\partial_t u + u\cdot\nabla u) - \operatorname{div} S(u,\rho) &= 0, & \text{in } (0,T)\times\Omega(t), \\ -S(u,\rho)\nu &= 0, & \text{on } (0,T)\times\partial\Omega(t), \\ u\cdot\nu &= V, & \text{on } (0,T)\times\partial\Omega(t), \end{cases}$$

where the stress tensor is

$$S(u,\rho) = 2\mu Du + \mu' \operatorname{div} u \,\mathrm{id} - P(\rho)\mathrm{id},$$

P is a smooth function of ρ with $P'(\rho) > 0$

Compressible Navier-Stokes flow with free boundary

TECHNISCHE UNIVERSITÄT DARMSTADT

Consider the free boundary problem

$$\begin{cases} \partial_t\rho + \mathrm{div}(\rho u) &= 0, & \text{in } (0,T)\times\Omega(t),\\ \rho(\partial_t u + u\cdot\nabla u) - \mathrm{div}S(u,\rho) &= 0, & \text{in } (0,T)\times\Omega(t),\\ -S(u,\rho)\nu &= 0, & \text{on } (0,T)\times\partial\Omega(t),\\ u\cdot\nu &= V, & \text{on } (0,T)\times\partial\Omega(t),\\ \rho(0) &= \rho_0, & \text{in } \Omega(0),\\ u(0) &= u_0, & \text{in } \Omega(0),\\ \Omega(0) &= \Omega_0. \end{cases}$$

where the stress tensor is

$$S(u,\rho) = 2\mu Du + \mu' \mathrm{div} u\, \mathrm{id} - P(\rho)\mathrm{id},$$

P is a smooth function of ρ with $P'(\rho) > 0$

Linearized model system

- transform to Lagrangian coordinates to receive problem on $\Omega(0)$

Linearized model system

TECHNISCHE UNIVERSITÄT DARMSTADT

- transform to Lagrangian coordinates to receive problem on $\Omega(0)$
- linearize around $u^* = 0$ and $\rho^* > 0$ constant

Linearized model system

TECHNISCHE UNIVERSITÄT DARMSTADT

- transform to Lagrangian coordinates to receive problem on $\Omega(0)$
- linearize around $u^* = 0$ and $\rho^* > 0$ constant
- consider half-space $\Omega(0) = \mathbb{R}^n_+$

Linearized model system

TECHNISCHE UNIVERSITÄT DARMSTADT

- transform to Lagrangian coordinates to receive problem on $\Omega(0)$
- linearize around $u^* = 0$ and $\rho^* > 0$ constant
- consider half-space $\Omega(0) = \mathbb{R}^n_+$

$$
(1) \quad \begin{cases}
\partial_t \rho + \gamma \operatorname{div} u & = & f_\rho, & \text{in } \mathbb{R}^n_+, \\
\partial_t u - \alpha \Delta u - (\alpha + \beta) \nabla \operatorname{div} u + \gamma \nabla \rho & = & f_u, & \text{in } \mathbb{R}^n_+, \\
2\alpha D u \, e_n + \beta \operatorname{div} u \, e_n - \gamma \rho \, e_n & = & g, & \text{on } \mathbb{R}^{n-1}, \\
u(0) & = & 0, & \text{in } \mathbb{R}^n_+ \\
\rho(0) & = & 0, & \text{in } \mathbb{R}^n_+
\end{cases}
$$

Main result:
Maximal L_p-regularity

Theorem

Let $1 < p, q < \infty$, $\alpha > 0$ *and* $\beta \in \mathbb{R}$ *such that* $\frac{\alpha}{2} + \beta > 0$. *Then for any number* $\delta > 0$ *such that*

$$f_\rho \in L_{p,0,\delta}(\mathbb{R}; H^1_q(\mathbb{R}^n_+)), \qquad f_u \in L_{p,0,\delta}(\mathbb{R}; L_q(\mathbb{R}^n_+)),$$

$$g \in H^{\frac{1}{2}}_{p,0,\delta}(\mathbb{R}; L_q(\mathbb{R}^n_+)) \cap L_{p,0,\delta}(\mathbb{R}; H^1_q(\mathbb{R}^n_+)),$$

Main result: Maximal L_p-regularity

TECHNISCHE UNIVERSITÄT DARMSTADT

Theorem

Let $1 < p, q < \infty$, $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $\frac{\alpha}{2} + \beta > 0$. Then for any number $\delta > 0$ such that

$$f_\rho \in L_{p,0,\delta}(\mathbb{R}; H_q^1(\mathbb{R}_+^n)), \qquad f_u \in L_{p,0,\delta}(\mathbb{R}; L_q(\mathbb{R}_+^n)),$$
$$g \in H_{p,0,\delta}^{\frac{1}{2}}(\mathbb{R}; L_q(\mathbb{R}_+^n)) \cap L_{p,0,\delta}(\mathbb{R}; H_q^1(\mathbb{R}_+^n)),$$

there is exactly one solution (ρ, u) of system (1) in the spaces

$$\rho \in L_{p,0,\delta}(\mathbb{R}; H_q^1(\mathbb{R}_+^n)),$$
$$u \in H_{p,0,\delta}^1(\mathbb{R}; L_q(\mathbb{R}_+^n)) \cap L_{p,0,\delta}(\mathbb{R}; H_q^2(\mathbb{R}_+^n)).$$

Reduction of the system

TECHNISCHE UNIVERSITÄT DARMSTADT

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \mathrm{div} u & = & f_\rho, & \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \mathrm{div} u + \gamma\nabla\rho & = & f_u, & \text{in } \mathbb{R}^n_+, \\ 2\alpha D u\, e_n + \beta \mathrm{div} u\, e_n - \gamma\rho\, e_n & = & g, & \text{on } \mathbb{R}^{n-1}. \end{cases}$$

Reduction of the system

TECHNISCHE UNIVERSITÄT DARMSTADT

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \operatorname{div} u = f_\rho, & \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \operatorname{div} u + \gamma\nabla\rho = f_u, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \beta \operatorname{div} u\, e_n - \gamma\rho\, e_n = g, & \text{on } \mathbb{R}^{n-1}. \end{cases}$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \operatorname{div} u)$,

$$\begin{cases} \lambda u - \alpha\Delta u - (\alpha+\beta+\frac{\gamma^2}{\lambda})\nabla \operatorname{div} u = f_u - \lambda^{-1}\gamma\nabla f_\rho, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + (\beta + \frac{\gamma^2}{\lambda}) \operatorname{div} u\, e_n = g + \lambda^{-1}\gamma f_\rho e_n, & \text{on } \mathbb{R}^{n-1}, \end{cases}$$

Reduction of the system

TECHNISCHE UNIVERSITÄT DARMSTADT

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \mathrm{div} u &= f_\rho, \quad \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \mathrm{div} u + \gamma\nabla\rho &= f_u, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \beta \mathrm{div} u\, e_n - \gamma\rho\, e_n &= g, \quad \text{on } \mathbb{R}^{n-1}. \end{cases}$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \mathrm{div} u)$,

$$\begin{cases} \lambda u - \alpha\Delta u - (\alpha + \beta + \frac{\gamma^2}{\lambda})\nabla \mathrm{div} u &= f_u - \lambda^{-1}\gamma\nabla f_\rho, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + (\beta + \frac{\gamma^2}{\lambda})\mathrm{div} u\, e_n &= g + \lambda^{-1}\gamma f_\rho e_n, \quad \text{on } \mathbb{R}^{n-1}, \end{cases}$$

Reduction of the system

TECHNISCHE
UNIVERSITÄT
DARMSTADT

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \operatorname{div} u = f_\rho, & \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \operatorname{div} u + \gamma\nabla\rho = f_u, & \text{in } \mathbb{R}^n_+, \\ 2\alpha D u\, e_n + \beta \operatorname{div} u\, e_n - \gamma\rho\, e_n = g, & \text{on } \mathbb{R}^{n-1}. \end{cases}$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \operatorname{div} u)$, $\eta_\lambda = \beta + \frac{\gamma^2}{\lambda}$

$$\begin{cases} \lambda u - \alpha\Delta u - (\alpha+\eta_\lambda)\nabla \operatorname{div} u = f_u - \lambda^{-1}\gamma\nabla f_\rho, & \text{in } \mathbb{R}^n_+, \\ 2\alpha D u\, e_n + \eta_\lambda \operatorname{div} u\, e_n = g + \lambda^{-1}\gamma f_\rho e_n, & \text{on } \mathbb{R}^{n-1}, \end{cases}$$

Reduction of the system

TECHNISCHE UNIVERSITÄT DARMSTADT

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \mathrm{div} u &= f_\rho, \quad \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \mathrm{div} u + \gamma\nabla\rho &= f_u, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \beta \mathrm{div} u\, e_n - \gamma\rho\, e_n &= g, \quad \text{on } \mathbb{R}^{n-1}. \end{cases}$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \mathrm{div} u)$, $\eta_\lambda = \beta + \frac{\gamma^2}{\lambda}$

$$\begin{cases} \lambda u - \alpha\Delta u - (\alpha+\eta_\lambda)\nabla \mathrm{div} u &= f_u - \lambda^{-1}\gamma\nabla f_\rho, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \eta_\lambda \mathrm{div} u\, e_n &= g + \lambda^{-1}\gamma f_\rho e_n, \quad \text{on } \mathbb{R}^{n-1}, \end{cases}$$

Reduction of the system

TECHNISCHE UNIVERSITÄT DARMSTADT

- apply (formally) Laplace transform to receive

$$\left\{\begin{array}{rcll} \lambda\rho + \gamma \operatorname{div} u &=& f_\rho, & \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \operatorname{div} u + \gamma\nabla\rho &=& f_u, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \beta \operatorname{div} u\, e_n - \gamma\rho\, e_n &=& g, & \text{on } \mathbb{R}^{n-1}. \end{array}\right.$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \operatorname{div} u)$, $\eta_\lambda = \beta + \frac{\gamma^2}{\lambda}$

$$\left\{\begin{array}{rcll} \lambda u - \alpha\Delta u - (\alpha+\eta_\lambda)\nabla \operatorname{div} u &=& f, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \eta_\lambda \operatorname{div} u\, e_n &=& h, & \text{on } \mathbb{R}^{n-1}, \end{array}\right.$$

Reduction of the system

- apply (formally) Laplace transform to receive

$$\left\{\begin{array}{rcll} \lambda\rho + \gamma \operatorname{div} u & = & f_\rho, & \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \operatorname{div} u + \gamma\nabla\rho & = & f_u, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \beta \operatorname{div} u\, e_n - \gamma\rho\, e_n & = & g, & \text{on } \mathbb{R}^{n-1}. \end{array}\right.$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \operatorname{div} u)$, $\eta_\lambda = \beta + \frac{\gamma^2}{\lambda}$

$$\left\{\begin{array}{rcll} \lambda u - \alpha\Delta u - (\alpha+\eta_\lambda)\nabla \operatorname{div} u & = & f, & \text{in } \mathbb{R}^n_+, \\ 2\alpha Du\, e_n + \eta_\lambda \operatorname{div} u\, e_n & = & h, & \text{on } \mathbb{R}^{n-1}, \end{array}\right.$$

- solve whole-space problem to reduce to $f = 0$

Reduction of the system

- apply (formally) Laplace transform to receive

$$\begin{cases} \lambda\rho + \gamma \operatorname{div} u &= f_\rho, \quad \text{in } \mathbb{R}^n_+, \\ \lambda u - \alpha\Delta u - (\alpha+\beta)\nabla \operatorname{div} u + \gamma\nabla\rho &= f_u, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha D u\, e_n + \beta \operatorname{div} u\, e_n - \gamma\rho\, e_n &= g, \quad \text{on } \mathbb{R}^{n-1}. \end{cases}$$

- substitute $\rho = \lambda^{-1}(f_\rho - \gamma \operatorname{div} u)$, $\eta_\lambda = \beta + \frac{\gamma^2}{\lambda}$

$$\begin{cases} \lambda u - \alpha\Delta u - (\alpha+\eta_\lambda)\nabla \operatorname{div} u &= 0, \quad \text{in } \mathbb{R}^n_+, \\ 2\alpha D u\, e_n + \eta_\lambda \operatorname{div} u\, e_n &= h, \quad \text{on } \mathbb{R}^{n-1}, \end{cases}$$

- solve whole-space problem to reduce to $f = 0$

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi')e^{-Ax_n} + Q_j(\xi')e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi')e^{-Ax_n} + Q_j(\xi')e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

- invert the resulting algebraic system for P_j and Q_j

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi')e^{-Ax_n} + Q_j(\xi')e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

- invert the resulting algebraic system for P_j and Q_j
-

$$P_j = \frac{2(\alpha + \eta_\lambda)^2 B i\xi_j}{\alpha(B - A)K} i\xi' \cdot \widehat{h}'(\xi', 0) - \frac{(\alpha + \eta_\lambda)\left((2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2\right)}{\alpha(B - A)K} i\xi_j \widehat{h}_n(\xi', 0)$$

Derivation of the solution formula

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi') e^{-Ax_n} + Q_j(\xi') e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

- invert the resulting algebraic system for P_j and Q_j
-

$$P_j = \frac{2(\alpha + \eta_\lambda)^2 B i\xi_j}{\alpha(B - A)K} i\xi' \cdot \widehat{h}'(\xi', 0) - \frac{(\alpha + \eta_\lambda)\left((2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2\right)}{\alpha(B - A)K} i\xi_j \widehat{h}_n(\xi', 0)$$

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi')e^{-Ax_n} + Q_j(\xi')e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

- invert the resulting algebraic system for P_j and Q_j
-

$$P_j = \frac{2(\alpha + \eta_\lambda)^2 B i\xi_j}{\alpha(B - A)K} i\xi' \cdot \widehat{h}'(\xi', 0) - \frac{(\alpha + \eta_\lambda)\left((2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2\right)}{\alpha(B - A)K} i\xi_j \widehat{h}_n(\xi', 0)$$

Derivation of the solution formula

TECHNISCHE UNIVERSITÄT DARMSTADT

- (formally) apply horizontal Fourier transform in x', introduce $v = \widehat{u}$
- consider the ansatz

$$v_j(\xi', x_n) = P_j(\xi')e^{-Ax_n} + Q_j(\xi')e^{-Bx_n}$$

for $A = \sqrt{(2\alpha + \eta_\lambda)^{-1}\lambda + |\xi'|^2}$ and $B = \sqrt{\alpha^{-1}\lambda + |\xi'|^2}$

- invert the resulting algebraic system for P_j and Q_j
-

$$P_j = \frac{2(\alpha + \eta_\lambda)^2 B i\xi_j}{\alpha(B - A)K} i\xi' \cdot \widehat{h}'(\xi', 0) - \frac{(\alpha + \eta_\lambda)\left((2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2\right)}{\alpha(B - A)K} i\xi_j \widehat{h}_n(\xi', 0)$$

Estimates on the polynomial K

TECHNISCHE UNIVERSITÄT DARMSTADT

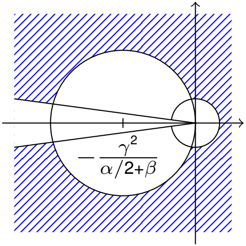
$$K = \eta_\lambda^2 B^3 + (2\alpha + \eta_\lambda)^2 AB^2 + (2\alpha + \eta_\lambda)(2\alpha + 3\eta_\lambda)A^2 B - (2\alpha + \eta_\lambda)^2 A^3$$

Estimates on the polynomial K

$$K = \eta_\lambda^2 B^3 + (2\alpha + \eta_\lambda)^2 AB^2 + (2\alpha + \eta_\lambda)(2\alpha + 3\eta_\lambda)A^2 B - (2\alpha + \eta_\lambda)^2 A^3$$

- it is $|K| > 0$ for all $\lambda \in \Sigma_\varepsilon$ satisfying $|\lambda| \geq \lambda_0 > 0$ and

$$\left(\mathrm{Re}\,\lambda + \frac{\gamma^2}{\alpha/2 + \beta}\right)^2 + (\mathrm{Im}\,\lambda)^2 \geq \left(\frac{\gamma^2}{\alpha/2 + \beta}\right)^2$$

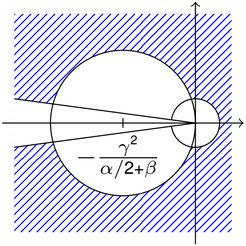


Estimates on the polynomial K

$$K = \eta_\lambda^2 B^3 + (2\alpha + \eta_\lambda)^2 AB^2 + (2\alpha + \eta_\lambda)(2\alpha + 3\eta_\lambda)A^2 B - (2\alpha + \eta_\lambda)^2 A^3$$

- it is $|K| > 0$ for all $\lambda \in \Sigma_\varepsilon$ satisfying $|\lambda| \geq \lambda_0 > 0$ and

$$\left(\mathrm{Re}\,\lambda + \frac{\gamma^2}{\alpha/2 + \beta}\right)^2 + (\mathrm{Im}\,\lambda)^2 \geq \left(\frac{\gamma^2}{\alpha/2 + \beta}\right)^2$$



- for $\gamma = 0$ it is with Taylor expansion

$$|K_0| \geq c(|\lambda|^{\frac{1}{2}} + |\xi'|)^3$$

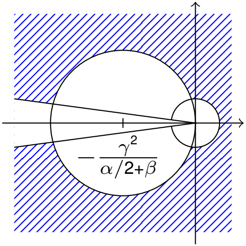
Estimates on the polynomial K

TECHNISCHE UNIVERSITÄT DARMSTADT

$$K = \eta_\lambda^2 B^3 + (2\alpha + \eta_\lambda)^2 AB^2 + (2\alpha + \eta_\lambda)(2\alpha + 3\eta_\lambda)A^2 B - (2\alpha + \eta_\lambda)^2 A^3$$

- it is $|K| > 0$ for all $\lambda \in \Sigma_\varepsilon$ satisfying $|\lambda| \geq \lambda_0 > 0$ and

$$\left(\operatorname{Re}\lambda + \frac{\gamma^2}{\alpha/2 + \beta}\right)^2 + (\operatorname{Im}\lambda)^2 \geq \left(\frac{\gamma^2}{\alpha/2 + \beta}\right)^2$$



- for $\gamma = 0$ it is with Taylor expansion

$$|K_0| \geq c(|\lambda|^{\frac{1}{2}} + |\xi'|)^3$$

- by a perturbation argument for large λ and ξ' and compactness one shows

$$|K| \geq c(|\lambda|^{\frac{1}{2}} + |\xi'|)^3$$

$\mathcal{R}$-boundedness by multiplier result

TECHNISCHE UNIVERSITÄT DARMSTADT

- the solution of the stationary problem is $\mathcal{T}(\lambda)(|\lambda|^{\frac{1}{2}}h, \nabla h)$ for

$$\mathcal{T}(\lambda)(f, g) = \mathcal{K}_1(\lambda)f + \mathcal{K}_2(\lambda)g$$

$\mathcal{R}$-boundedness by multiplier result

- the solution of the stationary problem is $\mathcal{T}(\lambda)(|\lambda|^{\frac{1}{2}}h, \nabla h)$ for

$$\mathcal{T}(\lambda)(f, g) = \mathcal{K}_1(\lambda)f + \mathcal{K}_2(\lambda)g$$

- $\mathcal{K}_1$ and $\mathcal{K}_2$ consist of terms like

$$\int_0^\infty \mathcal{F}^{-1}_{\xi'}\left[\frac{\alpha + \eta_\lambda}{\alpha} \; \frac{(2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2}{K} \quad e^{-B(x_n+y_n)} \quad \mathcal{F}_{\xi'}[g]\right](x', y_n)dy_n$$

$$\int_0^\infty \mathcal{F}^{-1}_{\xi'}\left[\frac{\alpha + \eta_\lambda}{\alpha} \frac{\left((2\alpha + \eta_\lambda)A^2 + \eta_\lambda B^2\right)A}{K} \mathcal{M}(x_n + y_n)\mathcal{F}_{\xi'}[g]\right](x', y_n)dy_n$$

$\mathcal{R}$-boundedness by multiplier result

TECHNISCHE UNIVERSITÄT DARMSTADT

- the solution of the stationary problem is $\mathcal{T}(\lambda)(|\lambda|^{\frac{1}{2}}h, \nabla h)$ for

$$\mathcal{T}(\lambda)(f,g) = \mathcal{K}_1(\lambda)f + \mathcal{K}_2(\lambda)g$$

- $\mathcal{K}_1$ and $\mathcal{K}_2$ consist of terms like

$$\int_0^\infty \mathcal{F}_{\xi'}^{-1}\left[\frac{\alpha+\eta_\lambda}{\alpha}\ \frac{(2\alpha+\eta_\lambda)A^2+\eta_\lambda B^2}{K}\quad e^{-B(x_n+y_n)}\quad \mathcal{F}_{\xi'}[g]\right](x',y_n)dy_n$$

$$\int_0^\infty \mathcal{F}_{\xi'}^{-1}\left[\frac{\alpha+\eta_\lambda}{\alpha}\ \frac{\left((2\alpha+\eta_\lambda)A^2+\eta_\lambda B^2\right)A}{K}\mathcal{M}(x_n+y_n)\mathcal{F}_{\xi'}[g]\right](x',y_n)dy_n$$

$\mathcal{R}$-boundedness by multiplier result

TECHNISCHE UNIVERSITÄT DARMSTADT

- the solution of the stationary problem is $\mathcal{T}(\lambda)(|\lambda|^{\frac{1}{2}}h, \nabla h)$ for

$$\mathcal{T}(\lambda)(f,g) = \mathcal{K}_1(\lambda)f + \mathcal{K}_2(\lambda)g$$

- $\mathcal{K}_1$ and $\mathcal{K}_2$ consist of terms like

$$\int_0^\infty \mathcal{F}_{\xi'}^{-1}\left[\frac{\alpha+\eta_\lambda}{\alpha} \; \frac{(2\alpha+\eta_\lambda)A^2+\eta_\lambda B^2}{K} \quad e^{-B(x_n+y_n)} \quad \mathcal{F}_{\xi'}[g]\right](x',y_n)dy_n$$

$$\int_0^\infty \mathcal{F}_{\xi'}^{-1}\left[\frac{\alpha+\eta_\lambda}{\alpha} \frac{\left((2\alpha+\eta_\lambda)A^2+\eta_\lambda B^2\right)A}{K}\mathcal{M}(x_n+y_n)\mathcal{F}_{\xi'}[g]\right](x',y_n)dy_n$$

- $\mathcal{R}$-boundedness by a multiplier result by Shibata and Shimizu

TECHNISCHE UNIVERSITÄT DARMSTADT

Thank you for your attention.