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独立行政法人日本学術振興会 日独共同大学院プログラム
JSPS-DFG Japanese-German Graduate Externship

第 8 回 日独流体数学国際研究集会
The 8th Japanese-German International Workshop on Mathematical Fluid Dynamics

June 17 - 20, 2013 at Waseda University, Nishi-Waseda Campus

63 Bldg. 2nd Floor - 03 Conference Room

■ Program ■

	Mon, June 17	Tue, June 18	Wed, June 19	Thu, June 20
9:30-10:40	Hi Jun CHOE ① (Yonsei Univ.)	Hi Jun CHOE ②	Hi Jun CHOE ③	Hi Jun CHOE ④
11:00-12:10	Juergen SAAL ① (TU Darmstadt)	Juergen SAAL ②	Juergen SAAL ③	Juergen SAAL ④
12:30-13:00	Tatsuo IGUCHI (Keio Univ.)	Teppei KOBAYASHI (Meiji Univ.)	Matthias GEISSERT (TU Darmstadt)	Hiroya ITO (Univ. of Electro-Communications)
13:00-15:00	Lunch break			
15:00-16:10	Yasushi TANIUCHI ① (Shinshu Univ.)	Yasushi TANIUCHI ②	Yasushi TANIUCHI ③	Yasushi TANIUCHI ④
16:40-17:10	Tsukasa IWABUCHI (Chuo Univ.)	Tohru OZAWA (Waseda Univ.)	Reinhard FARWIG (TU Darmstadt)	Erika USHIKOSHI (Tamagawa Univ.)
17:30-17:45	Lorenz von BELOW (TU Darmstadt)	Hana MIZEROVA (Univ. of Mainz)	Jonas SAUER (TU Darmstadt)	
17:50-18:05	Moritz REINHARD (TU Darmstadt)	Bangwei SHE (Univ. of Mainz)	Katharina SCHADE (TU Darmstadt)	＊18：00～ Reception
18:10-18:25	Hirokazu SAITO (Waseda Univ.)		Miho MURATA (Waseda Univ.)	

●→Main-course ●→30min talk ●→15min talk

Discussion room → open 9:30 - 17:00 63 Bldg. 1F
Meeting room for Mathematics and Applied Mathematics

Main-Course

Hi Jun CHOE

Yonsei University, Seoul

Title:

Regularity question of the incompressible Navier-Stokes equations

Abstract:

We are interested in the regularity questions of the Navier-Stokes equations. First we study the existence of Leray-Hopf solutions and the suitable weak solutions. We introduce Galerkin method and related compactness in solution spaces for Leray-Hopf solution, and approximation of nonlinearity and its convergence in Lesgues-Sobolev spaces and their embeddings.

For the regularity method, we first look over history including minimal surface, harmonic map, elliptic and parabolic systems and analyzed the main steps. Then, we move to partial regularity of the Navier-Stokes equations and introduce iteration scheme to get one dimensional Hausdorff measure. To improve Hausdorff measure by logarithmic factor, we follow higher integrability technique of reverse Holder for Ap weight.

Finally, we consider various known sresults including self similarity, scaling and vorticity direction. In particular, Serrin class, Constantin-Fefferman's vorticity direction and axis symmetric flow will be discussed.

Date:

① Monday, June 17 9:30-10:40 ② Tuesday, June 18 9:30-10:40

③ Wednesday, June 19 9:30-10:40 ④ Thursday, June 20 9:30-10:40

Juergen SAAL

Technical University of Darmstadt, Darmstadt

Title:

Some Progress on Fluid Dynamics in Singular Domains

Abstract:

Objective of this mini course is to present recent progress on models arising in fluid dynamics in non-smooth domains. Fluid flow in singular domains has quite some significance in applications. For instance, models of cyclones, such as hurricanes or tornados, are considered in a cylinder which is, by the presence of edges, a so-called weakly singular domain. Another important application are wetting and de-wetting phenomena. Here in general three-phase dynamic contact lines (gas/fluid/solid, fluid/fluid/solid, etc.) appear, which at the end lead to

systems of equations on domains having at most a Lipschitz boundary. In spite of their significance there exist still fundamental open problems concerning both, modeling and rigorous mathematical treatment. The aim of the course is not to give a full solution to all these interesting but also very difficult problems, but to introduce concepts which might lead to a better understanding of the difficulties and to present some new results for somewhat less intricate related fluid models.

Within the course we will - as far as possible on an elementary level - introduce and develop methods and techniques suitable for this purpose and different from classical methods merely working for smooth regions. In this context we will discuss, e.g., the (non-commuting) operator sum method and off-diagonal estimates. Based on these concepts we derive L^p maximal regularity for the linerized Stokes system subject to partial slip type boundary conditions. We will prove this basic result in optimal regularity classes for the case of cylindrical domains, wedges, and general (graph) Lipschitz domains. A basic ingredient here also is the fact that for perfect slip boundary conditions the Stokes equations reduce to a standard parabolic system. The results then extend to a more general class of partial slip boundary conditions by employing perturbation arguments to the corresponding reduced Stokes system.

Then we will demonstrate how the maximal regularity applies in order to prove well-posedness and stability of related non-linear Navier-Stokes type systems. In particular, we will obtain this type results for some of the models important for the applications mentioned above.

Date:

① Monday, June 17 11:00-12:10 ② Tuesday, June 18 11:00-12:10
③ Wednesday, June 19 11:00-12:10 ④ Thursday, June 20 11:00-12:10

Yasushi TANIUCHI

Shinshu University, Matsumoto

Title:

Uniqueness of mild solutions to the Navier-Stokes equations in unbounded domains

Abstract:

We discuss uniqueness of mild solutions bounded on the whole time axis to the Navier-Stokes equations in 3-dimensional unbounded domains. For example, we consider time-periodic and almost periodic-in-time solutions. Thus far, uniqueness of almost periodic-in-time solutions to the Navier-Stokes equations in unbounded domains, roughly speaking, is known only for a small almost periodic-in-time solution in $BC(R; L^3_w)$ within the class of solutions which have sufficiently

small $L^\infty(L^3_w)$-norm. We show that a small almost periodic-in-time solution in $BC(R; L^3_w \cap L^{6,2})$ is unique within the class of all almost periodic-in-time solutions in $BC(R; L^3_w \cap L^{6,2})$. Here $L^{p,q}$ denotes the Lorentz space. We also consider uniqueness of mild solutions u with precompact range $\{u(t); t \in \mathbb{R}\}$ in L^3_w.

The proof of our uniqueness theorems are based on the method of dual equations.

Date:

① Monday, June 17 15:00-16:10
② Tuesday, June 18 15:00-16:10
③ Wednesday, June 19 15:00-16:10
④ Thursday, June 20 15:00-16:10

30 minutes talks

Reinhard FARWIG

Technical University of Darmstadt, Darmstadt

with Hermann Sohr (Univ. Paderborn)

with Werner Varnhorn (Univ. Kassel)

Title:

Besov space regularity conditions for weak solutions of the Navier-Stokes equations

Abstract:

Consider a bounded domain $\Omega \subseteq \mathbb{R}^3$ with smooth boundary, an initial value $u_0 \in L^2_\sigma(\Omega)$, and a weak solution u of the Navier-Stokes system in $[0,T) \times \Omega$ with vanishing external force. Our aim is to develop regularity conditions for u which are based on the Besov space

$$\mathcal{B}^{q,s}(\Omega) := \left\{ v \in L^2_\sigma(\Omega);\ \|v\|_{\mathcal{B}^{q,s}(\Omega)} = \left(\int_0^\infty \|e^{-\tau A} v\|_q^s \, d\tau \right)^{1/s} < \infty \right\},$$

$2 < s < \infty$, $3 < q < \infty$, $\frac{2}{s} + \frac{3}{q} = 1$, where A means the Stokes operator. $\mathcal{B}^{q,s}(\Omega)$ has been introduced in [1,2] concerning regularity properties for u; it coincides with the well known Besov space $\mathbb{B}^{-2/s}_{q,s}(\Omega)$ of solenoidal vector fields in $B^{-2/s}_{q,s}(\Omega)$. Further, $\mathcal{B}^{q,s}(\Omega)$ coincides with the space $\mathcal{B}^{q,s}_\delta(\Omega)$ which is obtained replacing $\|v\|_{\mathcal{B}^{q,s}(\Omega)}$ by $\|v\|_{\mathcal{B}^{q,s}_\delta(\Omega)} = \left(\int_0^\delta \|e^{-\tau A} v\|_q^s d\tau \right)^{1,s}$, $\delta > 0$.

Our results extend regularity properties proved in [1,2]: E.g., the property $u \in L^\infty_{\text{loc}}([0,T); \mathcal{B}^{q,s}(\Omega))$ together with $\lim_{\delta \to 0} \|u\|_{L^\infty(0,S;B^{q,s}_\delta(\Omega))} = 0$ for each $0 < S < T$ is sufficient for regularity. Further, the property $u \in C([0,T); \mathcal{B}^{q,s}(\Omega))$ or the property $u \in L^\infty(0,T; \mathcal{B}^{q,s}_\delta(\Omega))$ with sufficiently small norm is sufficient.

The weak solution u is unique if $u \in L^\infty_{\text{loc}}([0,T); \mathcal{B}^{q,s}(\Omega))$ and satisfies the *strong energy inequality* on $[t_0, T)$ for *each* (!) $t_0 \in [0,T)$.

References:

[1] R. Farwig, H. Sohr, W. Varnhorn: *On optimal initial value conditions for local strong solutions of the Navier-Stokes equations.* Ann. Univ. Ferrara **55**, 89–110 (2009)

[2] R. Farwig, H. Sohr, W. Varnhorn: *Extensions of Serrin's uniqueness and regularity conditions for the Navier-Stokes equations.* J. Math. Fluid Mech. **14**, 529–540 (2012)

[3] R. Farwig, H. Sohr, W. Varnhorn: *Besov space regularity conditions for weak solutions of the Navier-Stokes equations.* TU Darmstadt, Preprint (2013)

Date:

Wednesday, June 19 16:40-17:10

Matthias GEISSERT

Technical University of Darmstadt, Darmstadt

with M. Nesensohn (TU Darmstadt)

with Y. Shibata (Waseda Univ.)

Title:

On global L^p-solutions for a fluid model of Oldroyd kind

Abstract:

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a sufficiently smooth boundary. We consider the following system describing the flow of a fluid of Oldroyd kind in a bounded domain $\Omega \subset \mathbb{R}^n$

$$(1)\quad \begin{cases} \rho(\partial_t u + u.\nabla u) - \alpha\Delta u + \nabla\pi = \beta \operatorname{Div}\tau & \text{in } (0,\infty)\times\Omega, \\ \operatorname{div} u = 0 & \text{in } (0,\infty)\times\Omega, \\ \partial_t\tau + u.\nabla\tau + \gamma\tau = \delta E u + g(\nabla u, \tau) & \text{in } (0,\infty)\times\Omega, \\ u = 0 & \text{on } (0,\infty)\times\partial\Omega, \\ u(0) = u_0 & \text{in } \Omega, \\ \tau(0) = \tau_0 & \text{in } \Omega. \end{cases}$$

Here, the function $u\colon [0,\infty)\times\Omega \to \mathbb{R}^n$ is the Eulerian velocity field, $\pi\colon [0,\infty)\times\Omega \to \mathbb{R}$ is the pressure, and $\tau\colon [0,\infty)\times\Omega \to \mathbb{R}^{n\times n}$ is the elastic part of the stress tensor. The symmetric part of the gradient is denoted by $Eu = \frac{1}{2}(\nabla u + \nabla u^T)$. Given are the constants $\rho, \alpha > 0$, $\beta, \delta > 0$, $\gamma \geq 0$ and the function

$$g(\nabla u, \tau) = \tau W u - W u \tau - a(\tau E u + E u \tau),$$

where $a \in [-1,1]$ and $Wu = \frac{1}{2}(\nabla u - \nabla u^T)$ denotes the antisymmetric part of the gradient.

We show existence and uniqueness of global strong solutions in the L_p-setting for small initial values $u_0 \in B^{2-2/p}_{q,p}(\Omega)$ and $\tau_0 \in H^1_{q,}(\Omega)$, where $1 < p, q < \infty$ satisfy $q > n$ and $\frac{1}{p} + \frac{n}{2q} < 1$. In the case $\gamma > 0$ we show that the solution converges exponentially fast to zero. This is not the case for $\gamma = 0$, since system (1) has non-trivial solutions. But we can show that even in this case the solution converges to some equilibrium of (1).

To prove our main result, we transform (1) to Lagrangian coordinates and discuss the *Oldroyd operator*

$$\mathcal{A}_\gamma = \begin{pmatrix} \alpha\rho^{-1}A_q & -\beta\rho^{-1}P_q \operatorname{Div} \\ -\delta E & \gamma \end{pmatrix}$$

where A_q denotes the Stokes operator and P_q the Helmholtz projection. We show that this operator matrix admits maximal L_p-L_q regularity and that the spectrum of $\mathcal{A}_\gamma$ is mainly determined by γ. More precisely, 0 is in the *resolvent* of $\mathcal{A}_\gamma$ if $\gamma > 0$, and 0 is in the spectrum of $\mathcal{A}_0$ if $\gamma = 0$. To prove global existence in the case

$\gamma = 0$, we improve the linear theory and show a maximal regularity result in spaces of non-decaying functions in time. A fixed point argument will then conclude the proof.

Date:

Wednesday, June 19 12:30-13:00

Tatsuo IGUCHI
Keio University, Yokohama

Title:

Solvability of the initial value problem to a model system for water waves

Abstract:

The water wave problem is mathematically formulated as a free boundary problem for an irrotational flow of an inviscid and incompressible fluid under the gravitational field. The basic equations for water waves are complicated due to the nonlinearity of the equations together with the presence of an unknown free surface. Therefore, until now many approximate equations have been proposed and analyzed to understand natural phenomena for water waves. Famous examples of such approximate equations are the shallow water equations, the Green–Nagdhi equations, Boussinesq type equations, the Korteweg–de Vries equation, the Kadomtsev–Petviashvili equation, the Benjamin–Bona–Mahony equation, the Camassa–Holm equation, the Benjamin–Ono equations, and so on. All of them are derived from the water wave problem under the shallowness assumption of the water waves, which means that the mean depth of the water is sufficiently small compared to the typical wavelength of the water surface.

On the other hand, it is well-known that the water wave problem has a variational structure. In fact, J. C. Luke (1967) gave a Lagrangian in terms of the velocity potential and the surface variation, and showed that the corresponding Euler–Lagrange equations are the basic equations for water waves. M. Isobe (1994) and T. Kakinuma (2000) derived model equations for water waves without any shallowness assumption. The model equations are the Euler–Lagrange equations to an approximated Lagrangian, which is obtained by approximating the velocity potential in Luke's Lagrangian.

In this talk, we consider one of the model equations and report the solvability of the initial value problem. The model are nonlinear dispersive equations and the hypersurface $t = 0$ is characteristic for the model equations. Therefore, the initial data have to be restricted in an infinite dimensional manifold in order to the existence of the solution, and we show that the manifold is invariant under the time evolution.

This talk is based on the joint research with my former student Yuuta Murakami.

Date:

Monday, June 17 12:30-13:00

Hiroya ITO

The University of Electro-Communications, Tokyo

Title:

Some remarks on generalized Korn's inequalities

Abstract:

Korn's inequality, well-known for its fundamental roll in dealing with various equations appearing in continuum mechanics, asserts that there exists a constant $C > 0$ such that

$$\|\varepsilon(u)\|^2_{L^2(\Omega)} + \|u\|^2_{L^2(\Omega)} \geq C\,\|\nabla u\|^2_{L^2(\Omega)}$$

for all H^1 vector fields u on a bounded Lipschitz domain Ω in $\mathbb{R}^n$, where $\varepsilon(u)$ represents the symmetric part of the gradient ∇u of u. We generalize Korn's inequality by replacing $\varepsilon(u)$ with a first-order homogeneous partial differential system in general form with constant coefficients for vector-valued functions; generalization yields not only inequalities applicable to continuum mechanics but also those applicable to differential geometry and general relativity. From J. Nečas's work we know some necessary and sufficient conditions (on the coefficients) for such an inequality to hold. The main purpose of this talk is to present a new necessary and sufficient condition, which is in general easier to check than the known conditions. Although the establishment of the new condition is not so easy in the sense that it relies on a certain tool in algebraic geometry, the condition gives us a lot of useful information about what are vague in Nečas's argument.

Date:

Thursday, June 20 12:30-13:00

Tsukasa IWABUCHI

Chuo University, Tokyo

Title:

Global solutions for the Navier-Stokes equations in the rotational framework

Abstract:

We consider the Cauchy problems for the Navier-Stokes equations with the Coriolis force in the homogeneous Sobolev spaces $\dot{H}^s(\mathbb{R}^3)$ and show the existence of

unique global solutions. Without the Coriolis force, it is known that the unique global solutions are obtained if the initial data is sufficiently small in the Sobolev space $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$. The regularity index 1/2 is scaling critical case from the view point of scaling invariant property for the Navier-Stokes equations without the Coriolis force. In this talk, the unique global solutions are obtained for large initial data if the speed of rotation is sufficiently large. In particular, we show the sufficient condition for the existence of global solutions by the use of the norm of initial data in $\dot{H}^s(\mathbb{R}^3)$ and the speed of the rotation in the case $s > 1/2$. In the case $s = 1/2$, the sufficient condition is not characterized by the norm of initial data but is given by the precompact sets of $\dot{H}^{\frac{1}{2}}(\mathbb{R}^3)$.

Date:

Monday, June 17 16:40-17:10

Teppei KOBAYASHI

Meiji University, Tokyo

Title:

Jeffery-Hamel's flows in the plane

Abstract:

Jeffery-Hamel's flow is an exact solution of the Navier-Stokes equations in a domain between two inclined lines. L. Rosenhead [1] treated such a problem. The solutions are given in the Jacobian elliptic functions and the behavior of the solutions is investigated.

In this talk, it is shown that in the domain there exists a solution of the Navier-Stokes equations applying the relation between the flux constant and the amplitude of the two lines.

References:

[1] L. Rosenhead, *The steady two-dimensional radial flow of viscous fluid between two inclined plane walls*, Proc. R. Soc. Lond. A, **175** (1940), 436-467.

Date:

Tuesday, June 18 12:30-13:00

Tohru OZAWA

Waseda University, Tokyo

with Shuji Machihara (Saitama Univ.)
with Hidemitsu Wadade (Gifu Univ.)

Title:

Hardy type inequalities on balls

Abstract:

This talk is based on my recent joint-work with Shuji Machihara and Hidemitsu Wadade. We revisit the Hardy inequalities on balls with radius R at the origin in $\mathbb{R}^n$ with $n \geq 2$. We describe how the behavior of functions on the boundary affects the Hardy type inequalities. A special attention is paid on the case $n = 2$ with logarithmic correction.

References:

[1] O. A. Ladyzhenskaya, *The mathematical theory of viscous incompressible flow, Second English edition, revised and enlarged. Translated from the Russian by Richard A. Silverman and John Chu. Mathematics and its Applications, Vol. 2*, Gordon and Breach, Science Publishers, New York-London-Paris, (1969).

[2] J. Leray, *Etude de diverses équations integrales non linéaires et de quelques problèmes que pose l'hydrodynamique*, J. Math. Pures Appl. **12** (1933), 1–82.

[3] S. Machihara, T. Ozawa and H. Wadade, *Hardy type inequalities on balls*, Tohoku Math. J. (in press)

Date:

Tuesday, June 18 16:40-17:10

Erika USHIKOSHI

Tamagawa University, Tokyo

Title:

New approach to the Hadamard variational formula for the Green function of the Stokes equations

Abstract:

We consider the stationary Stokes equations in the bounded domain $\Omega \subset \mathbb{R}^d$. The purpose of this talk is to give a new proof of the first variation of the Hadamard variational formula for the velocity and the pressure, and establish the second variation for them. More precisely, for every $\varepsilon > 0$, we define a family $\{\Omega_\varepsilon\}_{\varepsilon \geq 0}$ of perturbed domains defined by $\overline{\Omega}_\varepsilon = \Phi_\varepsilon(\Omega)$ with vol.$\Omega_\varepsilon =$ vol.Ω, where Φ_ε is the volume preserving diffeomorphism from the domain $\overline{\Omega}$ to $\overline{\Omega}_\varepsilon$. We consider the Green function of the Stokes equations which satisfy

$$\begin{cases} \Delta_x \boldsymbol{G}_{\varepsilon,n}(x,y) - \nabla_x R_{\varepsilon,n}(x,y) = e_n \delta(x-y), & (x,y) \in \Omega_\varepsilon \times \Omega_\varepsilon, \\ \operatorname{div} \boldsymbol{G}_{\varepsilon,n}(x,y) = 0, & (x,y) \in \Omega_\varepsilon \times \Omega_\varepsilon, \\ \boldsymbol{G}_{\varepsilon,n}(x,y) = 0, & x \in \partial\Omega_\varepsilon, y \in \Omega_\varepsilon \end{cases}$$

for $n = 1, \cdots, d$, where $\boldsymbol{G}_{\varepsilon,n}(x,y) = \{G^i_{\varepsilon,n}(x,y)\}_{i,n=1,\cdots,d}$ is the Green matrix for the velocity, $R_{\varepsilon,n}(x,y) = \{R_{\varepsilon,n}(x,y)\}_{n=1,\cdots,d}$ is the Green function for the pressure

and $\{e_n\}_{n=1,\cdots,d}$ denotes a canonical basis in $\mathbb{R}^d$. Under the volume preserving perturbation, we give a more refined proof of the the first variation formulas for the velocity $\delta\boldsymbol{G}$ and the pressure δR as $\delta G(x,y) := \lim_{\varepsilon\to 0}\frac{G_\varepsilon(x,y)-G(x,y)}{\varepsilon}$, $\delta R(x,y) := \lim_{\varepsilon\to 0}\frac{R_\varepsilon(x,y)-R(x,y)}{\varepsilon}$, and we derive the second variation of the Green function for the Stokes equations $\delta^2 G$ and $\delta^2 R$ as $\delta^2 G(y,z) := \lim_{\varepsilon\to 0}\frac{G_\varepsilon(y,z)-G(y,z)-\varepsilon\delta G(y,z)}{\varepsilon^2}$, $\delta^2 R(y,z) := \lim_{\varepsilon\to 0}\frac{R_\varepsilon(y,z)-R(y,z)-\varepsilon\delta R(y,z)}{\varepsilon^2}$. Our method gives a new systematic proof of the Hadamard variational formula, which enables us to deal with the higher derivatives with respect to the perturbation of domains.

Date:

Friday, June 20 16:40-17:10

15 minutes talks

Lorenz von BELOW

Technical University of Darmstadt, Darmstadt

Title:

The spin coating system in the singular limit of vanishing surface tension

Abstract:

In [1] Denk, Geißert, Hieber, Saal, and Sawada studied a model for the spin coating process. The model they proposed is a one-phase free boundary value problem for a Newtonian fluid and it is essentially given by the following set of equations:

$$\begin{cases} \partial_t u + u \cdot \nabla u - \Delta u + \nabla \theta = f, \quad \operatorname{div} u = 0 & \text{in } \Omega(t) \\ S(u,\theta)\nu - \sigma\kappa\nu = 0 & \text{on } \Gamma_+(t) \\ V = u \cdot \nu & \text{on } \Gamma_+(t) \\ u = 0 & \text{on } \Gamma_- \\ u(0) = u_0 & \text{in } \Omega(0) \end{cases} \tag{1}$$

for $t > 0$, where $\Omega(t)$ is a moving domain close to an infinite layer $\mathbb{R}^2 \times (0, h)$, with fixed lower boundary Γ_- and moving upper boundary $\Gamma_+(t)$ parametrised by an unknown function η. Here u denotes the velocity field, θ the pressure, $\sigma > 0$ the surface tension parameter, κ the mean curvature, $S(u, \theta)$ the stress tensor and V the velocity of the upper boundary in normal direction.

In this talk we present results concerning the influence of the surface tension parameter σ on solutions of (1) and its linearisation, in particular in the singular limit $\sigma \to 0$ of vanishing surface tension. Our results rely to a large extent on a careful analysis of an explicit solution formula for the linearised problem.

References:

[1] Robert Denk, Matthias Geissert, Matthias Hieber, Jürgen Saal, and Okihiro Sawada. *The spin-coating process: Analysis of the free boundary value problem. Communications in Partial Differential Equations*, **36**(7):1145–1192, 2011.

Date:

Monday, June 17 17:30-17:45

Hana MIZEROVA

Johannes Gutenberg University Mainz, Mainz

with Maria Lukacova (Mainz Univ.)

Title:

Error estimates for the Peterlin viscoelastic model

Abstract:

We consider the equation of a motion of an incompressible viscoelastic fluid

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = \eta \Delta \mathbf{v} + \text{div } \mathbf{T} - \nabla p$$
$$\text{div } \mathbf{v} = 0$$

on a bounded smooth domain $\Omega \subset \mathbb{R}^3$. The elastic stress tensor $\mathbf{T}$ can be expressed using the conformation tensor $\mathbf{C}$ in the following way

$$\mathbf{T} = \text{tr } \mathbf{C}.\mathbf{C},$$

where $\mathbf{C}$ is symmetric positive-definite tensor, which satisfies the equation of the form:

$$\frac{\partial \mathbf{C}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{C} - (\nabla \mathbf{v})\mathbf{C} - \mathbf{C}(\nabla \mathbf{v})^T = \text{tr } \mathbf{C}.\mathbf{I} - (\text{tr } \mathbf{C})^2 \, \mathbf{C} + \epsilon \Delta \mathbf{C}.$$

We impose homogenous Dirichlet condition for the velocity $\mathbf{v} = \mathbf{0}$ and no-flux condition for the conformation tensor $\frac{\partial \mathbf{C}}{\partial \mathbf{n}} = \mathbf{0}$ on the boundary $\partial\Omega$. We consider $\mathbf{v}_0$ and $\mathbf{C}_0$ to be the enough smooth initial data.

The existence of the weak solution

$$\mathbf{v} \in \mathbf{L^2}(0,T; \mathbf{H}^1_{0,div}(\Omega)) \cap \mathbf{L}^\infty(0,T; \mathbf{L}^2_{div}(\Omega))$$

$$\mathbf{C} \in \mathbf{L^2}(0,T; \mathbf{H^1}(\Omega)) \cap \mathbf{L}^\infty(0,T; \mathbf{L^2}(\Omega))$$

is proven using the Galerkin method and the classical energy estimates. Our aim is to analyze the error of a suitable approximation. We use a conforming finite element discretization. For sufficiently smooth solution and its approximation by $\mathcal{P}^k$ elements we get the order of convergence h^k, where h is a mesh size.

Date:

Tuesday, June 18 17:30-17:45

Miho MURATA

Waseda University, Tokyo

Title:

On the sectorial $\mathcal{R}$-boundedness of the Stokes operator for the compressible viscous fluid flow and its application

Abstract:

We consider the initial boundary value problem of the Stokes equations for the compressible viscous fluid flow with slip boundary condition in a bounded domain. In order to consider a global in time unique existence theorem for a nonlinear problem with some initial data close to a constant state in a bounded domain, the exponential stability of solutions to the Stokes equations is required. In this talk, we prove the exponential stability employing the sectorial $\mathcal{R}$-boundedness of the Stokes operator and a homotopic argument.

Date:

Wednesday, June 19 18:10-18:25

Moritz REINHARD

Technical University of Darmstadt, Darmstadt

Title:

Water entry of a flat elastic plate at high horizontal speed

Abstract:

Interactions of elastic structures with the fluid flow are difficult to compute since the elastic deformations of the body are coupled with the hydrodynamic forces. In problems where a body with small deadrise angle impacts onto an undisturbed water surface of infinite depth, hydroelastic effects can be accounted for using an asymptotic hydrodynamic model. In the asymptotic hydrodynamic model? where the fluid is supposed to be inviscid and incompressible ? the boundary conditions on the free surface and on the wetted part of the body are linearised and imposed on the initial equilibrium position of the liquid surface. The positions of the fluid spray-root regions are determined by Wagner's condition. In this talk we consider the problem where an inclined two-dimensional elastic plate enters a water surface at high horizontal speed. Newton's second law and Euler's beam equation subject to free-free boundary conditions govern both the rigid an elastic motions of the plate. The analysis is based on the normal-mode method.

We show that hydrodynamic pressures below atmospheric value can be found in the contact region for the rigid-plate impact. However, elastic vibrations of the plate lead to significant lower pressures. We conclude that elasticity of the plate may promote cavitation of the fluid, and the detachment of the fluid from the solid surface in the low-pressure regions during the early stage of impact.

Date:

Monday, June 17 17:50-18:05

Hirokazu SAITO

Waseda University, Tokyo

Title:

On some decay properties of solutions for the Stokes problem with surface tension

Abstract:

In this talk, we would like to consider decay properties of solutions for the Stokes problem which comes from a free boundary value problem in the half space. We divide the solution into three parts that is the low, middle and high frequency part in the Fourier space. The analysis of the low frequency part is the most important because the Lopatinski diterminant has roots which converge into the origin. We, therefore, concentrate the analysis of the low frequency part in this talk.

Date:

Monday, June 17 18:10-18:25

Jonas SAUER

Technical University of Darmstadt, Darmstadt

Title:

Navier-Stokes Flow in Infinite Cylinders with Non-Constant Cross Section

Abstract:

We investigate maximal regularity in L^q-spaces of the Stokes operator in infinite cylindrical domains of non-constant cross-section. Based on results by Farwig and Ri [1, 2] for the straight cylinder with bounded cross-section of class $C^{1,1}$ we obtain corresponding resolvent estimates also in the case of perturbed cylinders, given usual smallness assumptions on the perturbation. The calculations are carried out in cylindrical coordinates in order to describe the cross-sections in terms of the azimuthal angle and the height of the cylinder. Since there are resolvent estimates available in L^q-spaces even for non-homogeneous divergence, these estimates carry over to the perturbed cylindrical domains by classical perturbation arguments. Finally, we establish a bounded H^∞-calculus in L^q_σ by a general result for the shifted Stokes operator $A_q + c$, $c > 0$, due to Abels [3], implying in particular maximal regularity.

References:

[1] *Farwig, R. and Ri, M.-H.*: The resolvent problem and H^∞-calculus of the Stokes operator in unbounded cylinders with several exits to infinity. *J. Evol. Equ.*, **7**(3) (2007), 497–528.

[2] *Farwig, R. and Ri, M.-H.*: Stokes resolvent systems in an infinite cylinder. *Math. Nachr.*, **280**(9-10) (2008), 1061–1082.

[3] *Abels, H.*: Bounded imaginary powers and H_∞-calculus of the Stokes operator in unbounded domains. In: *Nonlinear elliptic and parabolic problems.* Progress in Nonlinear Differential Equations and Their Applications, *Birkhäuser*, **64** (2005), 1–15.

Date:

Wednesday, June 19 17:30-17:45

Katharina SCHADE

Technical University of Darmstadt, Darmstadt

Title:

L_p-theory for nematic liquid crystal systems

Abstract:

Liquid crystals are matter in a state between liquid and solid. They flow like a liquid, but their molecular structure is similar to that of a solid crystal. In the most common nematic phase, molecules possess an orientation but no positional order. As such, the Ericksen-Leslie model describes the flow of nematic liquid crystals with Navier-Stokes equations coupled with a diffusion equation governing the behaviour of the direction field, see [2].

We consider a simplified Ericksen-Leslie model going back to Lin and Liu in 1995 [3]. We are able to treat the problem in a quasilinear setting, which has not been considered before. We show the local existence of strong solutions using a variant of Clément-Li together with well-known maximal L_p-regularity results. We also investigate stability of equilibria with the generalized principle of linearized stability. We obtain global existence of strong solutions which are eventually bounded on their maximal interval of existence by exploiting the Lyapunov-properties of the system's energy. Furthermore, we obtain real-analyticity of the solution using Angenent's trick.

We give an outlook to a temperature-dependent extension of this model which satisfies the first and second law of thermodynamics.

References:

[1] M. Hieber, M. Nesensohn, J. Prüss and K. Schade, *Dynamics of Nematic Liquid Crystals: The Quasilinear Approach*, submitted.

[2] F.M. Leslie, *Some constitutive equations for liquid crystals*, Arch. Rational Mech. Anal. **28**(1968), 265–283.

[3] F.H. Lin and C. Liu, *Nonparabolic dissipative systems modeling the flow of liquid crystals*, Comm. Pure Appl. Math. **48**(1995), 501–537.

Date:

Wednesday, June 19 17:50-18:05

Bangwei SHE

Johannes Gutenberg University Mainz, Mainz

Title:

Numerical tests on some viscoelastic flows

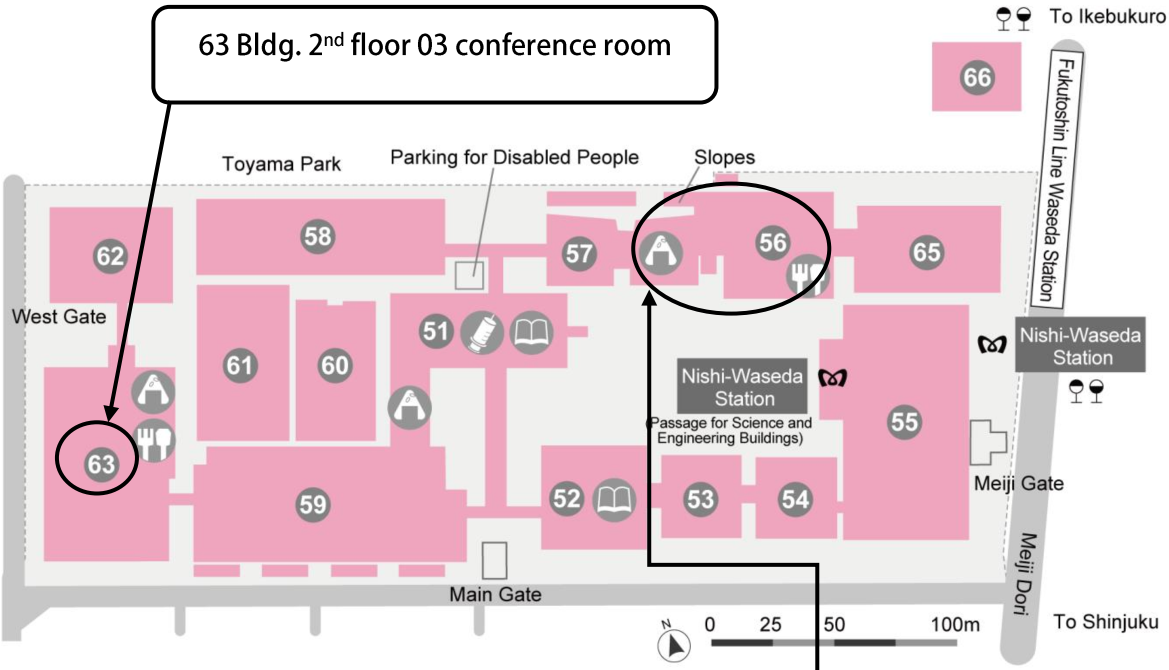
Abstract:

We presented a new combined finite volume and finite difference method for the viscoelastic flow. The high Weissenberg number problem is overcome by using the log-conformation representation. However, the convergence is still an open problem. As a complementary approach we use the kinetic theory to approximate complex viscoelastic stress tensor. The corresponding micro-model based on the Fokker-Plank equation is studied. We present a new multiscale method and illustrate flow behaviour through a series of numerical results.

Date:

Tuesday, June 18 17:50-18:05

Nishi-Waseda Campus Map



Basement floor
Shop and Cafeteria

Organizers : Reinhard FARWIG (TU Darmstadt)
Matthias HIEBER (TU Darmstadt)
Hideo KOZONO (Waseda University)
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