Water entry at high horizontal speed

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IRTG

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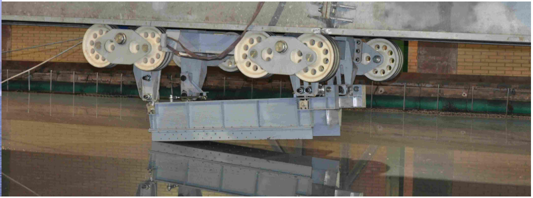
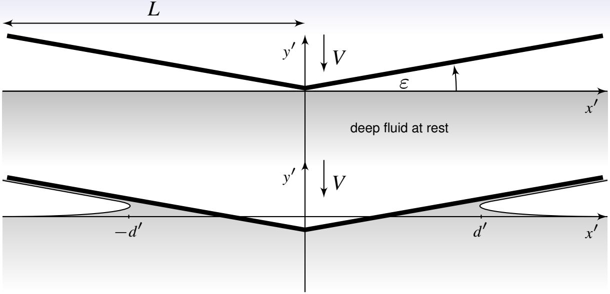
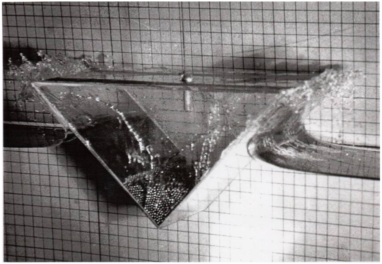
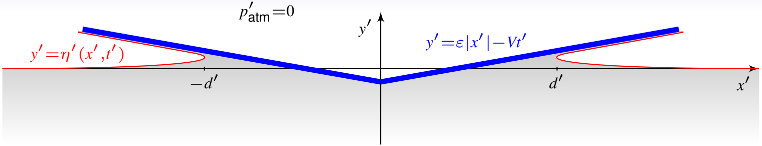


Wagner model for vertical impact





viscosity,
surface tension,
compressibility
are neglected



Nondimensional variables:

$x' = Lx,\ y' = Ly,\ t' = \frac{\varepsilon L}{V}t,\ \varphi' = LV\varphi,\ \eta' = \varepsilon L\eta$

$$\Delta\varphi(x,y,t) = 0 \qquad ((x,y) \in \Omega(t))$$

$$\varphi_y = -1 + \varepsilon\varphi_x\,\mathrm{sgn}(x) \qquad (y = \varepsilon(|x| - t))$$

$$\varphi_y = \eta_t + \varepsilon\eta_x\varphi_x \qquad (y = \varepsilon\eta(x,t))$$

$$\varphi_t = -\frac{\varepsilon}{2}|\nabla\varphi|^2 - \frac{\varepsilon g L}{V^2}y \qquad (y = \varepsilon\eta(x,t))$$

$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty)$$

initial conditions: $\eta(x,0) = 0,\ \varphi(x,y,0) = 0$

Hydrodynamic pressure: $p = -\varphi_t + \frac{\varepsilon}{2}|\nabla\varphi|^2 - \frac{\varepsilon g L}{V^2}y \qquad ((x,y) \in \Omega(t))$

Leading-order hydrodynamic problem for $\varepsilon \ll 1$

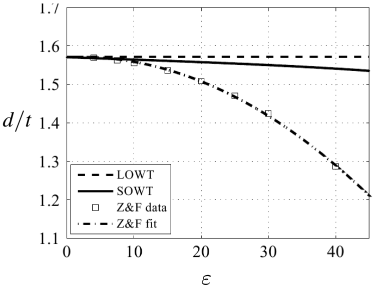
$$\Delta\varphi = 0 \qquad (y < 0)$$
$$\varphi_y = -1 \qquad (y = 0,\ |x| < d)$$
$$\varphi_y = \eta_t \qquad (y = 0,\ |x| > d)$$
$$\varphi_t = 0 \qquad (y = 0,\ |x| > d)$$
$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty)$$
$$p = -\varphi_t \qquad (y = 0\,,\, |x| < d)$$
$$\eta(x, 0) = 0 \qquad (x \in \mathbb{R})$$

Wagner's condition:

$$\eta(d, t) = d - t \qquad (d > 0)$$

$$\varphi = O(1) \qquad ((x, y) \to (\pm d, 0))$$

Solution: $d(t) = \frac{\pi}{2} t$

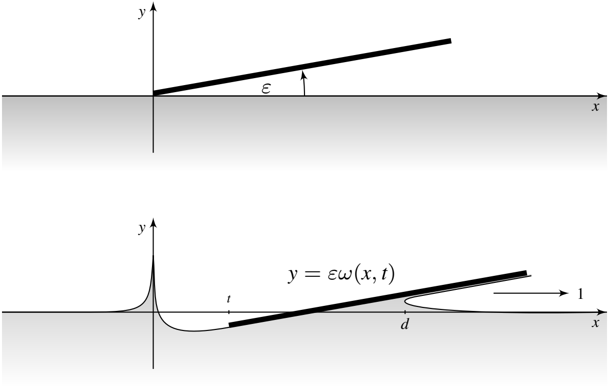


LOWT: Leading order Wagner theory

SOWT: Second order Wagner theory (Oliver, 2007)

Z&F: Numerical solution exploiting self-similarity (Zhao & Faltinsen, 1993)

Oblique impact of a plate



Oblique impact of a plate

$$
\begin{array}{ll}
\Delta\varphi = 0 & (y < 0) \\
\varphi_y = \omega_t(x,t) & (y = 0,\ t < x < d) \\
\varphi = 0 & (y = 0,\ x < 0, x > d) \\
\varphi_x = A(x) & (y = 0,\ 0 < x < t) \\
 & \\
\varphi \to 0 & (x^2 + y^2 \to \infty) \\
\varphi = O(1) & ((x,y) \to (d,0)) \\
|\nabla\varphi| = O(1) & ((x,y) \to (t,0))
\end{array}
$$

where $d = d(t)$ is given by:

$$
\begin{array}{ll}
\omega(d,t) = \eta(d,t) & (d > t) \\
\varphi_y = \eta_t & (y = 0,\ x < t,\ x > d) \\
\eta(x,0) = 0 &
\end{array}
$$

$$
\begin{array}{ll}
\eta(t,t) = \omega(t,t) & \\
p = -\varphi_t & (y = 0,\ t < x < d)
\end{array}
$$

$$
\text{MBVP} \Longrightarrow \int_0^t \sqrt{\frac{d-\xi}{t-\xi}}\, A(\xi)\, \mathrm{d}\xi = \int_t^d \sqrt{\frac{d-\xi}{\xi-t}}\, \omega_t(\xi,t)\, \mathrm{d}\xi
$$

displacement potential: $\Phi(x, y, t) = \int_0^\tau \varphi(x, y, \tau)\,\mathrm{d}\tau$

$$
\begin{aligned}
&\Delta\Phi = 0 && (y < 0) \\
&\Phi_y = \omega(x, t) && (y = 0,\ t < x < d) \\
&\Phi = 0 && (y = 0,\ x < 0 \text{ and } x > d) \\
&\Phi_x = tA(x) + B(x) && (y = 0,\ 0 < x < t)
\end{aligned}
$$

$$
\begin{aligned}
&|\nabla\Phi| = O(1) && ((x, y) \to (t, 0)) \\
&\Phi = O(((x-d)^2 + y^2)^{1/2}) && ((x, y) \to (d, 0)) \\
&\Phi \to 0 && (x^2 + y^2 \to \infty)
\end{aligned}
$$

$$
\int_0^t \sqrt{\tfrac{t-\xi}{d-\xi}}\,(t\,A(\xi) + B(\xi))\,\mathrm{d}\xi = -\int_t^d \sqrt{\tfrac{\xi-t}{d-\xi}}\omega(\xi, t)\,\mathrm{d}\xi
$$

$$
\int_0^t \sqrt{\tfrac{d-\xi}{t-\xi}}\,(t\,A(\xi) + B(\xi))\,\mathrm{d}\xi = \int_t^d \sqrt{\tfrac{d-\xi}{\xi-t}}\omega(\xi, t)\,\mathrm{d}\xi
$$

$$\int_0^t \sqrt{\tfrac{d-\xi}{t-\xi}}\, A(\xi)\, \mathsf{d}\xi = \int_t^d \sqrt{\tfrac{d-\xi}{\xi-t}} \omega_t(\xi, t)\, \mathsf{d}\xi$$

$$\int_0^t \sqrt{\tfrac{t-\xi}{d-\xi}}\, (t A(\xi) + B(\xi))\ \mathsf{d}\xi = - \int_t^d \sqrt{\tfrac{\xi-t}{d-\xi}} \omega(\xi, t)\, \mathsf{d}\xi$$

$$\int_0^t \sqrt{\tfrac{d-\xi}{t-\xi}}\, (t A(\xi) + B(\xi))\ \mathsf{d}\xi = \int_t^d \sqrt{\tfrac{d-\xi}{\xi-t}} \omega(\xi, t)\, \mathsf{d}\xi$$

For rigid-plate plate impact at constant velocity:

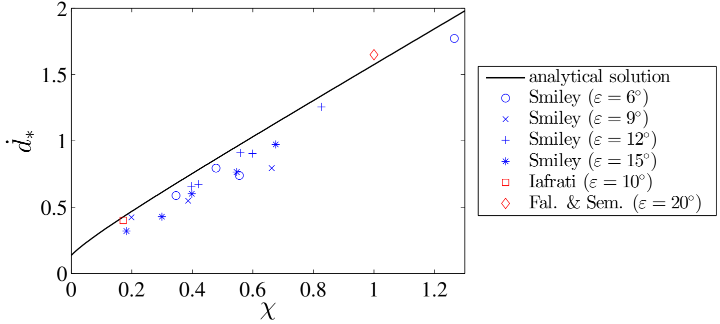
$$\omega(x, t) = x - t(1 + \chi)\,, \qquad \chi = \frac{V}{\varepsilon U}$$

$\Longrightarrow$ Solution is self-similar solution: $A(x) = A^*\,, \quad B(x) = B^* x\,, \quad \dot{d}(t) = \dot{d}_* t$

Solution of the problem:

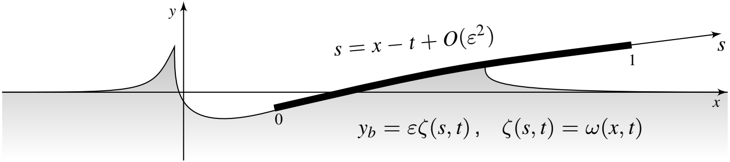
$$\dot{d}_* = q^{-2}\,, \qquad A^* = -\frac{\pi}{4q} \frac{\chi - 1 + 2q^2(1 + 2\chi)}{(q^2 + 1)^{3/2}}\,, \qquad B^* = -A^* \frac{4q^2 + 1}{2q^2 + 2}\,,$$

$$\mathsf{arsinh}(q) = q\sqrt{q^2 + 1}\, \frac{\chi + 3 - 2\chi q^2}{\chi - 1 + 2q^2(1 + 2\chi)}$$



$$\chi = \frac{V}{\varepsilon U}$$

Structural part of the problem for free elastic plate



Euler's beam equation:

$$\mu \frac{\partial^2}{\partial t^2}\zeta + \theta \frac{\partial^4}{\partial s^4}\zeta = p(s+t,0,t) - \mu\kappa$$

$$\frac{\partial^2}{\partial s^2}\zeta = \frac{\partial^3}{\partial s^3}\zeta = 0 \qquad (s=0,\ s=1)$$

$$\mu = \frac{\varrho_S h}{\varrho_F L} \qquad \theta = \frac{D}{\varrho_F L^3 U^2} \qquad \kappa = \frac{gL}{\varepsilon U^2}$$

h plate thickness

D flexural rigidity

ϱ_S plate density

ϱ_F fluid density

$$\zeta(s,t) = \sum_{k=0}^{\infty} a_k(t)\psi_k(s)$$

$$\frac{\partial^4}{\partial s^4}\psi_k = \lambda_k^4\psi_k \qquad (0 < s < 1)$$

$$\frac{\partial^2}{\partial s^2}\psi_k = \frac{\partial^3}{\partial s^3}\psi_k = 0 \qquad (s=0, s=1)$$

rigid modes of translation and rotation: $\psi_0(s), \quad \psi_1(s)$

normal modes for elastic deflection: $\psi_k(s)\,, \quad k \geq 2$

Coupling of structural and hydrodynamic part

$$\mu \frac{\mathrm{d}^2 a_k}{\mathrm{d}t^2} + \theta \lambda_k^4 a_k = \int_t^d p(x,0,t)\psi_k(x-t)\,\mathrm{d}x - \mu\kappa\delta_{0k}$$

Hydrodynamic pressure:

$$p(x,0,t) = -\frac{\dot{d}\,S(t)}{\pi(d-t)}\sqrt{\frac{x-t}{d-x}} - \frac{1}{\pi}\sqrt{(x-t)(d-x)}\,T(x,t)\,,$$

where

$$S(t) = \int_0^t \sqrt{\tfrac{t-\xi}{d-\xi}}\,A(\xi)\,\mathrm{d}\xi + \int_t^d \sqrt{\tfrac{\xi-t}{d-\xi}}\omega_t(\xi,t)\,\mathrm{d}\xi\,,$$

$$T(x,t) = ⨍_t^d \frac{\hat{\omega}_{tt}(\xi,t)}{(\xi-x)\sqrt{(\xi-t)(d-\xi)}}\,\mathrm{d}\xi\,,$$

$$\hat{\omega}_{tt}(x,t) = \int_t^x \omega_{tt}(\xi,t)\,\mathrm{d}\xi\,.$$

Final equations

Euler's beam equation: $$\frac{\mathrm{d}^2\vec{a}}{\mathrm{d}t^2} = \vec{F}\left(t, d, \vec{a}, \tfrac{\mathrm{d}\vec{a}}{\mathrm{d}t}, A(x)|_{x\in(0,t)}\right)$$

Wagner's condition: $$\frac{\mathrm{d}d}{\mathrm{d}t} = G\left(t, d, \vec{a}, \tfrac{\mathrm{d}\vec{a}}{\mathrm{d}t}, A(x)|_{x\in(0,t)}, B(x)|_{x\in(0,t)}\right)$$

update of A and B: $$\int_0^t \sqrt{\tfrac{d-\xi}{t-\xi}}\, A(\xi)\,\mathrm{d}\xi = K(t, d, \vec{a}, \tfrac{\mathrm{d}\vec{a}}{\mathrm{d}t})$$

$$\int_0^t \sqrt{\tfrac{d-\xi}{t-\xi}}\, B(\xi)\,\mathrm{d}\xi = L(t, d, \vec{a}, \tfrac{\mathrm{d}\vec{a}}{\mathrm{d}t})$$

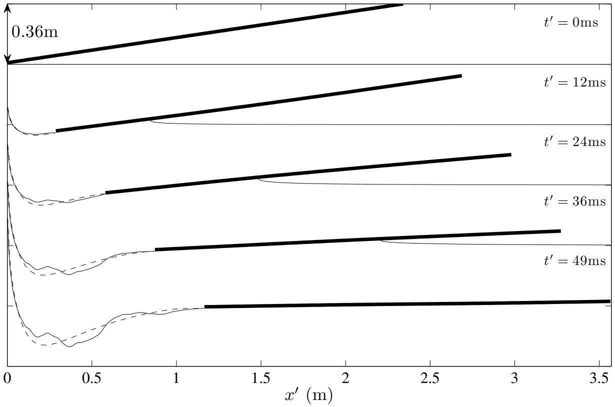
with initial conditions:

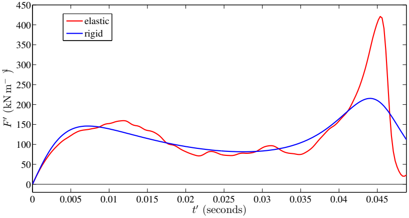
$$\vec{a}(0) = (\tfrac{1}{2}, \tfrac{1}{6}\sqrt{3}, 0, 0, \cdots), \qquad \tfrac{\mathrm{d}\vec{a}}{\mathrm{d}t}(0) = (-\tfrac{V}{\varepsilon U}, 0, 0, \ldots), \quad d(0) = 0$$

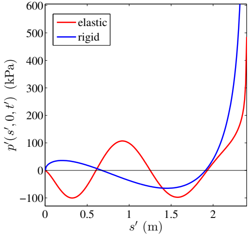
Results for steel plate:

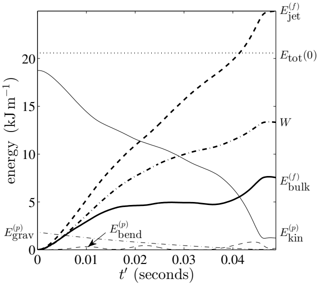
$L = 2.4$m, $h = 5$cm, $\varepsilon = 8.6^\circ$, $V = 6\text{ms}^{-1}$, $U = 24\text{ms}^{-1}$

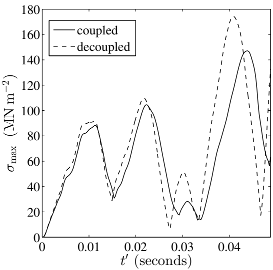
timestep $\Delta t = 5 \times 10^{-4}$, 8 elastic modes




elastic
rigid
F' (kN m^{-1})
t' (seconds)
0
50
100
150
200
250
300
350
400
450
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045


elastic
rigid
$p'(s', 0, t')$ (kPa)
s' (m)
−100
0
100
200
300
400
500
600
0.5
1
1.5
2


energy (kJ m^{-1})
t' (seconds)
$E^{(f)}_{\mathrm{jet}}$
$E_{\mathrm{tot}}(0)$
W
$E^{(f)}_{\mathrm{bulk}}$
$E^{(p)}_{\mathrm{kin}}$
$E^{(p)}_{\mathrm{grav}}$
$E^{(p)}_{\mathrm{bend}}$
0
5
10
15
20
0.01
0.02
0.03
0.04


coupled
decoupled
$\sigma_{\max}$ (MN m^{-2})
t' (seconds)
0
20
40
60
80
100
120
140
160
180
0.01
0.02
0.03
0.04

Summary

1. The Wagner model for impact problems of bodies with small deadrise angle into water was introduced.

2. A model for the impact of an elastic plate at high horizontal speed has been presented.

3. Influence of hydroelasticity on the force is significant. Large negative forces appear on the rear part of the plate during impact.

Thank you.

Any questions or comments?

References

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