FSI with Application in Hemodynamics
Analysis and Simulation

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II. Numerical Modelling
Numerical schemes for the FSI problems

- strong coupled schemes (inner subiterations or fully composite schemes)
- weakly coupled schemes (no subiterations needed)
Numerical schemes for the FSI problems

- **strong coupled schemes** (inner subiterations or fully composite schemes)
- **weakly coupled schemes** (no subiterations needed)

- for aerodynamical problems: weakly coupled schemes: OK
- for biomechanical problems: kind of stronger coupling, subiterations are typically needed
Numerical schemes for the FSI problems

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- weakly coupled schemes (no subiterations needed)

- for aerodynamical problems: weakly coupled schemes: OK
- for biomechanical problems: kind of stronger coupling, subiterations are typically needed

- many numerical approaches for the FSI: immersed boundary, Lagrangian, Arbitrary Eulerian-Lagrangian (ALE)
Quarteroni, Formaggia, Veneziani, Nobile et al. (’01–’07) . . . inner subiterations
FSI for hemodynamical applications

1. Quarteroni, Formaggia, Veneziani, Nobile et al. (’01-’07) . . . inner subiterations

2. Hron, Turek et al. (’09) monolithic scheme (strong coupling)
Quarteroni, Formaggia, Veneziani, Nobile et al. (’01-’07) ... inner subiterations

Hron, Turek et al. (’09) monolithic scheme (strong coupling)

Guidoboni, Glowinski, Cavallini, Čanić (’09): weak coupling: kinematical coupling
FSI for hemodynamical applications

1. Quarteroni, Formaggia, Veneziani, Nobile et al. (’01-’07) . . . inner subiterations
2. Hron, Turek et al. (’09) monolithic scheme (strong coupling)
4. Janela, Moura, Sequeira (’10): simulations for non-Newtonian FSI
FSI for hemodynamical applications

1. Quarteroni, Formaggia, Veneziani, Nobile et al. (’01-’07) ... inner subiterations
2. Hron, Turek et al. (’09) monolitic scheme (strong coupling)
4. Janela, Moura, Sequeira (’10): simulations for non-Newtonian FSI
5. Hundertmark, Lukáčová, Rusnáková (’08,’10, ’11): analysis and simulations for non-Newtonian FSI schemes (global and kinematical coupling)
Numerical methods
  Discretization
  FSI

Stability
  Energy estimates
  Energy estimate for the A operator
  Energy estimates for B operator

Numerical experiments
  Inflow data and parameters
  Experiment I: stenotic vessel
  Experiment II: bifurcation vessel

Convergence study
Discretization method

- Fluid equations
  - UG software toolbox

\[ \mu \left( |D(\nabla u)| \right) D(\nabla u) \approx \mu \left( |D(\nabla u_{\text{old}})| \right) D(\nabla u) \]

Structure equation

\[ \partial_2^2 \eta \partial_t^2 - a \partial_2^2 \eta \partial_x^2 + b \left( \frac{1}{\eta} \right) \eta - c \partial_3 \eta \partial_x^2 \partial_t = \text{RHS}(u, p, R_0) \]

- finite difference method in time (Newmark scheme)
- finite difference method in space (central approximations)
**Discretization method**

- **Fluid equations**
  - UG software toolbox
  - FV-type schemes

\[
\begin{align*}
\frac{\partial^2 \eta}{\partial t^2} - a \frac{\partial^2 \eta}{\partial x^2} + b \left( \frac{1}{\eta} \right) \eta - c \frac{\partial^3 \eta}{\partial x^2 \partial t} &= \text{RHS} (u, p, R_0) \\
\end{align*}
\]

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- finite difference method in space (central approximations)

Lukáčová (AG Numerik)  Modelling in Hemodynamics
Discretization method

- **Fluid equations**
  - UG software toolbox
  - FV-type schemes
  - pseudo compressibility stabilization

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Discretization method

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  - Newton/fixed point linearization of convective term
**Discretization method**

- Fluid equations
  - UG software toolbox
  - FV-type schemes
  - pseudo compressibility stabilization
  - Newton/fixed point linearization of convective term
- fixed point iterations of non-linear viscosity

\[
\mu(|D(\nabla u)|)D(\nabla u) \approx \mu(|D(\nabla u^{old})|)D(\nabla u)
\]
Discretization method

- Fluid equations
  - UG software toolbox
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\[ \mu(|D(\nabla \mathbf{u})|)D(\nabla \mathbf{u}) \approx \mu(|D(\nabla \mathbf{u}^{old})|)D(\nabla \mathbf{u}) \]

- Structure equation

\[ \frac{\partial^2 \eta}{\partial t^2} - a \frac{\partial^2 \eta}{\partial x_1^2} + b(1/\eta)\eta - c \frac{\partial^3 \eta}{\partial x_1^2 \partial t} = \text{RHS}(\mathbf{u}, p, R_0) \]

- finite difference method in time (Newmark scheme)
- finite difference method in space (central approximations)

... second order

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Convergence study
I. Strong coupling - global iteration method $k=1,2,$ 

\[ \ldots \]

- solve fluid eq. & structure eq. on the domain $\Omega(\eta^k)$ for $t^n$, $n = 1, 2, 3 \ldots$
- homogeneous Dirichlet or Neumann boundary conditions
Fluid-Structure Coupling 1

- **I. Strong coupling** - global iteration method $k=1,2,\ldots$
  - solve fluid eq. & structure eq. on the domain $\Omega(\eta^k)$ for $t^n, n = 1, 2, 3\ldots$
  - homogeneous Dirichlet or Neumann boundary conditions

$(\xi = \partial_t \eta)$

\[
\frac{\xi^{n+1} - \xi^n}{\Delta t} - a\alpha \frac{\partial^2 \eta^{n+1}}{\partial x_1^2} + b\alpha \eta^{n+1} - c\alpha \frac{\partial^2 \xi^{n+1}}{\partial x_1^2} = RHS(p^n, u^n, R_0) + (1 - \alpha) \left( a \frac{\partial^2 \eta^n}{\partial x_1^2} - b\eta^n + c \frac{\partial^2 \xi^n}{\partial x_1^2} \right)
\]

\[
\frac{\eta^{n+1} - \eta^n}{\Delta t} = \alpha \xi^{n+1} + (1 - \alpha) \xi^n, \quad \alpha = 0.5
\]
Global iteration with respect to the domain

\[ k=0, \ h=\eta^k \]

**initial condition, solve NS and deformation problem**

\[ t_1, \ (x,t_1) \]

\[ t_2, \ (x,t_2) \]

\[ \ldots \]

\[ t_n, \ (x,t_n) \]

if convergence

if not

**update domain geometry**

\[ \Omega(h), \ h=\eta(x,t) \]

\[ \Omega^{k+1}(h), \ h=\eta^{k+1}(x,t) \]

\[ u^{k+1}(x,t) \]

\[ \eta^{k+1}(x,t) \]

\[ \ldots \]

STOP
II. Kinematical coupling [Guidoboni et al. ('09)]
- solve Fluid eq. + Structure eq. on $\Omega(\eta^n)$ for $t \in [t^n, t^{n+1}]$
- **split structure eq.** into 2 parts
- kinematical lateral B.C.

\[
 u_2|_{y=H} = \frac{\partial \eta}{\partial t} = \xi
\]
II. Kinematical coupling [Guidoboni et al. (’09)]
- solve Fluid eq. + Structure eq. on \( \Omega(\eta^n) \) for \( t \in [t^n, t^{n+1}] \)
- **split structure eq.** into 2 parts
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\[
\begin{align*}
\left. u_2 \right|_{y=H} &= \frac{\partial \eta}{\partial t} = \xi \\
\end{align*}
\]

- **1. hydrodynamic part**
  fluid eq. \quad + \quad \xi = \left. u_2 \right|_{y=H}

\[
\frac{\partial \xi}{\partial t} = c \frac{\partial^2 \xi}{\partial x_1^2} + \text{RHS}(u, p)
\]

- **2. elastic part**

\[
\begin{align*}
\frac{\partial \eta}{\partial t} &= \xi \\
\frac{\partial \xi}{\partial t} &= a \frac{\partial^2 \eta}{\partial x_1^2} - b(1/\eta)\eta + \text{RHS}(R_0)
\end{align*}
\]
Fluid-Structure Coupling 3

- Kinematical coupling / Discretization

**A operator: hydrodynamic part**
Fluid solver +

\[
\frac{\xi^{n+1/2} - \xi^n}{\Delta t} - c\alpha \frac{\partial^2 \xi^{n+1/2}}{\partial x_1^2} = RHS(p^n, u^n) + c(1 - \alpha) \frac{\partial^2 \xi^n}{\partial x_1^2},
\]

**B operator: elastic part**

\[
\frac{\eta^{n+1} - \eta^n}{\Delta t} = \beta \xi^{n+1} + (1 - \beta)\xi^{n+1/2}, \quad \beta = 0.5
\]

\[
\frac{\xi^{n+1} - \xi^{n+1/2}}{\Delta t} - a\alpha \frac{\partial^2 \eta^{n+1}}{\partial x_1^2} + b(1/\eta^n)\alpha \eta^{n+1}
= RHS(R_0) + a(1 - \alpha)\frac{\partial^2 \eta^n}{\partial x_1^2} - b(1/\eta^n)(1 - \alpha)\eta^n \quad \alpha = 1, 0.5
\]
Types of splitting

- kinematical splitting scheme of the **Marchuk-Yanenko type**
  - 1st order accurate
  - $A_{\Delta t} \rightsquigarrow B_{\Delta t}$
  - in the $(n + 1)$-th time-step on $\Omega(t^n)$
Types of splitting

- kinematical splitting scheme of the **Marchuk-Yanenko type**
  - 1st order accurate
  - \( A_{\Delta t} \bowtie B_{\Delta t} \)
    in the \((n + 1)\)th time-step on \(\Omega(t^n)\)

- kinematical splitting scheme of the **Strang-type kinematical splitting**
  - 2nd order accurate
  - \( B_{\Delta t/2} \bowtie A_{\Delta t} \bowtie B_{\Delta t/2} \)
    in the \((n + 1)\)th time-step on \(\Omega(t^n)\)
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Arbitrary Lagrangian-Eulerian Mapping

\[ A_t : \Omega_0 \rightarrow \Omega_t \]

\[ A_t(X) = x \in \Omega_t \]

\[ f(t, x) \equiv \tilde{f}(t, A_t^{-1}x) \equiv \tilde{f}(t, X) \]

\[ \frac{D^A u}{Dt} \text{ ALE derivative} \quad w := \frac{\partial x}{\partial t} \text{ "grid velocity"} \]

\[ \frac{D^A u(t, x)}{Dt} := \frac{\partial \tilde{u}(t, X)}{\partial t} = \frac{\partial u(t, x)}{\partial t} + w \cdot \nabla u(t, x) \]
Weak formulation of coupled problem

- test functions: $v \in V$, where

$$V \equiv \left\{ v \in W^{1,p}(\Omega(t))^2 : v_1|_{\Gamma_{wall}} = 0, v_2|_{x_2=0} = 0 \right\}$$

$\frac{\mathcal{D}A}{Dt}u$ ALE derivative

$w := \frac{\partial X}{\partial t}$ grid velocity

$$\frac{\mathcal{D}A}{Dt}u := \frac{\partial u}{\partial t} + w \cdot \nabla u(x, t)$$
Weak formulation of coupled problem

- test functions: \( \mathbf{v} \in V \), where

\[
V \equiv \left\{ \mathbf{v} \in W^{1,p}(\Omega(t))^2 : v_1|_{\Gamma_{wall}} = 0, v_2|_{x_2=0} = 0 \right\}
\]

\[
\frac{D^A u}{D t} \quad \text{ALE derivative} \quad \mathbf{w} := \frac{\partial \mathbf{x}}{\partial t} \quad \text{grid velocity}
\]

\[
\frac{D^A u}{D t} := \frac{\partial \mathbf{u}}{\partial t} + \mathbf{w} \cdot \nabla \mathbf{u}(\mathbf{x}, t)
\]

\[
\int_{\Omega(t)} \frac{D^A u}{D t} \mathbf{v} \, d\mathbf{x} + \int_{\Omega(t)} ((\mathbf{u} - \mathbf{w}) \cdot \nabla \mathbf{u}) \mathbf{v} \, d\mathbf{x} -
\]

\[
\int_{\Omega(t)} \rho \text{ div } \mathbf{v} \, d\mathbf{x} + \frac{1}{\rho} \int_{\Omega(t)} 2\mu(|D(\mathbf{u})|)D(\mathbf{u}) : D(\mathbf{v}) \, d\mathbf{x} +
\]

\[
\int_0^L \left( \frac{\partial^2 \eta}{\partial t^2} + b\eta \right) v_2|_{\Gamma_{wall}} \, dx_1 + \int_0^L \left( a \frac{\partial \eta}{\partial x_1} + c \frac{\partial^2 \eta}{\partial t \partial x_1} \right) \frac{\partial (v_2|_{\Gamma_{wall}})}{\partial x_1} \, dx_1
\]

\[
= \int_0^{R_0} P_{in} v_1|_{x_1=0} \, dx_2, \quad \forall \mathbf{v} \in V, \ a.e. \ t \in I.
\]
• with G. Rusnáková
  • weak formulation of the semi-discrete coupled problem
    • $p \geq 2$
    • test with $u^{n+1}$
    • kinematic pressure boundary conditions on infow / outflow
    • symmetry boundary conditions $x_2 = 0$
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Convergence study
Energy estimate for $\xi^{n+1/2} := u_2^{n+1}|_{\Gamma_{wall}}$

\[
\frac{1}{2} \| u^{n+1} \|_{L^2(\Omega(t_{n+1}))}^2 + \frac{c^*}{2\rho} \Delta t \| u^{n+1} \|_{W^{1,2}(\Omega(t_{n+1}/2))}^2 \\
+ \frac{1}{2} \| \xi^{n+1/2} \|_{L^2(0,L)}^2 + c\Delta t \| \xi^{n+1/2} \|_{L^2(0,L)}^2 \leq \\
\leq \frac{1}{2} \| u^n \|_{L^2(\Omega(t_n))}^2 + \frac{1}{2} \| \xi^n \|_{L^2(0,L)}^2 + c\Delta t \| P_{in}^{n+1} \|_{L^2(0,L)}^2
\]

- we obtain the following estimate for the A-operator

\[
\| \xi^{n+1/2} \|_{L^2(0,L)}^2 + \| u^{n+1} \|_{L^2(\Omega(t_{n+1}))}^2 \\
\leq \| \xi^n \|_{L^2(0,L)}^2 + \| u^n \|_{L^2(\Omega(t_n))}^2 + 2c\Delta t \| P_{in}^{n+1} \|_{L^2(0,L)}^2
\]
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Convergence study
Energy of the solution

Energy of A-operator:

\[
\|\xi^{n+\frac{1}{2}}\|_{L^2(0,L)}^2 \leq \|\xi^n\|_{L^2(0,L)}^2 + \|u^n\|_{L^2(\Omega(t_n))}^2 - \|u^{n+1}\|_{L^2(\Omega(t_{n+1}))}^2 + 2c\Delta t|P_{\text{in}}^{n+1}|^2
\]

Energy of B-operator:

\[
\begin{align*}
a \|\eta^{N+1}_{x_1}\|_{L^2(0,L)}^2 &+ b \|\eta^{N+1}\|_{L^2(0,L)}^2 + \|\xi^{N+1}\|_{L^2(0,L)}^2 \\
&= a \|\eta^0_{x_1}\|_{L^2(0,L)}^2 + b \|\eta^0\|_{L^2(0,L)}^2 + \|\xi^0\|_{L^2(0,L)}^2 \\
&+ \sum_{n=0}^{N} \left( \|\xi^{n+\frac{1}{2}}\|_{L^2(0,L)}^2 - \|\xi^n\|_{L^2(0,L)}^2 \right)
\end{align*}
\]
Energy of the solution

Energy of A-operator:

\[ \left\| \xi^{n+1/2} \right\|_{L^2(0,L)}^2 \leq \left\| \xi^n \right\|_{L^2(0,L)}^2 + \left\| u^n \right\|_{L^2(\Omega(t_n))}^2 - \left\| u^{n+1} \right\|_{L^2(\Omega(t_{n+1}))}^2 + 2c\Delta t \left| P_{in}^{n+1} \right|^2 \]

Energy of B-operator:

\[ a \left\| \eta_{x_1}^{N+1} \right\|_{L^2(0,L)}^2 + b \left\| \eta^{N+1} \right\|_{L^2(0,L)}^2 + \left\| \xi^{N+1} \right\|_{L^2(0,L)}^2 \]
\[ = a \left\| \eta_{x_1}^0 \right\|_{L^2(0,L)}^2 + b \left\| \eta^0 \right\|_{L^2(0,L)}^2 + \left\| \xi^0 \right\|_{L^2(0,L)}^2 \]
\[ + \sum_{n=0}^{N} \left( \left\| \xi^{n+1/2} \right\|_{L^2(0,L)}^2 - \left\| \xi^n \right\|_{L^2(0,L)}^2 \right) \]

Energy estimate of the solution:

\[ E^{N+1} \leq E^0 + \sum_{n=0}^{N} 2c\Delta t \left| P_{in}^{n+1} \right|^2 \]

\[ E^k = \left\| u^k \right\|^2_{L^2(\Omega(t_k))} + a \left\| \eta_{x_1}^k \right\|^2_{L^2(0,L)} + b \left\| \eta^k \right\|^2_{L^2(0,L)} + \left\| \xi^k \right\|^2_{L^2(0,L)} \]
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Convergence study
Wall Shear Stress

\[ WSS := \tau_w = -T_f n \cdot \tau, \quad (1) \]

negative values of the WSS - indicate existence of large recirculation zones

Oscillatory Shear Index

\[ OSI := \frac{1}{2} \left( 1 - \frac{\int_0^T \tau_w \, dt}{\int_0^T |\tau_w| \, dt} \right), \quad (2) \]

measures temporal oscillations of the shear stress pointwisely - indicates areas with large stenotic plug danger

Reynolds number

\[ RE_0 = \frac{\rho Vl}{\mu_0} \quad \text{or} \quad RE_\infty = \frac{\rho Vl}{\mu_\infty} \quad \text{or} \quad RE = \frac{\rho Vl}{\frac{1}{2}(\mu_0 + \mu_\infty)} \]
hemodynamical parameters

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✓ **Oscillatory Shear Index**

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inflow velocity

pulsatile parabolic profile

\[ u_{inflow}(-L, x_2) = V(1 - x_2)(1 + x_2) \sin^2 \left( \frac{\pi t}{\omega} \right) \quad \text{on} \quad \Gamma_{in}, \]

or some \( f(t) \), e.g., iliac artery measurements
Inflow data

inflow velocity

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or some \( f(t) \), e.g., iliac artery measurements

\[
Q(t) = \int_{\Gamma_{\text{in}}} u_{\text{inflow}}(x_2) \, dx \]

\[
\Rightarrow f(t) = Q(t) / 3^{4/5} VR(t)
\]
Inflow data

inflow velocity

pulsatile parabolic profile

\[ u_{\text{inflow}}(-L, x_2) = V(1 - x_2)(1 + x_2)\sin^2\left(\frac{\pi t}{\omega}\right) \quad \text{on} \quad \Gamma_{\text{in}}, \]

or some \( f(t) \), e.g., iliac artery measurements

\[
Q(t) = -\int_{\Gamma_{\text{in}}} u_{\text{inflow}} \, dx_2 \quad \Rightarrow \quad f(t) = Q(t)\frac{3}{4VR(t)}
\]
Some hemodynamical quantities

domain deformation: stenosed and straight channel

WSS-stenosed channel

OSI

M.L. & A. Hundertmark [Comp. & Math. Appl. ('10)]

Lukáčová (AG Numerik)

Modelling in Hemodynamics
Some hemodynamical quantities

WSS-stenosed channel

\[ WSS-stenosed \text{ channel} \]

\[ \text{dyne cm}^{-2} \]

- \[ \ldots \text{NS} \]
- \[ \text{Carreau, } q = -0.32 \]
- \[ \text{Yeleswarapu} \]
- \[ \text{Carreau, } q = -10 \]

\[ \text{time } 0.36 \text{ s} \]

\[ \text{cm} \]

Lukáčová (AG Numerik)  Modelling in Hemodynamics
Some hemodynamical quantities

M.L. & A. Hundertmark [Comp.& Math.Appl. ('10)]
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1. Effects due to complex geometry:
   wall deformation, extrema of WSS, OSI in stenosed /
   bifurcation regions

2. Effects due to the Non-Newtonian rheology:
   **WSS**: larger negative absolute value \( \Rightarrow \) recirculation zones,
   **OSI**: larger extremes \( \Rightarrow \) more sensible prediction of
   stenotic plug

3. Difference between Carreau and Yeleswarapu model:
   negligible
Effects due to complex geometry:
wall deformation, extrema of WSS, OSI in stenosed / bifurcation regions

Effects due to the Non-Newtonian rheology:
WSS: larger negative absolute value $\Rightarrow$ recirculation zones,
OSI: larger extremes $\Rightarrow$ more sensible prediction of stenotic plug

Difference between Carreau and Yeleswarapu model: negligible

FSI & kinematical coupling: efficient, stable FSI algorithm