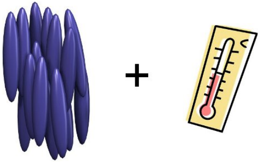
A thermodynamically consistent extension of the simplified Ericksen-Leslie Model

Katharina Schade, TU Darmstadt

jointly with M. Hieber and J. Prüss



9th Japanese-German International Workshop on Mathematical Fluid Dynamics

Waseda University

Nov 5, 2013

1. The *old* Model 

- Ω: bounded C^2-domain.

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$u : (0,\infty)\times\Omega \to \mathbb{R}^n$: velocity,
$\pi : (0,\infty)\times\Omega \to \mathbb{R}$: pressure,
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How to realize this in an extended system  ?

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- **Velocity equation**

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$$\begin{cases} \partial_t u + (u \cdot \nabla) u + \nabla \pi & = \operatorname{div} \mathbb{S}, \\ \operatorname{div} u & = 0, \end{cases}$$

where $D := \frac{1}{2}([\nabla u]^T + \nabla u)$, $\quad \mathbb{S} := 2\mu D - \lambda [\nabla d]^T \nabla d$.

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Total energy

$$E = E_{kin} + E_{int} = \int_\Omega \frac{1}{2}|u|^2 + e\,dx,$$

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Internal relations

$$\begin{cases} e = \psi + \vartheta\eta, \\ \eta = -\partial_1\psi, \quad \kappa_0 = \partial_1 e, \quad \lambda = \partial_2\psi, \\ \psi'' > 0. \end{cases}$$

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Connection to (-d) and (-ϑ)

$$(\partial_t + u\cdot\nabla)e = \partial_1 e(\partial_t + u\cdot\nabla)\vartheta + \partial_2 e(\partial_t + u\cdot\nabla)(|\nabla d|^2/2).$$

$$= \kappa_0(\partial_t + u\cdot\nabla)\vartheta + (\lambda - \vartheta\partial_1\lambda)(\partial_t + u\cdot\nabla)(|\nabla d|^2/2).$$

5. Clever RHS for ( -ϑ) leads to  §1

Idea

- Choose right-hand side of ( -ϑ) such that

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Suitable right-hand side

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$$\kappa_0(\partial_t\vartheta + (u\cdot\nabla)\vartheta) + \operatorname{div} q = 2\mu|D|^2 - \partial_1\lambda\vartheta([\nabla d]^T\nabla d) : \nabla u$$
$$- (\lambda - \vartheta\partial_1\lambda)\nabla d : \nabla[\gamma(\vartheta \operatorname{div}(\frac{\lambda}{\vartheta}\nabla d) + \lambda|\nabla d|^2 d)]$$

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$$\begin{cases} \int_\Omega (\partial_t + u \cdot \nabla)\eta + \operatorname{div}\left(\frac{q}{\vartheta}\right) dx \geq 0, \\ \text{with equality } \textit{iff} \text{ the system is in an equilibrium state.} \end{cases}$$

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- (-d) + (-ϑ) + (-CL) $\rightarrow$ §2.

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