Kinetic flux vector splitting(KFVS) method for compressible two-phase flow containing non-conservative products

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Baer-Nunziato(BN) equtions

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Baer-Nunziato equtions

$$\frac{\partial}{\partial t}(\phi_s) + u_s \frac{\partial}{\partial x}(\phi_s) = 0$$

$$\frac{\partial}{\partial t}(\phi_s \rho_s) + u_s \frac{\partial}{\partial x}(\phi_s \rho_s u_s) = 0$$

$$\frac{\partial}{\partial t}(\phi_s \rho_s u_s) + u_s \frac{\partial}{\partial x}\left(\phi_s\left(\rho_s u_s^2 + P_s\right)\right) = P_g \frac{\partial \phi_s}{\partial x}$$

$$\frac{\partial}{\partial t}(\phi_s \rho_s E_s) + u_s \frac{\partial}{\partial x}\left(\phi_s u_s\left(\rho_s E_s + P_s\right)\right) = u_s P_g \frac{\partial \phi_s}{\partial x} \qquad (1)$$

$$\frac{\partial}{\partial t}(\phi_g \rho_g) + u_g \frac{\partial}{\partial x}(\phi_g \rho_g u_g) = 0$$

$$\frac{\partial}{\partial t}(\phi_g \rho_g u_g) + u_g \frac{\partial}{\partial x}\left(\phi_g\left(\rho_g u_g^2 + P_g\right)\right) = -P_g \frac{\partial \phi_s}{\partial x}$$

$$\frac{\partial}{\partial t}(\phi_g \rho_g E_g) + u_g \frac{\partial}{\partial x}\left(\phi_g u_g\left(\rho_g E_g + P_g\right)\right) = -u_s P_g \frac{\partial \phi_s}{\partial x}$$

ϕ, ρ, u, P are the volume fraction, phase density, velocity, phase pressure

$$E = e + u^2 / 2$$

backgrounds

application of two-phase flow

deflagration to detonation transition

solid explosive detonation

two-phase models

- average
- mixture
  - Baer-Nunziato model(1986)

Numerical research status

- Andrianov, Warnecke-- weak solution, Riemann problem(2004)
- Schwendeman, Wahle, Kapila--Godunov method(2006)
- Deledicque, Papalexandris--exact Riemann solver(2007)
- Tokareva, Toro -- HLLC-type Riemann solver(2010)

For simplicity, we do the following sign change:

$$\rho^{(1)} = \rho_s\phi_s, \rho^{(2)} = \rho_g\phi_g, P^{(1)} = P_s\phi_s, P^{(2)} = P_g\phi_g$$

and $E^{(1)} = E_s, E^{(2)} = E_g$ for symbolic uniform.

Equation can be written as:

$$W_t^{(i)} + F_x^{(i)} = S^{(i)}, i = 0,1,2. \qquad (2)$$

$$i = 0: \quad , W^{(0)} = \rho^{(1)} Z \quad F^{(0)} = \rho^{(1)} U^{(1)} Z, Z = 1/\phi_s;$$

$$i = 1,2: \ , W^{(i)} = \left(\rho^{(i)} \quad \rho^{(i)} U^{(i)} \quad \rho^{(i)} E^{(i)}\right)^T$$

$$F^{(i)} = \left(\rho^{(i)} U^{(i)} \quad \rho^{(i)} U^{(i)} U^{(i)} + P^{(i)} \quad U^{(i)}\left(\rho^{(i)} E^{(i)} + P^{(i)}\right)\right)^T,$$

$$S^{(0)} = 0, S^{(1)} = \left(0 \quad P_g \frac{\partial \phi_s}{\partial x} \quad U^{(1)} P_g \frac{\partial \phi_s}{\partial x}\right)^T, S^{(2)} = -S^{(1)}.$$

?

Here we introduce distribution function $f^{(i)}, i = 1,2.$

Relationship between macro- and micro- quantities:

$$W^{(i)} = \int \psi_\alpha^{(i)} f^{(i)} du d\xi, F^{(i)} = \int u \psi_\alpha^{(i)} f^{(i)} du d\xi.$$

What can we do on the non-conservative terms?

The idea is to treat the non-conservative terms, which discribe the interaction between two phases, as the external force for each phase.

GKS model for BN equations:

$$\begin{aligned} f_t^{(1)} + u f_x^{(1)} + a f_u^{(1)} &= 0 \\ f_t^{(2)} + u f_x^{(2)} - a f_u^{(1)} &= 0 \end{aligned} \qquad (3)$$

It's easy to get: $a = \dfrac{P_g}{\rho^{(1)}} \dfrac{\partial \phi_s}{\partial x} = \dfrac{P^{(2)}}{\left(1-\phi_s\right)\rho^{(1)}} \dfrac{\partial \phi_s}{\partial x}$

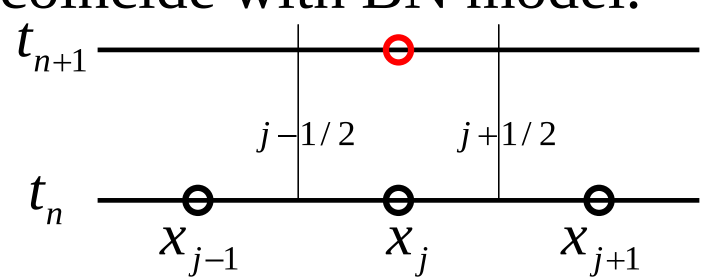
Then the non-conservative terms can be written as:

$$S^{(1)} = \begin{pmatrix} 0 & \rho^{(1)} a & U^{(1)} \rho^{(1)} a \end{pmatrix}^T, S^{(2)} = -S^{(1)}$$

But we have to check whether equations (3) coincide with BN model.

Integral over the following domain :

$$\Omega = \left[x_{j-1/2}, x_{j+1/2} \right] \times \left[t_n, t_{n+1} \right]$$



$$\int \psi_\alpha^{(i)} \left(f_t^{(1)} + u f_x^{(1)} + a f_u^{(1)} \right) du\, d\xi\, dx\, dt = 0, i = 0,1;$$

$$\int \psi_\alpha^{(2)} \left(f_t^{(2)} + u f_x^{(2)} - a f_u^{(1)} \right) du\, d\xi\, dx\, dt = 0.$$

$$\Rightarrow \left.\begin{aligned} W_j^{n+1,(0)} - W_j^{n,(0)} &= \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left(F_{j-1/2}^{(0)}(t) - F_{j+1/2}^{(0)}(t) \right) dt + \Delta t S_j^{(0)} \\ W_j^{n+1,(1)} - W_j^{n,(1)} &= \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left(F_{j-1/2}^{(1)}(t) - F_{j+1/2}^{(1)}(t) \right) dt + \Delta t S_j^{(1)} \\ W_j^{n+1,(2)} - W_j^{n,(2)} &= \frac{1}{\Delta x} \int_{t^n}^{t^{n+1}} \left(F_{j-1/2}^{(2)}(t) - F_{j+1/2}^{(2)}(t) \right) dt + \Delta t S_j^{(2)} \end{aligned}\right\} \quad (4)$$

Where, $\psi^{(0)} = Z; \psi^{(i)} = \left(1, u, \frac{1}{2}\left(u^2 + \xi^2 \right) \right)^T, i = 1, 2.$

To solve the equations, we need to know the flux and special source term.

$$F_{j+1/2}^{(i)} = \int u \psi_\alpha^{(i)} f_{j+1/2}^{(i)} du\, d\xi, S^{(1)} = \left(0 \quad \rho^{(1)} a \quad U^{(1)} \rho^{(1)} a \right)^T.$$

$$f_{j+1/2}^{(1)}\left(x_{j+1/2}, t, u \right) = \left(1 - 2\lambda a t (u - U) \right) f_0^{(1)}\left(x_{j+1/2} - ut \right),$$

$$f_{j+1/2}^{(2)} = f_0^{(2)}\left(x_{j+1/2} - ut \right) + a f_{j+1/2,u}^{(1)} t.$$

update

$$\left(\rho^{(1)}Z^{(1)}\right)^{n+1}=\left(\rho^{(1)}Z^{(1)}\right)^{n}+\frac{\Delta t}{\Delta x}\left(F^{(0)}_{j-1/2}-F^{(0)}_{j+1/2}\right)$$

$$\begin{pmatrix}\rho^{(1)}\\ \rho^{(1)}U^{(1)}\\ \rho^{(1)}E^{(1)}\end{pmatrix}^{n+1}=\begin{pmatrix}\rho^{(1)}\\ \rho^{(1)}U^{(1)}\\ \rho^{(1)}E^{(1)}\end{pmatrix}^{n}+\frac{\Delta t}{\Delta x}\begin{pmatrix}F^{(1)}_{\rho,j-1/2}-F^{(1)}_{\rho,j+1/2}\\ F^{(1)}_{\rho U,j-1/2}-F^{(1)}_{\rho U,j+1/2}\\ F^{(1)}_{\rho E,j-1/2}-F^{(1)}_{\rho E,j+1/2}\end{pmatrix}+\Delta t S^{(1)} \qquad (5)$$

$$\begin{pmatrix}\rho^{(2)}\\ \rho^{(2)}U^{(2)}\\ \rho^{(2)}E^{(2)}\end{pmatrix}^{n+1}=\begin{pmatrix}\rho^{(2)}\\ \rho^{(2)}U^{(2)}\\ \rho^{(2)}E^{(2)}\end{pmatrix}^{n}+\frac{1}{\Delta x}\int_{t^n}^{t^{n+1}}\begin{pmatrix}F^{(2)}_{\rho,j-1/2}-F^{(2)}_{\rho,j+1/2}\\ F^{(2)}_{\rho U,j-1/2}-F^{(2)}_{\rho U,j+1/2}\\ F^{(2)}_{\rho E,j-1/2}-F^{(2)}_{\rho E,j+1/2}\end{pmatrix}-\Delta t S^{(1)}$$

Numerical tests

CFL=0.6, L=1, M=300, t=0.1,initial discontinuity locate at x=0.5.

Initial conditions

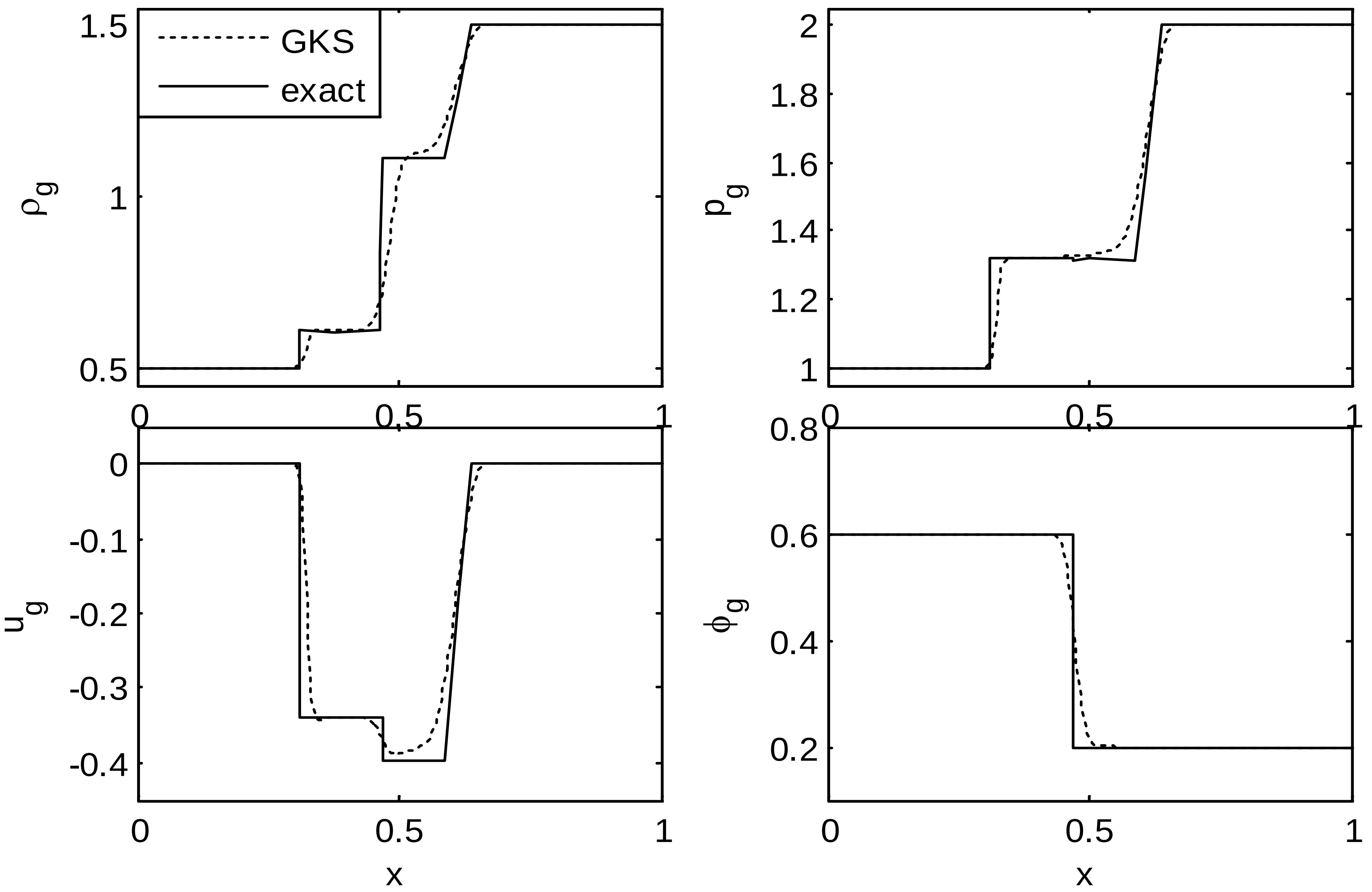
Test-1

	ϕ_L	u_L	ρ_L	p_L	ϕ_R	u_R	ρ_R	p_R
solid	0.4	0	1	1	0.8	0	2	2
gas	0.6	0	0.5	1	0.2	0	1.5	2

Test-2

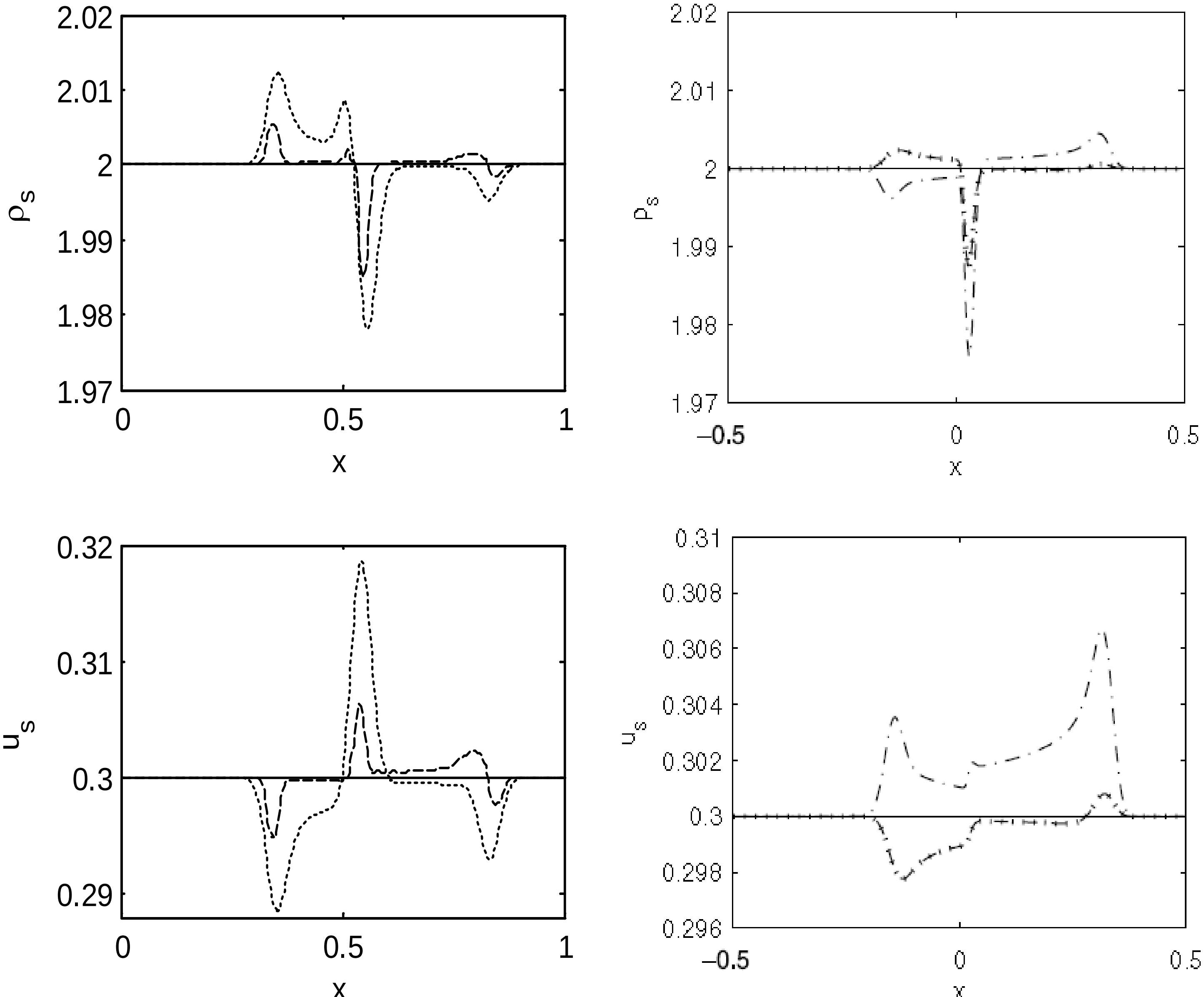
	ϕ_L	u_L	ρ_L	p_L	ϕ_R	u_R	ρ_R	p_R
solid	0.8	0.3	2	5	0.3	0.3	2	12.8567
gas	0.2	2	1	1	0.7	2.8011	0.1941	0.1

Test-1

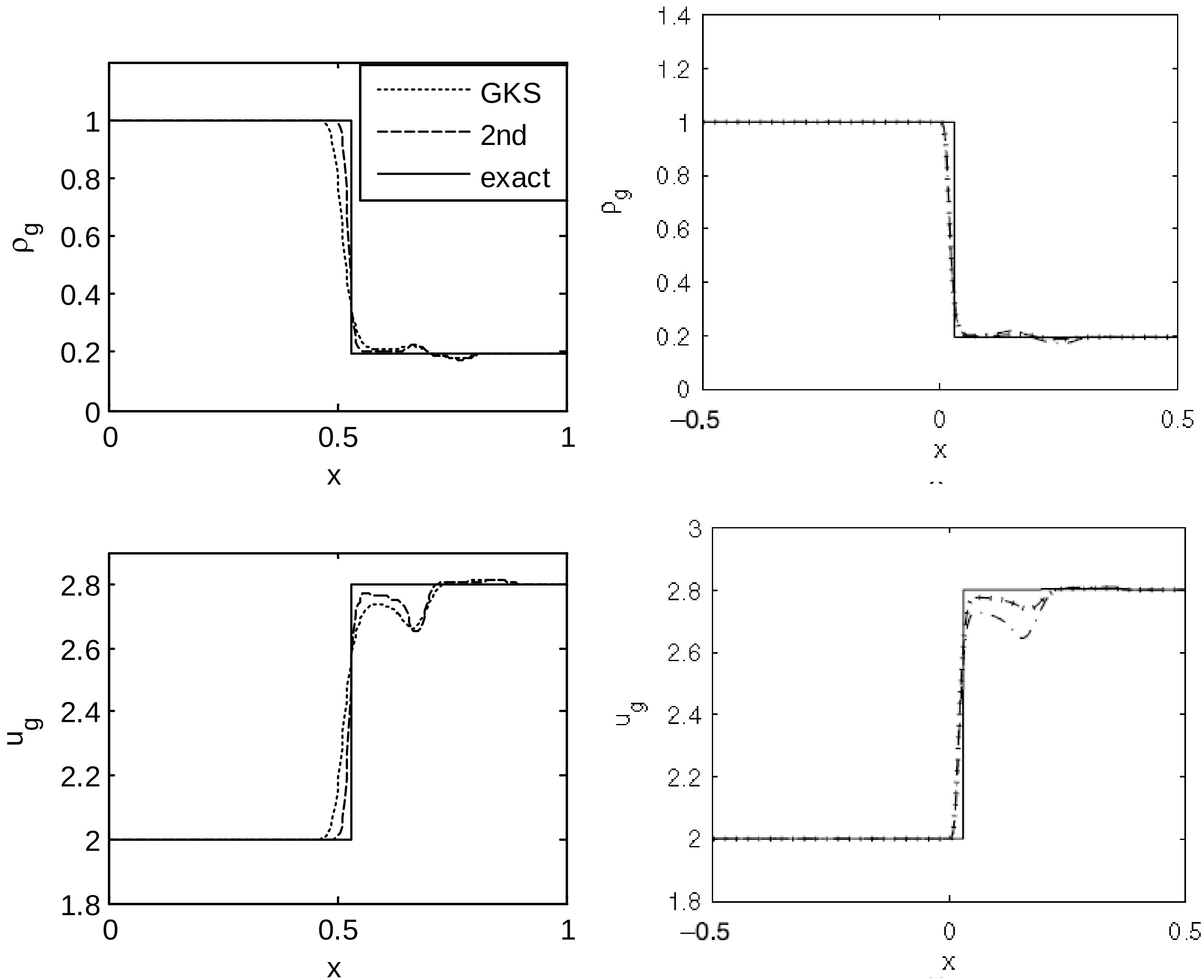


GKS
exact
ρ_g
p_g
u_g
ϕ_g
1.5
1
0.5
2
1.8
1.6
1.4
1.2
1
0
-0.1
-0.2
-0.3
-0.4
0.8
0.6
0.4
0.2
0
0.5
1
x

Test-2



reference result


GKS
2nd
exact
ρg
ug
x
0
0.5
1
1.8
2
2.2
2.4
2.6
2.8
3
0.2
0.4
0.6
0.8
1.2
1.4
−0.5

Thank you very much for your attention!