Decay rates towards traveling waves for a model system of radiating gas

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1. Introduction & Mathematical formulation

System of equations for polytropic gas with radiative heat flow:

$$\rho_t + (\rho u)_x = 0, \tag{R.a}$$
$$\rho(u_t + u u_x) + p_x = 0, \tag{R.b}$$
$$\rho\theta(s_t + u s_x) + q_x = 0, \tag{R.c}$$
$$p = \rho R \theta, \tag{R.d}$$
$$\theta = \frac{A}{R}\rho^{\gamma-1}\exp((\gamma-1)s/R), \tag{R.e}$$
$$-q_{xx} + 3\alpha^2 q + 4\alpha\sigma(\theta^4)_x = 0. \tag{R.f}$$

ρ	:	density	s	:	entropy
u	:	velocity	q	:	radiative heat flux
p	:	pressure	θ	:	absolute temperature

Expansion around $(\rho, u, s, q) = (\rho_0, 0, s_0, 0)$:

$$\begin{aligned} &\rho = \rho_0 + \varepsilon\bar{\rho}(\bar{t}, \bar{x}), && s = s_0 + \varepsilon^2 \bar{s}(\bar{t}, \bar{x}), \\ &u = \varepsilon\bar{u}(\bar{t}, \bar{x}), && q = \varepsilon^2 \bar{q}(\bar{t}, \bar{x}), \\ &\bar{t} = \varepsilon t, && \bar{x} = x - C_s t, \quad C_s := \sqrt{\gamma R \theta_0} \end{aligned}$$

Retain $\mathcal{O}(\varepsilon^2)$ terms and neglect $\mathcal{O}(\varepsilon^3)$ terms to obtain $\cdots$

- Governing Equations [K. Hamer, 1971]

$$u_t + uu_x + q_x = 0, \qquad \text{(E.a)}$$

$$-q_{xx} + q + u_x = 0. \qquad \text{(E.b)}$$

$$u = u(t,x),\ \ q = q(t,x) \in \mathbb{R} \text{ for } t > 0,\ \ x \in \mathbb{R}$$

$$\lim_{x\to\pm\infty} q(x) = 0.$$

- Initial data

$$u(0,x) = u_0(x), \qquad \text{(I.a)}$$

$$\lim_{x\to\pm\infty} u_0(x) = u_\pm. \qquad \text{(I.b)}$$

$$\text{(E.b)} \ \Rightarrow\ q = K(-u_x),\ \ q_x = u - Ku$$

$$\text{where} \quad K := (1-\Delta)^{-1}, \quad Kf(x) = \int_{-\infty}^{+\infty} \frac{1}{2} e^{-|x-y|} f(y)dy.$$

$$\text{(E.a)}, \text{(E.b)} \ \Rightarrow\ u_t + uu_x + u - Ku = 0$$

• Comparison with Inviscid/Viscous Burgers Equation

Hamer's Radiating Gas Model	Viscous Burgers Equation
$u_t + uu_x + \underline{u - Ku} = 0$	$u_t + uu_x \underline{- u_{xx}} = 0$
$u - Ku = \mathcal{F}^{-1}\left(\dfrac{\xi^2}{1+\xi^2}\,\widehat{u}(\xi)\right)$	$-u_{xx} = \mathcal{F}^{-1}\left(\xi^2\,\widehat{u}(\xi)\right)$

similar to viscosity, but weaker effect especially for $|\xi| \gg 1$ (short waves)

Degree of dissipation

Inviscid Burgers Eq. $<$ Radiating Gas $<$ Viscous Burgers Eq.

Proposition 1 (Kawashima, Nishibata '99) *Suppose* $u_0 \in B^1(\mathbb{R})$.

i) Blow up criterion

If
$$\inf_x u_0'(x) < -\left(1 + \sqrt{1+4k_0}\right)/2,$$
then sol. blows up : $\quad {}^{\exists}T < \infty \ s.t. \ \lim_{t\to T-0} \inf_x u_x(t,x) = -\infty.$

$$\left(k_0 := \min\{\,\delta_0\,/\,2,\ \sup_x u_0'(x)\,\} \quad \delta_0 := \sup_x u_0(x) - \inf_x u_0(x)\right)$$

ii) Non-Blow up criterion

If
$$\delta_0 \le 1/2 \quad \textit{and} \quad \inf_x u_0'(x) \ge -\left(1 + \sqrt{1-2\delta_0}\right)/2,$$
then ${}^{\exists}1$ *global classical sol.*

Moreover, 'maximum principle' holds:

$$\inf_x u_0(x) \le u(t,x) \le \sup_x u_0(x),$$
$$\min\{\inf_x u_0'(x), v^*\} \le u_x(t,x) \le \sup_x u_0'(x).$$
$$\left(v^* := -\left(1 - \sqrt{1-2\delta_0}\right)/2.\right)$$

Definition 2 by Kruzkov('70) (method of vanishing viscosity)

Admissible solution : $(u,q) \in L^\infty([0,T) \times \mathbb{R})$ *which satisfies*

$$\int_0^T \int_{-\infty}^{+\infty} \left[|u-k|\phi_t + \mathrm{sign}(u-k) \left\{ \left(\frac{1}{2}u^2 - \frac{1}{2}k^2 \right) \phi_x - (u - Ku)\phi \right\} \right] dxdt \geq 0,$$

$$0 \leq {}^\forall \phi \in C_0^\infty((0,T) \times \mathbb{R}), \ {}^\forall k \in \mathbb{R},$$

and

$$\int_{-\infty}^{+\infty} (-q\psi_{xx} + q\psi - u\psi_x)\, dx = 0, \quad {}^\forall \psi \in \mathcal{S}(\mathbb{R}),$$

with the initial condition

$$u(0,x) = u_0(x) \ \textit{a.e. } x \in \mathbb{R}.$$

cf. For piecewise smooth data, ($d(t)$: location of discontinuity)

Rankine-Hugoniot conditions : $\begin{cases} [q] = 0, \ [u] = [q_x], \\ \dot{d}(t) = \big(u(t,d(t)-0) + u(t,d(t)+0)\big)/2. \end{cases}$

entropy condition : $u(t,d(t)-0) > u(t,d(t)+0)$

- Traveling Wave Solution

Def :

$$(u,q)(t,x) = (U,Q)(x-st), \quad {}^{\exists}s \in \mathbb{R},\ U \in B^1(\mathbb{R}),\ Q \in B^2(\mathbb{R})$$

satisfying

$$-sU' + UU' + Q' = 0, \tag{T.a}$$

$$-Q'' + Q + U' = 0, \tag{T.b}$$

$$\lim_{x\to\pm\infty} U(x) = u_\pm. \tag{T.c}$$

Proposition 3 (Kawashima, Nishibata '98)

i) ${}^{\exists}TW \quad \Rightarrow \quad u_- > u_+, \quad s = (u_- + u_+)/2, \quad \lim_{x\to\pm\infty} Q(x) = 0.$

ii) $0 < u_- - u_+ < 2\sqrt{2n}/(n+1) \quad (n = 1,2,3\cdots)$

$\Rightarrow {}^{\exists}$ *TW (unique up to shift) s.t.*

$$U \in B^n,\ Q \in B^{n+1},\ s = (u_- + u_+)/2,\ \lim_{x\to\pm\infty} Q(x) = 0,$$

$$|U(\eta) - u_\pm| \le \frac{1}{2}\delta_S \exp(-c|\eta|), \quad \left|\frac{d^n}{d\eta^n}U(\eta)\right| \le C_n \delta_S^{n+1}, \quad \delta_S := u_- - u_+$$

2. Previous results on asymptotic analysis

Asymptotic stability of rarefaction wave, constant state, BV

- **[Kawashima, Tanaka '04]** stability of rarefaction wave
- **[Ito '96]** stability of BV data around const. or rarefaction wave

Asymptotic stability of traveling wave

- **[Kawashima, Nishibata '98]** u_0: continuous, $\delta_S < \frac{\sqrt{6}}{2}$
- **[Kawashima, Nishibata '99]** u_0: Riemann data $u_- > u_+$, $\delta_S \le \frac{1}{2}$
- **[Nishibata '00]** $u_0 \in B^1(\mathbb{R} \setminus \{0\}), u_0(-0) > u_0(+0)$, $\delta_S \le \frac{1}{2}$

$$\left.\begin{array}{l}\text{initial perturbation } \phi_0 := u_0 - U \in L^1 \\ \text{anti-derivative } \Phi_0(x) := \int_{-\infty}^{x} \phi_0(y)dy \in H^3\end{array}\right\} \Rightarrow \text{uniform convergence}$$

$$+\,\Phi_0 \in L^1 \qquad \Rightarrow \mathcal{O}(t^{-1/4}) \text{ decay}$$

- **[Nishikawa, Nishibata '07]**

  u_0: Riemann data $u_- > u_+, \ \delta_S \leq \frac{1}{2}$ $\qquad \Rightarrow \mathcal{O}(e^{-ct})$ decay

$$\phi_0 \in L^1, \ \langle x \rangle^\alpha \Phi_0 \in H^3(\mathbb{R}) \ (\alpha > 0), \ \delta_S < \frac{\sqrt{6}}{2} \ \Rightarrow \ \mathcal{O}((1+t)^{-\frac{\alpha}{2}}) \text{ decay}$$

$$\left(\langle x \rangle = (1+x^2)^{1/2}\right)$$

- **[Many more recent works]**

H. Freistuhler, D.Serre, P. Marcati, C. Lattanzio, C. Mascia, C. Rhode, T. Nguyen, R.Plaza, K.Zumbrun, R. Duan, K. Fellner, C. Zhu $\cdots$

3. Main results

We study the asymptotic stability of TW with discontinuous IV. In particular, show that convergence rate reflects the spatial decay structure of the initial perturbation.

[M.O.]: *Convergence rates towards the traveling waves for a model system of radiating gas with discontinuities*, submitted

3.1. Preliminaries

- We can set $s = 0$ $\quad i.e. \quad u_- + u_+ = 0$ WLOG.
  $\left(\tilde{u}(t,x) = u(t, x + s_0 t) - s_0,\ \tilde{q}(t,x) = q(t, x + s_0 t),\ s_0 = (u_- + u_+)/2\right)$

- 'center of mass' x_0: (If $u_0 - u_S \in L^1,\ u_S(x) := u_\pm\ (x \gtrless 0)$.)
  $$x_0 := \frac{1}{u_- - u_+} \int_{-\infty}^{\infty} (u_0(x) - U(x))dx, \text{ where } U(0) = (u_- + u_+)/2 = 0.$$
  $$\left(\Leftrightarrow \int_{-\infty}^{\infty} (u_0(x) - U(x - x_0))dx = 0 \right)$$

Proposition 4 *(Behavoir of discontinuities)* (Nishibata '00)

Suppose

$$u_0 \in B^1(\mathbb{R} \setminus \{0\}), \quad u_0(-0) > u_0(+0),$$
$$u_0 - u_S \in L^1 \cap L^\infty,$$
$$\delta_0 \le 1/2, \quad v_* \le u_0'(x) \text{ for } x \ne 0.$$

Then $^\exists 1$ *global admissible solution*

$$u \in B^1(\Omega),\ q \in B^2(\Omega),\ \Omega := [0,\infty) \times \mathbb{R} \setminus \{x = d(t)\},$$

where trajectory of jump $\{x = d(t)\}$ *is* C^1*-curve in* (t,x) *space with*

$$\dot{d}(t) = (u_L(t) + u_R(t))/2, \quad d(0) = 0$$
$$d(t) \to x_0 \quad (t \to \infty).$$

Gap at the jump verifies

$$0 < u_L(t) - u_R(t) \le (u_0(-0) - u_0(+0)) \exp(-ct), \quad ^\exists c = c(u_0) > 0.$$

$$u_L(t) := u(t, d(t) - 0),\ u_R(t) := u(t, d(t) + 0)$$
$$\delta_0 := \sup_x u_0(x) - \inf_x u_0(x),\ v_* := -\left(1 + \sqrt{1 - 2\delta_0}\right)/2.$$

Define perturbation $(\phi,\psi)(\tau,\xi)$ ($\tau \geq 0$, $\xi \in \mathbb{R}_0 := \mathbb{R} \setminus \{0\}$) by

$$(\phi,\psi)(\tau,\xi) := (u,q)(\tau,\xi+d(\tau)) - (U,Q)(\xi+d(\tau)-x_0).$$

Then $(\phi,\psi)(\tau,\xi)$ satisfies

$$\left.\begin{array}{c} \phi_\tau + \partial_\xi \left[\Big(U(\xi+d(\tau)-x_0) - \dot{d}(\tau) \Big) \phi + \frac{1}{2}\phi^2 \right] + \psi_\xi = 0, \\ -\psi_{\xi\xi} + \psi + \phi_\xi = 0, \end{array} \right\} \qquad \text{(P)}$$

with initial and boundary data to (P)

$$\phi(0,\xi) = \phi_0(\xi) := u_0(\xi) - U(\xi - x_0), \qquad \text{(PI)}$$

$$\lim_{\xi \to \pm\infty} \psi(t,\xi) = 0, \quad {}^\forall t \geq 0. \qquad \text{(PB)}$$

If $\phi(\tau,\cdot) \in L^1$, define anti-derivative Φ of ϕ by

$$\Phi(\tau,\xi) := \int_{-\infty}^{\xi} \phi(\tau,\eta) d\eta.$$

Equation for Φ :

$$\Phi_\tau + (U - \dot{d}(\tau))\Phi_\xi + \frac{1}{2}\Phi_\xi^2 + \psi = 0.$$

3.2.a. Decay rate for exponentially decaying I.V.

Theorem 5 *Suppose*

$$0 < u_- - u_+ < \sqrt{6}/2,$$
$$[u_0] := u_0(-0) - u_0(+0) > 0,$$
$$\phi_0 \in L^1,$$
$$e^{\lambda|\xi|/2}\Psi_0 \in H^3(\mathbb{R}_0) \text{ for } \lambda \in (0, \lambda_0), \ (\lambda_0 < \sqrt{2} : \text{const.})$$
$$\|e^{\lambda|\xi|/2}\Psi_0\|_{H^3} \ll 1.$$

Then $^{\exists}1$ *time global solution* (ϕ, ψ) *to (P) satisfying*

$$e^{\lambda|\xi|/2}\phi \in \bigcap_{i=0}^{2} C^i\left([0,\infty); H^{2-i}(\mathbb{R}_0)\right), e^{\lambda|\xi|/2}\psi \in \bigcap_{i=0}^{2} C^i\left([0,\infty); H^{3-i}(\mathbb{R}_0)\right)$$

and $^{\exists}C, \gamma > 0$ *s.t.*

$$\|\phi(t)\|_{H^2}^2 + \|\psi(t)\|_{H^3}^2 \le C\left\{\|e^{\lambda|\xi|/2}\Psi_0(\xi)\|_{H^3}^2 + [u_0]\left(1 + \|u_0\|_{L^\infty} + \|U\|_{L^\infty}\right)\right\} e^{-\gamma t}.$$

3.2.b. Decay rate for algebraically decaying I.V.

Theorem 6 *Suppose*

$$0 < u_- - u_+ < \sqrt{6}/2,$$
$$[u_0] := u_0(-0) - u_0(+0) > 0,$$
$$\phi_0 \in L^1,$$
$$\langle \xi \rangle^{\alpha/2} \Psi_0 \in H^3(\mathbb{R}_0) \text{ for } \alpha > 0,$$
$$\| \langle \xi \rangle^{\alpha/2} \Psi_0 \|_{H^3} \ll 1.$$

Then $^\exists 1$ *time global solution* (ϕ, ψ) *to (P) satisfying*

$$\langle \xi \rangle^{\alpha/2} \phi \in \bigcap_{i=0}^{2} C^i \Big([0,\infty); H^{2-i}(\mathbb{R}_0) \Big), \langle \xi \rangle^{\alpha/2} \psi \in \bigcap_{i=0}^{2} C^i \Big([0,\infty); H^{3-i}(\mathbb{R}_0) \Big)$$

and $^\exists C, \gamma > 0$ *s.t.*

$$\begin{aligned} &\|\phi(t)\|_{H^2}^2 + \|\psi(t)\|_{H^3}^2 \\ &\quad \le C \Big\{ \| \langle \xi \rangle^{\alpha/2} \Psi_0(\xi) \|_{H^3}^2 + [u_0] \left(1 + \|u_0\|_{L^\infty} + \|U\|_{L^\infty}\right) \Big\} (1+t)^{-\alpha}. \end{aligned}$$

3.3. Outline of the proof

(Local existence) + (A-priori estimate) $\Rightarrow$ (Global existence)

Lemma 7 (Local existence) $W = e^{\lambda|\xi|}$ *or* $\langle\xi\rangle^{\alpha}$.

If $W^{1/2}\phi_0 \in H^2(\mathbb{R}_0)$,

then $^{\exists}T > 0$ *and* $^{\exists}1$ *solution* (ϕ, ψ) *to (P) over* $[0, T]$

satisfying

$$W^{1/2}\phi \in C^0(H^2) \cap C^1(H^1),$$
$$W^{1/2}\psi \in C^0(H^3) \cap C^1(H^2).$$

Moreover, if $\phi_0 \in L^1$ *and* $W^{1/2}\Phi_0 \in L^2$ *(if* $W = e^{\lambda|\xi|}, \lambda < 2$*),*

then $\phi_0 \in C^0(L^1)$ *and* $W^{1/2}\Phi \in C^0(H^1) \cap C^1(L^2)$.

Construction of local solution (without weight)

Rewrite (P) to $\phi_\tau + \Big(U\big(\xi + d(\tau) - x_0\big) + \phi - \dot{d}(\tau)\Big)\phi_\xi + U_\xi\phi + \phi - K\phi = 0.$

For $\xi > 0$, set $\quad \phi_1(\tau,\xi) := \phi(\tau,\xi), \quad \phi_2(\tau,\xi) := \phi(\tau,-\xi).$

System of governing eqs. for $(\phi_1,\phi_2)(\tau,\xi)$ $(\xi \in \mathbb{R}_+)$ is

$$\partial_\tau\phi_1 + \Big(U_1(\tau,\xi) + \phi_1 - \dot{d}(\tau)\Big)\partial_\xi\phi_1 + \partial_\xi U_1\phi_1 + \phi_1 - K_1\phi_1 - K_2\phi_2 = 0,$$
$$\partial_\tau\phi_2 - \Big(U_2(\tau,\xi) + \phi_2 - \dot{d}(\tau)\Big)\partial_\xi\phi_2 - \partial_\xi U_2\phi_2 + \phi_2 - K_2\phi_1 - K_1\phi_2 = 0,$$

where $\; U_1(\tau,\xi) := U(d(\tau) + \xi - x_0), \quad U_2(\tau,\xi) := U(d(\tau) - \xi - x_0)$

and $\; K_1 f(\xi) := \int_0^\infty K(\xi - \eta)f(\eta)d\eta, \quad K_2 f(\xi) := \int_0^\infty K(\xi + \eta)f(\eta)d\eta.$

Entropy condition: $u_L > u_R \Rightarrow$

$$(U_1 + \phi_1)(\tau, 0+0) - \dot{d}(\tau),\; -(U_2 + \phi_2)(\tau, 0+0) - \dot{d}(\tau) < 0.$$

We can extend the space to $\mathbb{R}$. $\Rightarrow$ Kato's theory does apply.

Proposition 8 (A-priori estimate)

If

$$N(T) := \sup_{0\le t\le T} \|W^{1/2}\Phi(t)\|_{H^3} \ll 1$$

then

$$^{\exists}C, \gamma > 0 \textit{ indep. of } T$$

s.t.

i) $$\|e^{\lambda|\xi|/2}\Phi(t,\xi)\|_{H^3}^2 + \|e^{\lambda|\xi|/2}\psi(t,\xi)\|_{H^3}^2 \le C\left\{\|e^{\lambda|\xi|/2}\Phi_0(\xi)\|_{H^3}^2 + g_0(1+\|u_0\|_{L^\infty} + \|U\|_{L^\infty})\right\} e^{-\gamma t},$$

for $\lambda \in (0, \lambda_0)$, $\lambda_0 < \sqrt{2}$,

ii) $$\|\langle\xi\rangle^{\beta/2}\Phi(t,\xi)\|_{H^3}^2 + \|\langle\xi\rangle^{\beta/2}\psi(t,\xi)\|_{H^3}^2 \le C\left\{\|\langle\xi\rangle^{\alpha/2}\Phi_0(\xi)\|_{H^3}^2 + g_0(1+\|u_0\|_{L^\infty} + \|U\|_{L^\infty})\right\}(1+t)^{-(\alpha-\beta)}$$

for $\alpha > 0$ *and* $\beta \in [0, \alpha]$.

A-priori estimate (Basic estimate for exponential weight)

$$\partial_\tau \left(\frac{1}{2}\phi^2\right) + \partial_\xi \left\{ (U - \dot{d}(\tau))\frac{1}{2}\phi^2 + \frac{1}{3}\phi^3 - \psi(\psi_\xi - \phi) \right\}$$
$$\underline{+\frac{1}{2}U_\xi \phi^2} + \psi^2 + \psi_\xi^2 = 0. \quad (1)$$

$$\partial_\tau \left(\frac{1}{2}\Phi^2\right) + \partial_\xi \left\{ (U - \dot{d}(\tau))\frac{1}{2}\Phi^2 + (\Phi + \psi)(\psi_\xi - \phi) \right\}$$
$$-\frac{1}{2}U_\xi \Phi^2 \underline{+ \left(1 + \frac{1}{2}\Phi\right)\phi^2} - (\psi^2 + \psi_\xi^2) = 0. \quad (2)$$

(1) $\times$ 2 + (2) yields

$$\partial_\tau \left(\frac{1}{2}\Phi^2 + \phi^2\right) + \partial_\xi \left\{ (U - \dot{d}(\tau))\left(\frac{1}{2}\Phi^2 + \phi^2\right) + \frac{2}{3}\phi^3 + (\Phi - \psi)(\psi_\xi - \phi) \right\}$$
$$-\frac{1}{2}U_\xi \Phi^2 \underline{+ \left(1 + \frac{1}{2}\Phi + U_\xi\right)\phi^2} + \psi^2 + \psi_\xi^2 = 0. \quad (3)$$

$\int_{-\infty}^{\infty} e^{\lambda|\xi|} \times (3)\, d\xi \quad (W := e^{\lambda|\xi|})$

$$
\begin{aligned}
\Rightarrow \quad & \frac{d}{d\tau}\int_{-\infty}^{\infty} W\left(\frac{1}{2}\Phi^2+\phi^2\right)(\tau,\xi)d\xi \\
& + \frac{1}{2}\int_{-\infty}^{\infty} W\Big(-U_\xi(\xi+d(\tau)-x_0)+P(\tau,\xi)+Q(\tau,\xi)\Big)\Phi^2(\tau,\xi)d\xi \\
& + \int_{-\infty}^{\infty} W\left(1+P(\tau,\xi)+Q(\tau,\xi)+U_\xi(\xi+d(\tau))+\frac{\Phi(\tau,\xi)}{2}\right)\phi^2(\tau,\xi)d\xi \\
& + \int_{-\infty}^{\infty} W(\psi^2+\psi_\xi^2)(\tau,\xi)d\xi + \frac{1}{6}(u_L(\tau)-u_R(\tau))^3 \\
& = \lambda\int_{-\infty}^{\infty} \mathrm{sign}(\xi) W\left\{(\Phi-\psi)(\psi_\xi-\phi)+\frac{2}{3}\phi^3\right\}(\tau,\xi)d\xi. \qquad (4)
\end{aligned}
$$

$$P := \lambda \mathrm{sign}(\xi)\Big(-U(\xi+d(\tau)-x_0)+U(d(\tau)-x_0)\Big) \geq 0,\ Q := \lambda \mathrm{sign}(\xi)\frac{\phi_L+\phi_R}{2}(\tau).$$

Note $\quad -U_\xi(\xi+d(\tau))+P(\tau,\xi) \geq c\lambda,\ {}^{\exists}c>0,\ {}^{\forall}\xi,\tau,$

$$u_L(\tau)-u_R(\tau)>0, \quad \|U'\|_{L^\infty} \leq 3/8.$$

If $N(T)+\lambda+\gamma/\lambda \ll 1$, then $\int_0^t e^{\gamma\tau} \times (4)\, d\tau$ yields

$$e^{\gamma t}|\Phi(t)|_{\lambda,1}^2 + \int_0^t e^{\gamma\tau}\big(\lambda|\Phi(\tau)|_\lambda^2 + |\phi(\tau)|_\lambda^2 + |\psi(\tau)|_{\lambda,1}^2\big)d\tau \leq C|\Phi_0|_{\lambda,1}^2.$$

Summary

1. Introduction

    - Mathematical formulation of the radiating gas model
    - Dissipative properties, blow up criterion
    - Kruzkov's weak solution
    - Traveling waves

2. Review on previous results

    Asymptotic stability of traveling waves & convergence rates

3. Main results

    Obtain convergence rates subject to decay structure of initial data