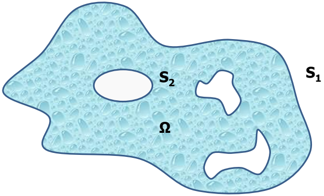
$$\begin{cases} -\nu\Delta\mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p = \mathbf{f} \quad \text{in } \Omega, \\ div\mathbf{v} = 0 \quad \text{in } \Omega, \\ \mathbf{v} = \mathbf{h} \quad \text{on } \partial\Omega, \end{cases} \tag{NS}$$

$\mathbf{v}$ – velocity of the fluid, p -pressure.
$\Omega \subset \mathbb{R}^n, n = 2, 3$,–multi-connected domain:

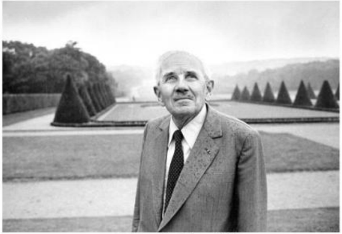


Incompressibility of the fluid ($div\mathbf{v} = 0$) implies the necessary compatibility condition for the solvability of problem (NS):

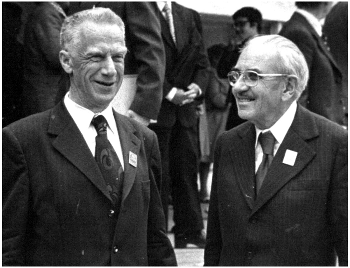
$$\int\limits_{\partial\Omega} \mathbf{h}\cdot\mathbf{n}\,dS = \sum_{j=1}^{N} \int\limits_{\Gamma_j} \mathbf{h}\cdot\mathbf{n}\,dS = \sum_{j=1}^{N} F_j = 0, \qquad (F)$$

$\mathbf{n}$ is a unit vector of the outward normal to $\partial\Omega$.

J. Leray (1933)



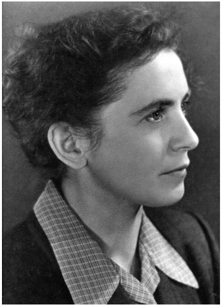
Jean Leray - Parc de Sceaux (1985)



$$F_j = \int_{\Gamma_j} \mathbf{h} \cdot \mathbf{n} \, dS = 0 \quad j = 1, \ldots, N. \qquad (F_0)$$

E. Hopf (1941)

O.A. Ladyzhenskaya (1959)

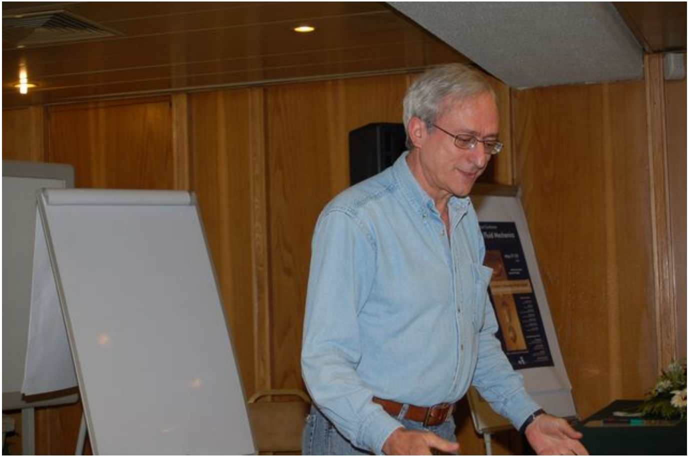


I.I. Vorovich and V.I. Judovich (1961)

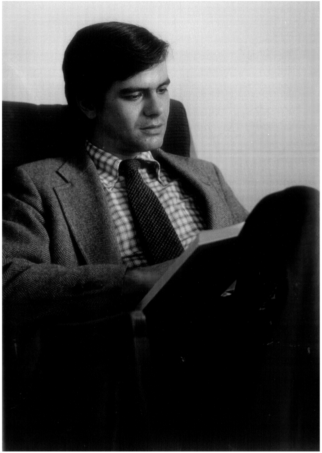
H. Fujita (1961), R. Finn (1961)

The solvability (for arbitrary domains) but assuming that fluxes F_j are "sufficiently small".

G.P. Galdi (1991), W. Borchers & K.P. (1994),
H. Kozono & T. Yanagisawa (2009)

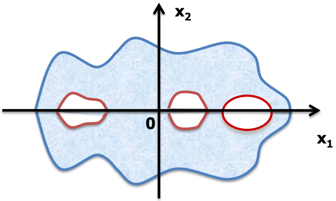


Ch.J. Amick (1984)



He solved the problem in a symmetric plane domain for arbitrary values of fluxes

$$F_j = \int\limits_{\Gamma_j} \mathbf{h} \cdot \mathbf{n} \, dS, \quad \sum_{j=1}^{N} F_j = 0.$$



Ch.J. Amick obtained a priori estimate by contradiction.

A. Takeshita (1993), J. Heywood (2010), R. Farwig, H. Kozono (2012) proved (constructing a counterexample) that

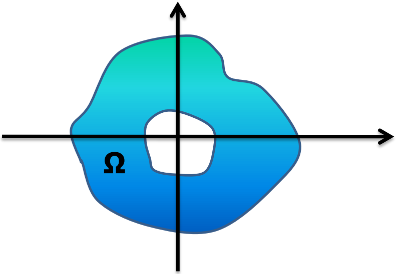
$$\left| \int_\Omega (\mathbf{u} \cdot \nabla)\mathbf{u} \cdot \mathbf{A} dx \right| \leq \varepsilon c \int_\Omega |\nabla \mathbf{u}|^2 dx \quad \forall \mathbf{u} \in H(\Omega), \qquad (LHI)$$

(Leray-Hopf inequality) in general, is possible if

$$\int_{\Gamma_j} \mathbf{h} \cdot \mathbf{n} \, dS = 0, \quad j = 1, \ldots, N.$$

In 1993 L.I. Sazonov rediscovered Amick's results constructing in a symmetrical plane domain an extension satisfying Leray's inequality (LHI).
Further results of constructing an extensions in symmetric plane domains (virtual drains) were obtained by H. Fujita and H. Morimoto (1997), (2007).

In 1997 H. Fujita and H. Morimoto studied problem (NS) in a domain Ω with two components of the boundary S_1 and S_2. Assuming that $\mathbf{h} = F\nabla u_0|_{\partial\Omega} + \boldsymbol{\alpha}$, where $F \in \mathbb{R}$, u_0 is a harmonic function, and $\boldsymbol{\alpha}$ has zero fluxes, they proved that there is a countable subset $\mathbb{N} \subset \mathbb{R}$ such that if $F \notin \mathbb{N}$ and $\boldsymbol{\alpha}$ is small (in a suitable norm), then (NS) has a weak solution. Moreover, if $\Omega \subset \mathbb{R}^2$ is an annulus and $u_0 = \log|x|$, then $\mathbb{N} = \emptyset$.




flow past a (rotating) body


ПУХНАЧЕВ