

**独立行政法人日本学術振興会 日独共同大学院プログラム**  
**JSPS-DFG Japanese-German Graduate Externship**  
**第9回 日独流体数学国際研究集会**  
**The 9th Japanese-German International**  
**Workshop on Mathematical Fluid Dynamics**

November 5 - 8, 2013 at Waseda University, Nishi-Waseda Campus

63 Bldg. 2<sup>nd</sup> Floor - 05 Conference Room

■ Program ■

	Tue, Nov. 5	Wed, Nov. 6	Thu, Nov. 7	Fri, Nov. 8
9:30-10:40	<b>John G. HEYWOOD</b> ① (Univ. of British Columbia)	<b>PILECKAS</b> ②	<b>PILECKAS</b> ③	<b>HEYWOOD</b> ③
10:55-12:05	<b>Konstantinas PILECKAS</b> ① (Vilnius Uni.)	<b>HEYWOOD</b> ②	<b>TABATA</b> ③	<b>HEYWOOD</b> ④
12:20-13:00	<b>Cameron TROPEA</b> (TU Darmstadt)	<b>Carlo R. GRISANTI</b> (Univ. of Pisa)	<b>Francesca CRISPO</b> (2nd Univ. of Naples)	<b>Maria LUKACOVA</b> (Univ. of Mainz)
13:00-15:00	Lunch break			
15:00-16:10	<b>Masahisa TABATA</b> ① (Waseda Univ.)	<b>TABATA</b> ②	<b>PILECKAS</b> ④	<b>TABATA</b> ④
16:40 (-16:55)	<b>Shinya NISHIBATA</b> (Tokyo Tech)	<b>Takayuki KUBO</b> (Univ. of Tsukuba)	<b>Martin RAPP</b> (TU Darmstadt)	<b>Bangwei SHE</b> (Univ. of Mainz)
(17:05-) -17:20			<b>Pen-yuan HSU</b> (Univ. of Tokyo)	<b>Hana MIZEROVA</b> (Univ. of Mainz)
17:30-17:45	<b>Katharina SCHADE</b> (TU Darmstadt)	<b>Yuto IMAI</b> (Waseda Univ.)	<b>Naofumi MORI</b> (Kyusyu Univ.)	
17:50-18:05	<b>Jonas SAUER</b> (TU Darmstadt)	<b>Tomoyuki NAKATSUKA</b> (Nagoya Univ.)	<b>Takahito KASHIWABARA</b> (Univ. of Tokyo)	* 18 : 00~ Reception

●→Main-course      ●→40min talk      ●→15min talk

**Discussion room → open on Wed and Fri / 9:30 - 17:00 / 63 Bldg 1F**  
**Meeting room for Mathematics and Applied Mathematics**  
**Closed on Tue and Thu!!**

## Main-Course

**John G. HEYWOOD**

University of British Columbia, Vancouver

Title:

**On a Conjectured Pointwise Bound for Solutions of the Stokes Equations in Nonsmooth Domains**

Abstract:

Let  $\Omega$  be an arbitrary open subset of  $R^3$ . Let  $\mathbf{D}(\Omega) = \{\varphi \in \mathbf{C}_0^\infty(\Omega) : \nabla \cdot \varphi = 0\}$ . And let  $\mathbf{J}(\Omega)$  and  $\mathbf{J}_0(\Omega)$  be the completions of  $\mathbf{D}(\Omega)$  in the  $\mathbf{L}^2$ -norm  $\|\cdot\|$  and in the Dirichlet-norm  $\|\nabla \cdot\|$ , respectively. Given  $\mathbf{u} \in \mathbf{J}_0(\Omega)$ , there is at most one  $\mathbf{f} \in \mathbf{J}(\Omega)$  such that  $(\nabla \mathbf{u}, \nabla \varphi) = -(\mathbf{f}, \varphi)$  for all  $\varphi \in \mathbf{D}(\Omega)$ . If such a function  $\mathbf{f}$  exists, it is denoted by  $\tilde{\Delta} \mathbf{u}$ , and Wenzheng Xie has conjectured that

$$\sup_{\Omega} |\mathbf{u}|^2 \leq \frac{1}{3\pi} \|\nabla \mathbf{u}\| \|\tilde{\Delta} \mathbf{u}\|. \quad (1)$$

Xie proved (1) for  $\Omega = R^3$ , and proved that its general validity would follow from its validity for smoothly bounded domains. He also proved that the constant is optimal, for any domain.

For general domains, Xie proved an analogue of (1) for the Laplacian, considering it as a model problem. However, at one point of the proof he used the maximum principle, in showing that for every point  $y$  in a smoothly bounded domain  $\Omega$ ,

$$\|G_\mu(\cdot, y)\|_{L^2(\Omega)} \leq \|g_\mu(\cdot, y)\|_{L^2(R^3)}, \quad (2)$$

where  $G_\mu$  is the Green's function for the Helmholtz operator  $-\Delta + \mu$  in  $\Omega$ , and  $g_\mu$  is the corresponding fundamental singularity. Although he conjectured an analogue of (2) for the Stokes operator, it has yet to be proven. However, I have proven, even for the Stokes operator, that the ratio of the two sides of (2) tends to 1 as  $\mu \rightarrow \infty$ . Consequently, to complete the proof of (1), it is now enough to consider smoothly bounded domains and to show a tendency toward singularity, in the sense that

$$\mu_n \equiv \|\tilde{\Delta} u_n\|^2 / \|\nabla u_n\|^2 \rightarrow \infty, \quad (3)$$

if  $\{u_n\}$  is a sequence of functions such that the ratio of the left to right sides of (1) tends to its supremum. My efforts to prove (3) have led to many further conjectures.

It is well understood that if (1) is valid for arbitrary domains, then so is most of the current existence and regularity theory for the Navier-Stokes equations.

Date:

- ① Tuesday, Nov. 5 9:30-10:40      ② Wednesday, Nov. 6 10:55-12:05  
 ③ Friday, Nov. 8 9:30-10:40      ④ Friday, Nov. 8 10:55-12:05

# Konstantin PILECKAS

Vilnius University, Vilnius

Faculty of Mathematics and Informatics, Vilnius University

Title:

**Stationary Navier–Stokes equations in bounded domains with multiply connected boundaries. Leray’s problem**

Abstract:

## Lecture 1

During this lecture we study in a bounded domain  $\Omega \subset \mathbb{R}^n$  the stationary Navier–Stokes system with homogeneous boundary conditions

$$\begin{cases} -\nu\Delta\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla p = \mathbf{f} & \text{in } \Omega, \\ \operatorname{div} \mathbf{v} = 0 & \text{in } \Omega, \\ \mathbf{v} = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where  $\nabla = (\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n})$ ,  $\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^n a_i b_i$ ,  $\operatorname{div} \mathbf{v} = \nabla \cdot \mathbf{v}$ ,  $\Delta_x = \nabla \cdot \nabla$  is the Laplacian,  $\mathbf{v}$  and  $p$  stand for the velocity vector and the pressure,  $\mathbf{f}$  is the density of external forces:

$$\mathbf{v} = (v_1, \dots, v_n), \quad \mathbf{f} = (f_1, \dots, f_n),$$

$\nu > 0$  means the constant viscosity of the liquid.

Let  $H(\Omega)$  be a subspace of solenoidal vector fields belonging to  $\dot{W}_2^1(\Omega)$ . By a weak solution of problem (1) we understand a vector function  $\mathbf{v} \in H(\Omega)$  satisfying the integral identity

$$\nu \int_{\Omega} \nabla \mathbf{v} \cdot \nabla \boldsymbol{\eta} \, dx + \int_{\Omega} (\mathbf{v} \cdot \nabla) \mathbf{v} \cdot \boldsymbol{\eta} \, dx = \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\eta} \, dx \quad \forall \boldsymbol{\eta} \in H(\Omega), \quad (2)$$

where

$$\nabla \mathbf{v} \cdot \nabla \boldsymbol{\eta} = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial v_i}{\partial x_j} \frac{\partial \eta_i}{\partial x_j}.$$

It will be shown that the integral identity (2) is equivalent to the operator equation in the space  $H(\Omega)$

$$\mathbf{v} = \mathcal{A}\mathbf{v} \quad (3)$$

with the compact operator  $\mathcal{A}$ . In order to apply the Leray–Schauder Fixed Point Theorem, we will prove that all possible solutions  $\mathbf{v}^\lambda$  of the equation

$$\mathbf{v}^\lambda = \lambda \mathcal{A}\mathbf{v}^\lambda, \quad \lambda \in [0, 1], \quad (4)$$

are uniformly (with respect to  $\lambda$ ) bounded. Then from the Leray–Schauder it follows that equation (3) has at least one solution.

The solution of the integral identity (2) (or, equivalently, of the operator equation (3)) in general could be non-unique. However we prove the uniqueness of this solution for small data.

We also show that there exists a pressure function  $p \in L_2(\Omega)$  such that  $\int_{\Omega} p(x) dx = 0$  and

$$\begin{aligned} \nu \int_{\Omega} \nabla \mathbf{v} \cdot \nabla \boldsymbol{\eta} dx + \int_{\Omega} (\mathbf{v} \cdot \nabla) \mathbf{v} \cdot \boldsymbol{\eta} dx &= \int_{\Omega} p \operatorname{div} \boldsymbol{\eta} dx \\ &+ \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\eta} dx \quad \forall \boldsymbol{\eta} \in \dot{W}_2^1(\Omega). \end{aligned} \quad (5)$$

Moreover,

$$\mathbf{v} \in W_{2,loc}^2(\Omega), \quad p \in W_{2,loc}^1(\Omega)$$

and the pair  $(\mathbf{v}, p)$  satisfies the Navier–Stokes equations (1) almost everywhere in  $\Omega$ .

## Lecture 2

The stationary Navier-Stokes system with nonhomogeneous boundary conditions

$$\begin{cases} -\nu \Delta \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p = 0 & \text{in } \Omega, \\ \operatorname{div} \mathbf{v} = 0 & \text{in } \Omega, \\ \mathbf{v} = \mathbf{a} & \text{on } \partial\Omega, \end{cases} \quad (6)$$

will be studied in the multi-connected domain  $\Omega = \Omega_0 \setminus \cup_{j=1}^N \Omega_j$ , where  $\bar{\Omega}_j \subset \Omega$ ,  $\Omega_j \cap \Omega_i = \emptyset$ ,  $j \neq i$ .

The continuity equation  $\operatorname{div} \mathbf{v} = 0$  implies the necessary compatibility condition for the solvability of problem (6):

$$\int_{\partial\Omega} \mathbf{a} \cdot \mathbf{n} dS = \sum_{j=1}^N \int_{\Gamma_j} \mathbf{a} \cdot \mathbf{n} dS = \sum_{j=1}^N F_j = 0, \quad (7)$$

where  $\mathbf{n}$  is a unit vector of the outward (with respect to  $\Omega$ ) normal to  $\partial\Omega$ ,  $\Gamma_j = \partial\Omega_j$ . The compatibility condition (7) means that the net flux of the fluid over the boundary  $\partial\Omega$  is zero.

In this lecturer we shall prove the existence of the solution under the stronger than (7) condition which requires the all fluxes  $F_j$  of the boundary value  $\mathbf{a}$  to be zero separately across each component  $\Gamma_j$  of the boundary  $\partial\Omega$ :

$$F_j = \int_{\Gamma_j} \mathbf{a} \cdot \mathbf{n} dS = 0, \quad j = 1, 2, \dots, N, \quad (8)$$

Notice that the condition (8) does not allow the presence of sinks and sources.

In this lecture we will study the method based on of the Leray–Hopf extension of the boundary data. If the condition (8) is valid, then there exists a function  $\mathbf{b}$  such that

$$\operatorname{rot} \mathbf{b}(x)|_{\partial\Omega} = \mathbf{a}(x).$$

We construct the Leray–Hopf cut-off function  $\zeta(x, \varepsilon)$  which has following properties:

(i)  $\zeta(x, \varepsilon) = 1$  for  $x \in \partial\Omega$ ,  $\zeta(x, \varepsilon) = 0$  for  $\operatorname{dist}(x, \partial\Omega) \geq \delta = \delta(\varepsilon)$ ,

(ii)  $0 \leq \zeta(x, \varepsilon) \leq 1$ ,

(iii)  $|\nabla\zeta(x, \varepsilon)| \leq \frac{c\varepsilon}{\operatorname{dist}(x, \partial\Omega)}$  with the constant  $c$  independent of  $\varepsilon$ .

The Leray–Hopf’s extension function has the form

$$\mathbf{B}(x, \varepsilon) = \operatorname{rot}(\zeta(x, \varepsilon)\mathbf{b}(x)). \quad (9)$$

Then

$$\operatorname{div} \mathbf{B}(x, \varepsilon) = 0, \quad \mathbf{B}(x, \varepsilon)|_{\partial\Omega} = \mathbf{a}(x).$$

We look for a weak solution of the problem (6) in the form  $\mathbf{v} = \mathbf{u} + \mathbf{B}$ , where  $\mathbf{u} \in H(\Omega)$ . Then for  $\mathbf{u}$  we get the integral identity

$$\begin{aligned} & \nu \int_{\Omega} \nabla \mathbf{u} \cdot \nabla \boldsymbol{\eta} \, dx + \int_{\Omega} (\mathbf{u} \cdot \nabla) \mathbf{u} \cdot \boldsymbol{\eta} \, dx + \int_{\Omega} (\mathbf{u} \cdot \nabla) \mathbf{B} \cdot \boldsymbol{\eta} \, dx + \int_{\Omega} (\mathbf{B} \cdot \nabla) \mathbf{u} \cdot \boldsymbol{\eta} \, dx \\ & = \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\eta} \, dx - \int_{\Omega} (\mathbf{B} \cdot \nabla) \mathbf{B} \cdot \boldsymbol{\eta} \, dx - \nu \int_{\Omega} \nabla \mathbf{B} \cdot \nabla \boldsymbol{\eta} \, dx \quad \forall \boldsymbol{\eta} \in H(\Omega). \end{aligned} \quad (10)$$

The integral identity (10) is equivalent to the operator equation

$$\mathbf{u} = \mathcal{B}\mathbf{u}$$

with the compact operator  $\mathcal{B}$  in the space  $H(\Omega)$ . In order to show that all possible solutions of the operator equation with the parameter  $\lambda$  are uniformly bounded, we prove and apply the following Leray–Hopf’s inequality

$$\left| \int_{\Omega} (\mathbf{u} \cdot \nabla) \mathbf{B} \cdot \mathbf{u} \, dx \right| \leq c\varepsilon \int_{\Omega} |\nabla \mathbf{u}|^2 \, dx \quad \forall \mathbf{u} \in H(\Omega), \quad (11)$$

where the constant  $c$  is independent of  $\varepsilon$  and  $\mathbf{u}$ . We get the desired estimate by choosing in (11) the parameter  $\varepsilon > 0$  sufficiently small.

### Lecture 3

We shall show that the Leray–Hopf’s extension of the boundary data is not possible, if the condition (8) is violated (the counterexample of A. Takeshita will be presented), i.e., we prove that if the fluxes  $F_i$  of the boundary value  $\mathbf{a}$  throw the connected components  $\Gamma_i$  of the boundary  $\partial\Omega$  are nonzero, then the Leray–Hopf’s inequality (11), in general, is not valid. Thus in this case it is not possible to apply the same as in the previous lecture method.

In this lecture we consider the method of getting an a priori estimate by a contradiction. In order to simplify the proofs, we still assume for a while that the condition (8) is fulfilled. The main idea of the last method consist in the following. Consider the integral identity corresponding to the operator equation with the parameter  $\lambda$ :

$$\begin{aligned} & \nu \int_{\Omega} \nabla \mathbf{u}^\lambda \cdot \nabla \boldsymbol{\eta} \, dx - \lambda \int_{\Omega} ((\mathbf{u}^\lambda + \mathbf{B}) \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{u}^\lambda \, dx - \lambda \int_{\Omega} (\mathbf{u}^\lambda \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{B} \, dx \\ &= \lambda \left( \int_{\Omega} (\mathbf{B} \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{B} \, dx - \nu \int_{\Omega} \nabla \mathbf{B} \cdot \nabla \boldsymbol{\eta} \, dx + \nu \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\eta} \, dx \right) \quad \forall \boldsymbol{\eta} \in H(\Omega). \end{aligned} \quad (12)$$

Here  $\mathbf{B}$  is an arbitrary divergence free extension of the boundary value  $\mathbf{a}$ .

Assume that the solutions of (12) are not uniformly bounded in  $H(\Omega)$  with respect to  $\lambda \in [0, 1]$ . Then there exist sequences  $\{\lambda_k\}_{k \in \mathbb{N}} \subset [0, 1]$  and  $\{\mathbf{u}^{\lambda_k} = \mathbf{u}_k\}_{k \in \mathbb{N}} \in H(\Omega)$  such that

$$\begin{aligned} & \nu \int_{\Omega} \nabla \mathbf{u}_k \cdot \nabla \boldsymbol{\eta} \, dx - \lambda_k \int_{\Omega} ((\mathbf{u}_k + \mathbf{B}) \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{u}_k \, dx - \lambda_k \int_{\Omega} (\mathbf{u}_k \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{B} \, dx \\ &= \lambda_k \left( \int_{\Omega} (\mathbf{B} \cdot \nabla) \boldsymbol{\eta} \cdot \mathbf{B} \, dx - \nu \int_{\Omega} \nabla \mathbf{B} \cdot \nabla \boldsymbol{\eta} \, dx + \nu \int_{\Omega} \mathbf{f} \cdot \boldsymbol{\eta} \, dx \right) \quad \forall \boldsymbol{\eta} \in H(\Omega), \end{aligned} \quad (13)$$

and

$$\lim_{k \rightarrow \infty} \lambda_k = \lambda_0 \in [0, 1], \quad \lim_{k \rightarrow \infty} J_k = \lim_{k \rightarrow \infty} \|\nabla \mathbf{u}_k\|_{L_2(\Omega)} = \infty.$$

First, taking in (13)  $\boldsymbol{\eta} = J_k^{-2} \mathbf{u}_k$  and passing to a limit we obtain the equality

$$\nu = \lambda_0 \int_{\Omega} (\hat{\mathbf{u}} \cdot \nabla) \hat{\mathbf{u}} \cdot \mathbf{B} \, dx, \quad (14)$$

where  $\hat{\mathbf{u}}$  is a weak limit in  $H(\Omega)$  of the sequence  $\{J_k^{-1} \mathbf{u}_k\}$ .

Second, taking in (13)  $\boldsymbol{\eta} = J_k^{-2} \boldsymbol{\xi}$  with arbitrary  $\boldsymbol{\xi} \in H(\Omega)$  we obtain that  $\hat{\mathbf{u}}$  and the corresponding pressure function  $\hat{p}$  satisfy the Euler equations:

$$\begin{cases} \lambda_0 (\hat{\mathbf{u}} \cdot \nabla) \hat{\mathbf{u}} + \nabla \hat{p} = 0, \\ \operatorname{div} \hat{\mathbf{u}} = 0, \\ \hat{\mathbf{u}}|_{\partial\Omega} = 0. \end{cases} \quad (15)$$

It follows from (15) that

$$\widehat{p}|_{\Gamma_i} = \widehat{p}_i, i = 1, \dots, N,$$

where  $\widehat{p}_i$  are constants.

Multiplying (15) by  $\mathbf{B}$  and integrating by parts yields

$$\lambda_0 \int_{\Omega} (\widehat{\mathbf{u}} \cdot \nabla) \widehat{\mathbf{u}} \cdot \mathbf{B} \, dx = \sum_{i=1}^N \widehat{p}_i F_i. \quad (16)$$

If the condition (8) is valid, i.e.  $F_i = 0, i = 1, \dots, N$ , then the right-hand side of (16) is equal to zero and (16) contradicts to (14). Thus, all possible solutions of (12) are bounded and by the Leray-Schauder Fixed Point Theorem there exists at least one weak solution of problem (6).

Since by incompressibility of the fluid

$$\sum_{i=1}^N F_i = 0,$$

the right-hand side of (16) will be zero also if

$$\widehat{p}_1 = \widehat{p}_2 = \dots = \widehat{p}_N.$$

It was shown by Ch. Amick constructing a counterexample that, in general, it is not the case. However, if the domain  $\Omega \subset \mathbb{R}^2$  and the boundary data satisfy certain symmetry conditions then

$$\widehat{p}_1 = \widehat{p}_2 = \dots = \widehat{p}_N$$

holds and we get the needed a priori estimate by the contradiction. Thus, For the symmetric case the existence of the solution to (6) is proved (by Ch. Amick) only under the necessary condition (7).

Finally, we prove the existence of the solution to problem (6) in the case of "sufficiently small" fluxes  $F_j$ .

## Lecture 4

We will study the nonhomogeneous boundary value problem for Navier–Stokes equations in a two–dimensional bounded multiply connected domain  $\Omega \subset \mathbb{R}^2$ . We will prove that this problem admits at least one solution only under the necessary condition (7) without imposing any smallness assumptions on the value of  $|F|$ . The proof of the main result uses the Bernoulli law for a weak solution to the Euler equations. The detailed proof of the weak version of the Bernoulli law will be presented. A priori estimate for possible solutions of (12) is proved by a contradiction method. We get the contradiction by using the co-area formula. This

proof is essentially based on results concerning "fine properties" of Sobolev functions and level sets of  $W^{2,1}$ -functions (a weak version of the Morse-Sard theorem) which were obtained recently by J. Bourgain, M.V. Korobkov and J. Kristensen. A short introduction to these topics will be presented in the lecture.

Date:

- ① Tuesday, Nov. 5 10:55-12:05      ② Wednesday, Nov. 6 9:30-10:40
- ③ Thursday, Nov. 7 9:30-10:40      ④ Thursday, Nov. 7 15:00-16:10



**Masahisa TABATA**

Waseda University, Tokyo

Department of Mathematics, Waseda University  
3-4-1, Ohkubo, Shinjuku, Tokyo, 169-8555 Japan

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Title:

**Galerkin-Characteristics Finite Element Methods for Flow Problems**

Abstract:

A remarkable feature of flow problems is that the governing equations include the material derivative term

$$\frac{D\phi}{Dt} \equiv \frac{\partial\phi}{\partial t} + u \cdot \nabla\phi,$$

where  $u$  is a function expressing the flow field and  $\phi$  is an unknown physical quantity such as the density, the velocity, or the energy. It makes the problems asymmetric and nonlinear when the velocity field is unknown, e.g., in the Navier-Stokes equations  $\phi$  stands for each component  $u_i$  of unknown velocity  $u$ , which leads to the nonlinear term  $u \cdot \nabla u_i$ . The combination of this term with the diffusion term  $-\nu\Delta u$  describes many important phenomena in sciences and engineering. It produces various fruitful and interesting results, especially when the diffusion constant  $\nu$  is small, e.g., high Reynolds number problems in the Navier-Stokes equations. In devising numerical schemes for the solution of those phenomena, it is well-known that the discretization of this term is crucial because the conventional Galerkin finite element method and the centered difference method easily produce unphysical oscillating solutions. Among remedies for the instability the method of characteristics seems to be natural from the physical point of view since it approximates the particle movement along the trajectory.

In this course we consider finite element methods based on characteristics, i.e., Galerkin-characteristics FEMs. Those methods lead to symmetric schemes which are robust even for convection-dominated problems. Beginning with the basic idea and fundamental properties of Galerkin-characteristics method, we discuss recent results on the stability and convergence of the schemes for flow problems.

Date:

- ① Tuesday, Nov. 5 15:00-16:10      ② Wednesday, Nov. 6 15:00-16:10  
③ Thursday, Nov. 7 10:55-12:05      ④ Friday, Nov. 8 15:00-16:10

## 40 minutes talks

**Francesca CRISPO**

Second University of Naples, Caserta      francesca.crispo@unina2.it  
with Paolo Maremonti (Seconda Università degli Studi di Napoli)

Title:

**High regularity results of solutions to modified  $p$ -Navier-Stokes equations**

Abstract:

We consider the following *modified*  $p$ -Navier-Stokes system

$$-\nabla \cdot (|\nabla u|^{p-2} \nabla u) + (u \cdot \nabla)u + \nabla \pi = f, \quad \nabla \cdot u = 0 \quad \text{in } \mathbb{R}^3, \quad (1)$$

in the sub-quadratic case  $p \in (1, 2)$ . This system was considered for the first time in the sixties by Lions. Subsequently the system obtained by replacing  $\nabla u$  with its symmetric part has been much more studied, due to its connection with the motion of shear-thinning fluids. As this last system is usually called  $p$ -Navier-Stokes system, we refer to system (1) as *modified*  $p$ -Navier-Stokes.

We find sufficient conditions for the existence of high regular solutions, in the sense of second derivatives in  $L^q(\mathbb{R}^3)$ , for  $q \in (\frac{3p}{3-p}, +\infty)$ . By embedding, we get  $C^{1,\alpha}$ -regularity of solutions.

As for the Navier-Stokes system in  $\mathbb{R}^3$ , the uniqueness in the existence class is not achieved.

These results are part of a joint work with Paolo Maremonti (Second University of Naples).

Date:

Thursday, Nov. 7 12:20-13:00

**Carlo Romano GRISANTI**

University of Pisa, Pisa      grisanti@dma.unipi.it  
with Giovanni P. Galdi (University of Pittsburgh)

Title:

**An inverse problem with time periodicity for a non-newtonian liquid in an infinite pipe.**

Abstract:

We study the motion of a non-newtonian liquid in a straight pipe of infinite length. The time-periodic case under consideration has an analogue in the newtonian framework since 1955 with the work of Womersley concerning the blood motion in large arteries. In an infinite pipe of constant cross section  $\Sigma$ , a velocity field parallel to the axis of the pipe and not depending on the axial coordinate

has to be found. The only constraints are the no-slip boundary condition and the flow-rate, which is a prescribed, time periodic function  $\alpha(t)$ . The inverse problem for the velocity  $v$  and the pressure gradient  $\Gamma$  is the following

$$\left\{ \begin{array}{ll} \frac{\partial v}{\partial t} = \mu_0 \Delta v + \nabla \cdot S(\nabla v) + \Gamma(t) & \text{in } \Sigma \times (0, T) \\ \int_{\Sigma} v(x, t) dx = \alpha(t) & \text{in } [0, T] \\ v|_{\partial \Sigma} = 0 & \text{in } [0, T] \end{array} \right.$$

Under suitably assumptions of coercivity, growth and monotonicity on the extra-stress tensor  $S(\nabla v)$ , we give a positive answer to the well posedness of the problem. Our result takes into account both the shear-thinning and shear-thickening behavior of the liquid.

Date:

Wednesday, Nov. 6 12:20-13:00

## Takayuki KUBO

University of Tsukuba, Tsukuba

Title:

**On two phase problem: compressible-compressible model problem**

Abstract:

We consider the model problem for the two phase problem in cases of compressible - compressible fluid flows without surface tension. In order to prove the local in time existence of this problem, the generation of analytic semigroup for linearized problem and its maximal  $L_p - L_q$  regularity are needed in our method. The key step of our method is to prove the existence of  $\mathcal{R}$ -bounded solution operator to the generalized resolvent problem corresponding the linearized problem:

$$\begin{aligned} \lambda \rho_{\pm} + \gamma_{\pm}^1 \operatorname{div} \vec{u}_{\pm} &= f_{\pm} && \text{in } \mathbb{R}_{\pm}^N, \\ \lambda \vec{u}_{\pm} - \operatorname{Div} S_{\pm}(\vec{u}_{\pm}, \rho_{\pm}) &= g_{\pm} && \text{in } \mathbb{R}_{\pm}^N, \\ \vec{u}_{+}|_{x_N=0+} - \vec{u}_{-}|_{x_N=0-} &= \vec{k} && \text{on } \mathbb{R}_0^N, \\ S_{+}(\vec{u}_{+}, \rho_{+}) \mathbf{n}_{+}|_{x_N=0+} - S_{-}(\vec{u}_{-}, \rho_{-}) \mathbf{n}_{-}|_{x_N=0-} &= -\vec{h} && \text{on } \mathbb{R}_0^N. \end{aligned}$$

Here,  $\rho_{\pm}, \vec{u}_{\pm} = (u_{\pm,1}, \dots, u_{\pm,N}) (N \geq 2)$  are unknown mass density and unknown velocity fields.  $S_{\pm}(\vec{u}_{\pm}, \rho_{\pm}) = 2\mu_{\pm}^1 D(\vec{u}_{\pm}) + (\mu_{\pm}^2 \operatorname{div} \vec{u}_{\pm} - \gamma_{\pm}^2 \rho_{\pm}) I$  is stretching tensor,  $D(\vec{u}) = (\nabla \vec{u} + {}^T \nabla \vec{u})/2$  is  $N \times N$  matrix called the Cauchy deformation tensor and  $I$  denotes the  $N \times N$  identity matrix. Moreover for  $N \times N$  matrix function  $M =$

$(M_{ij})$ , the  $i$  th component of  $\text{Div}M$  is defined by  $\sum_{j=1}^N \partial_j M_{ij}$ .  $\vec{n}_+ = (0, \dots, 0, -1)$  is the unit outward normal to  $\mathbb{R}_+^n$  and  $\mu_{\pm}^i, \gamma_{\pm}^i$  ( $i = 1, 2$ ) are constants such that

$$\mu_{\pm}^1 > 0, \mu_{\pm}^1 + \mu_{\pm}^2 > 0, \gamma_{\pm}^1, \gamma_{\pm}^2 \geq 0.$$

The resolvent parameter  $\lambda$  varies in  $\Lambda_{\varepsilon, \lambda_0} = \Sigma_{\varepsilon, \lambda_0} \cap K_{\varepsilon}$ , where  $\Sigma_{\varepsilon, \lambda_0} = \{\lambda \in \mathbb{C} \mid |\arg \lambda| \leq \pi - \varepsilon, |\lambda| \geq \lambda_0\}$  and

$$K_{\varepsilon} = \left\{ \lambda \in \mathbb{C} \mid (\text{Re}\lambda + \gamma_m + \varepsilon)^2 + (\text{Im}\lambda)^2 \geq (\gamma_m + \varepsilon)^2 \right\}$$

with  $\gamma_m = \max\left(\frac{\gamma_1^+ \gamma_2^+}{\mu_+^1 + \mu_+^2}, \frac{\gamma_1^- \gamma_2^-}{\mu_-^1 + \mu_-^2}\right)$ .

In this talk, we will report the existence of  $\mathcal{R}$ -bounded solution operator to this generalized resolvent problem. This is joint work with Yoshihiro Shibata and Kouhei Soga (Waseda Univ.).

Date:

Wednesday, Nov. 6 16:40-17:20

## Maria LUKACOVA

Johannes Gutenberg University Mainz, Mainz

Title:

**Asymptotic preserving schemes for weakly compressible flows**

Abstract:

Many geophysical flows exhibit complex multiscale phenomena in particular when low Mach or Froude number limits are considered. Consequently, construction of efficient problem-suited schemes for weakly compressible flows is a challenging problem.

In this talk we present new asymptotic preserving finite volume and discontinuous Galerkin schemes within the framework of bicharacteristic schemes. The methods couple a finite volume or discontinuous Galerkin formulation with approximate evolution operators. The latter are constructed using the bicharacteristics of multidimensional hyperbolic systems, such that all of the infinitely many directions of wave propagation are considered explicitly. In order to take into account multiscale phenomena nonlinear fluxes are split into a linear part governing the acoustic and gravitational waves and to the rest nonlinear part that models advection. Time integration is realized by the IMEX type approximation using the semi-implicit second-order backward differentiation formulas (BDF2) scheme. We prove that the proposed large time step schemes are asymptotic preserving for small Mach or Froude number flows. Numerical experiments demonstrate stability, accuracy and robustness of these new large time step with respect to small Mach or Froude numbers. Comparisons with the standard one-dimensional approximate Riemann

solvers used for the flux integration demonstrate better stability, accuracy as well as reliability of our new multidimensional methods.

This work has been done in cooperation with S. Noelle (Aachen), K.R. Arun (Trivandrum), L. Yelash (Mainz), G. Bispen (Mainz), F. Giraldo (Monterey) and A. Mueller (Monterey).

Date:

Friday, Nov. 8 12:20-13:00

## **Shinya NISHIBATA**

Tokyo Institute of Technology, Tokyo

Department of Mathematical and Computing Sciences,  
Tokyo Institute of Technology

shinya@is.titech.ac.jp

Title:

**Stationary waves to symmetric hyperbolic-parabolic systems in half space**

Abstract:

In this talk, we consider the large-time behavior of solutions to hyperbolic-parabolic coupled systems in the half line. Assuming that the systems admit the entropy function, we may rewrite them to symmetric forms. For these symmetrizable hyperbolic-parabolic systems, we first prove the existence of the stationary solution. In the case where one eigenvalue of Jacobian matrix appeared in a stationary problem is zero, we assume that the characteristics field corresponding to the zero eigenvalue is genuine non-linear in order to show the existence of a degenerate stationary solution. We also prove that the stationary solution is time asymptotically stable under a smallness assumption on the initial perturbation. The key to the proof is to derive the uniform a priori estimates by using the energy method in half space developed by Matsumura and Nishida as well as the stability condition of Shizuta–Kawashima type. These theorems for the general hyperbolic-parabolic system cover the compressible Navier–Stokes equation for heat conductive gas.

These results are obtained through the joint work with **Dr. Tohru Nakamura** at Kyushu University.

Date:

Tuesday, Nov. 5 16:40-17:20

## **Cameron TROPEA**

Technical University of Darmstadt, Darmstadt

Institute of Fluid Mechanics and Aerodynamics

Title:

**New Horizons for Optimization in Fluid Mechanics**

**Abstract:**

Improved capabilities in both experimental and numerical fluid dynamics suggest that closer interaction between ‘mathematical’ and ‘engineering’ interests in fluid dynamics, pursuits which have often been followed quite remote from one another, is becoming indispensable. This applies to a wide range of problems, starting from the elusive description of a dynamic contact angle to problems of flow geometry optimization or closed-loop feedback control of flow control devices. The focus of this lecture will be on several experimental developments over the very recent past which, while not being game-changing in character, have significantly broadened our ability to elucidate flow physics in the laboratory and open up new avenues of analysis, especially for time-dependent flow fields. The examples presented have been chosen to be representative of fields in which considerable scope for collaboration between the engineer and mathematician can immediately be recognized.

One main topic will be addressed, relating to our ability to capture three velocity components of a flow field over three dimensions, and the consequences this capability brings to our understanding of flow fields. Two examples in this topical area will be given; one involving time resolved measurements, the other time averaged. The very complete flow field descriptions at heretofore uncharted time and length scales allow models to be formulated which are much more universal in their validity than models based only on ‘engineering’ correlations. It is at this model formulation stage and/or for purposes of optimization where an accompanying rigorous mathematical description of the fluid dynamics can be invaluable.

**Date:**

Tuesday, Nov. 5 12:20-13:00

## 15 minutes talks

**Pen-Yuan Hsu**

The University of Tokyo, Tokyo

Title:

**On Liouville problems for the planer Navier-Stokes equations with the no-slip boundary condition**

Abstract:

In this work we prove a Liouville type result for the Navier-Stokes equations in the half plane with the no-slip boundary condition. As an application, we extend a geometric regularity criterion [?] of solutions to the Navier-Stokes equations in  $\mathbb{R}^3$  to the case when the domain is a half space  $\mathbb{R}_+^3$  with the Dirichlet condition by applying our Liouville type theorem.

This talk is based on the joint work with Professor Yoshikazu Giga and Professor Yasunori Maekawa.

References:

- [1] Y. Giga and H. Miura, On vorticity directions near singularities for the Navier-Stokes flows with infinite energy, *Comm. Math. Phys.* **303** (2011) 289-300.

Date:

Thursday, Nov. 7 17:05-17:20

**Yuto IMAI**

Waseda University, Tokyo

Title:

**Quaternifications and extensions of current algebras on  $S^3$** 

Abstract:

Let  $(\mathfrak{g}, [\cdot, \cdot]_{\mathfrak{g}})$  be a complex Lie algebra and  $U(\mathfrak{g})$  be the enveloping algebra of  $\mathfrak{g}$ . Let  $\mathbf{H}$  be the quaternions and  $S^3\mathbf{H}$  be the space of  $\mathbf{H}$ -valued mappings on  $S^3$ . We introduce a Lie algebra structure on  $S^3\mathfrak{g}^{\mathbf{H}} = S^3\mathbf{H} \otimes U(\mathfrak{g})$ . Then we introduce a 2-cocycle on  $S^3\mathfrak{g}^{\mathbf{H}}$  and the corresponding central extension  $S^3\mathfrak{g}^{\mathbf{H}} \oplus (\mathbf{C}a)$ . As a Lie subalgebra of  $S^3\mathbf{H}$  we have the Lie algebra of Laurent polynomial spinors  $\mathbf{C}[\phi^{\pm(m,l,k)}]$  spanned by a complete orthogonal system of eigenspinors  $\{\phi^{\pm(m,l,k)}\}_{m,l,k}$  of the tangential Dirac operator on  $S^3$ . Then  $\mathbf{C}[\phi^{\pm(m,l,k)}] \otimes U(\mathfrak{g})$  is a Lie subalgebra of  $S^3\mathfrak{g}^{\mathbf{H}}$ . Its central extension  $\widehat{\mathfrak{g}}(a)$  is obtained as a Lie subalgebra of  $S^3\mathfrak{g}^{\mathbf{H}} \oplus (\mathbf{C}a)$ . Finally we have a Lie algebra  $\widehat{\mathfrak{g}}$  which is obtained by adding to  $\widehat{\mathfrak{g}}(a)$  a derivation  $d$ . When  $\mathfrak{g}$  is a simple Lie algebra,  $\widehat{\mathfrak{g}}$  is an infinite dimensional simple Lie algebra. We shall investigate the root space decomposition and the Chevalley generators of  $\widehat{\mathfrak{g}}$ .

Date:

Wednesday, Nov. 6 17:30-17:45

## **Takahito KASHIWABARA**

The University of Tokyo, Tokyo

Title:

**On the Stokes equation with a generalized Robin boundary condition arising from fluid-structure interaction**

Abstract:

We consider some “generalized” Robin boundary condition for the incompressible Stokes problem which reads:

$$\begin{cases} -\nu\Delta\mathbf{u} + \nabla p = \mathbf{f}, & \operatorname{div} \mathbf{u} = 0 & \text{in } \Omega, \\ \boldsymbol{\sigma}(\mathbf{u}, p)\mathbf{n} + \alpha\mathbf{u} - \beta\operatorname{div}_\Gamma\mathbf{\Pi}_\Gamma(\mathbf{u}) + \beta\kappa\mathbf{\Pi}_\Gamma(\mathbf{u})\mathbf{n} = \mathbf{h} & & \text{on } \Gamma = \partial\Omega, \end{cases}$$

where  $\boldsymbol{\sigma}$  and  $\mathbf{\Pi}_\Gamma$  are fluid and membrane stress tensors given by

$$\begin{aligned} \boldsymbol{\sigma}(\mathbf{u}, p) &= -p\mathbf{I} + \nu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T), \\ \mathbf{\Pi}_\Gamma(\mathbf{u}) &= \lambda\operatorname{div}_\Gamma\mathbf{u}\mathbf{I} + \mu(\nabla_\Gamma\mathbf{u} + (\nabla_\Gamma\mathbf{u})^T). \end{aligned}$$

Here,  $\alpha, \beta, \lambda, \mu, \nu$  are positive constants;  $\mathbf{n}$  is the outer unit normal on  $\Gamma$ ;  $\nabla_\Gamma$  and  $\operatorname{div}_\Gamma$  are the surface gradient and divergence operators;  $\kappa = \operatorname{div}_\Gamma\mathbf{n}$  is the mean curvature of  $\Gamma$ . Such a boundary condition is motivated from some fluid-structure interaction problems in hemodynamics.

We establish the well-posedness of the PDE problem above and its finite element approximation. The key feature is the introduction of Sobolev spaces which admit equal-order regularity both in  $\Omega$  and on  $\Gamma$ , i.e.,  $H^m(\Omega; \Gamma) = \{v \in H^m(\Omega); v|_\Gamma \in H^m(\Gamma)\}$ ,  $m = 1, 2, \dots$ . This is a joint work with C. M. Colciago (EPFL) and Prof. A. Quarteroni (EPFL and MOX).

Date:

Thursday, Nov. 7 17:50-18:05

## **Hana Mizerová**

Johannes Gutenberg University Mainz, Mainz

with Mária Lukáčová (Mainz Univ.)

Title:

**Numerical analysis of a viscoelastic fluid flow**

Abstract:

We consider the viscoelastic model describing the behavior of some polymeric fluids. The polymer molecules are treated as two beads connected by a nonlinear



spring. The Peterlin approximation of the spring force is used to derive the equation for the conformation tensor. The first aim of this short talk is to present the existence results for this model. The second part of the talk is dedicated to the error estimates for the suitable approximation of some viscoelastic models. We apply the method of characteristics on the convective term to get the numerical results.

Date:

Friday, Nov. 8 17:05-17:20

## Naofumi MORI

Title: Kyusyu University, Fukuoka

**Dissipative structure for the Timoshenko system with Cattaneo's type heat conduction**

Abstract:

In this talk we consider the Timoshenko system by introducing the heat conduction satisfying the Cattaneo law in the one-dimensional whole space. The system is written in the form

$$\begin{cases} \varphi_{tt} - (\varphi_x - \psi)_x = 0, \\ \psi_{tt} - a^2\psi_{xx} - (\varphi_x - \psi) + b\theta_x = 0, \\ \theta_t + q_x + b\psi_{tx} = 0, \\ \tau_0 q_t + q + \kappa\theta_x = 0, \end{cases}$$

where  $t \geq 0$  is the time variable,  $x \in \mathbb{R}$  is the spacial variable which denotes the point on the center line of the beam,  $\varphi$  is the transversal displacement,  $\psi$  is the rotation angle of the beam,  $\theta$  denotes the temperature,  $q$  denotes the heat flow, and  $a, b, \kappa$  and  $\tau_0$  are positive constants.

We observe that the dissipative structure of the system is of the regularity-loss type which is a little different from that of the dissipative Timoshenko system studied in [?], though they are both regarded as symmetric hyperbolic systems. Besides, we also see that the dissipative structure of the system is the same as that of the system with Fourier's type heat conduction but the condition in which the decay property is the regularity-loss type is different.

Moreover, we show the optimal  $L^2$  decay estimates of the solution in a general situation. The key of the proof is to show the detailed pointwise estimates of the solution in the Fourier space.

References:

- [1] K. Ide, K. Haramoto and S. Kawashima, Decay property of regularity-loss type for dissipative Timoshenko system, *Math. Models Meth. Appl. Sci.*, **18** (2008), 647-667.

Date:

Thursday, Nov. 7 17:30-17:45

## **Tomoyuki NAKATSUKA**

Nagoya University, Nagoya

Title:

**On uniqueness of symmetric Navier-Stokes flows around a body in the plane**

Abstract:

In this talk, we consider the uniqueness of symmetric weak solutions to the stationary Navier-Stokes equation in two-dimensional exterior domains. It is known that, under suitable symmetry conditions on the domain and the data, the problem admits at least one symmetric weak solution tending to zero at infinity. Thus far, the existence of a weak solution tending to zero at infinity is not obtained without symmetry.

Given two symmetric weak solutions  $u$  and  $v$ , we show that if  $u$  satisfies the energy inequality  $\|\nabla u\|_2^2 \leq (f, u)$  and  $\sup(|x| + 1)|v(x)|$  is sufficiently small, then  $u = v$ . As an application, our uniqueness theorem, together with the recent result of Yamazaki(2011), gives some information on the asymptotic behavior of a symmetric weak solution corresponding to small data. The proof of the uniqueness theorem relies upon the density property for the solenoidal vector field  $C_{0,\sigma}^\infty$  and the Hardy inequality for symmetric functions.

Date:

Wednesday, Nov. 6 17:50-18:05

## **Martin RAPP**

Technical University of Darmstadt, Darmstadt

Title:

**Variable Exponent Spaces and Their Applications to Fluid Dynamics**

Abstract:

We are interested in solving equations modelling the behaviour of electrorheological fluids in a bounded domain  $\Omega$ . These fluids have a viscosity which depends on the strength of an electrical field which we assume to be non-constant in space such that the diffusive term is modelled by the  $p(\cdot)$ -Laplacian

$$-\operatorname{div}(\delta + \nabla u)^{p(\cdot)-2} \nabla u,$$

where  $p : \Omega \rightarrow [1, \infty)$  is a variable exponent and  $\delta \geq 0$ .

First the theory of Lebesgue and Sobolev Spaces with variable exponents will be

introduced. Then we will solve the Navier-Stokes equations for electrorheological fluids and the following convection-diffusion equation

$$\begin{aligned} -\operatorname{div}(\delta + \nabla u)^{p(\cdot)-2} \nabla u + \mathbf{a} \cdot \nabla u &= f && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega. \end{aligned}$$

for some given  $\mathbf{a} \in L_{\sigma}^{r(\cdot)}(\Omega)$  by the developed theory.

Date:

Thursday, Nov. 7 16:40-16:55

## Jonas SAUER

Technical University of Darmstadt, Darmstadt

Fachbereich Mathematik, Technische Universität Darmstadt, 64283 Darmstadt,  
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Title:

### **Navier-Stokes Flow in Spatially Periodical Domains**

Abstract:

We investigate maximal regularity in  $L^q(G)$ -spaces of the abstract Stokes operator  $\mathcal{A}_{G,q}$  in the locally compact abelian group  $G := \mathbb{R}^{n-1} \times T$ , where  $T$  is the one-dimensional torus and the abstract Stokes operator is defined in an obvious way on  $L_{\sigma}^q(G)$ . This is achieved by using abstract harmonic analysis in order to establish a concept of the class of Muckenhoupt weights  $A_p(G)$  similar to (and including) the classical concept in the setup of  $\mathbb{R}^n$ . In fact, this concept carries over to a much larger class of locally abelian groups. With the Muckenhoupt weights at hand, we prove a group version of the extrapolation theorem, corresponding to the classical extrapolation theorem due to García-Cuerva and Rubio de Francia [1]. Using this, we show the  $\mathcal{R}$ -boundedness of the operator family  $\lambda(\lambda - \mathcal{A}_{G,q})^{-1}$  in the weighted space  $L_{\omega,\sigma}^q(G)$  of solenoidal vector functions.

The group ansatz also furnishes us with unique solvability of the linear Stokes equations in  $G$  even for non-homogeneous divergence data. This result may be transferred in the fashion of Farwig and Sohr [2,3] to the periodic half space  $\mathbb{R}_+^{n-1} \times T$  and due to the non-homogeneous divergence data also to periodical cylindrical domains with non-constant cross-section.

References:

- [1] *García-Cuerva, J. and Rubio de Francia, J. L.*: Weighted norm inequalities and related topics. *North-Holland Publishing Co., Amsterdam*, Volume 116, 1985
- [2] *Farwig, R. and Sohr, H.*: Weighted  $L^q$ -theory for the Stokes resolvent in exterior domains. *J. Math. Soc. Japan*, **49**(2) (1997), 251–288

- [3] *Farwig, R. and Sohr, H.*: Generalized resolvent estimates for the Stokes system in bounded and unbounded domains. *J. Math. Soc. Japan*, **46**(4) (1994), 607–643

Date:

Tuesday, Nov. 5 17:50-18:05

## **Katharina SCHADE**

Technical University of Darmstadt, Darmstadt

with M. Hieber and J. Prüss

Title:

**A thermodynamically consistent extension of the simplified Ericksen-Leslie model**

Abstract:

The simplified Ericksen-Leslie model [?, ?] describes the flow of nematic liquid crystals. By imposing the first and second law of thermodynamics we obtain a temperature-dependent extension which is hence *a priori* thermodynamically consistent.

By treating the problem in a quasilinear setting similarly to [?], we subsequently show the local existence of strong solutions and global existence of strong solutions near equilibria as well as global existence of solutions which are eventually bounded. Furthermore, we obtain real-analyticity of the solution using Angenent's trick.

References:

- [1] M. Hieber, M. Nesensohn, J. Prüss and K. Schade, *Dynamics of Nematic Liquid Crystals: The Quasilinear Approach*, submitted.
- [2] F.M. Leslie, *Some constitutive equations for liquid crystals*, Arch. Rational Mech. Anal. **28**(1968), 265–283.
- [3] F.H. Lin and C. Liu, *Nonparabolic dissipative systems modeling the flow of liquid crystals*, Comm. Pure Appl. Math. **48**(1995), 501–537.

Date:

Tuesday, Nov. 5 17:30-17:45

## **Bangwei SHE**

Johannes Gutenberg University Mainz, Mainz

Title:

**Numerical simulation on some viscoelastic flows**

Abstract:

The aims of this short presentation are twofolds:

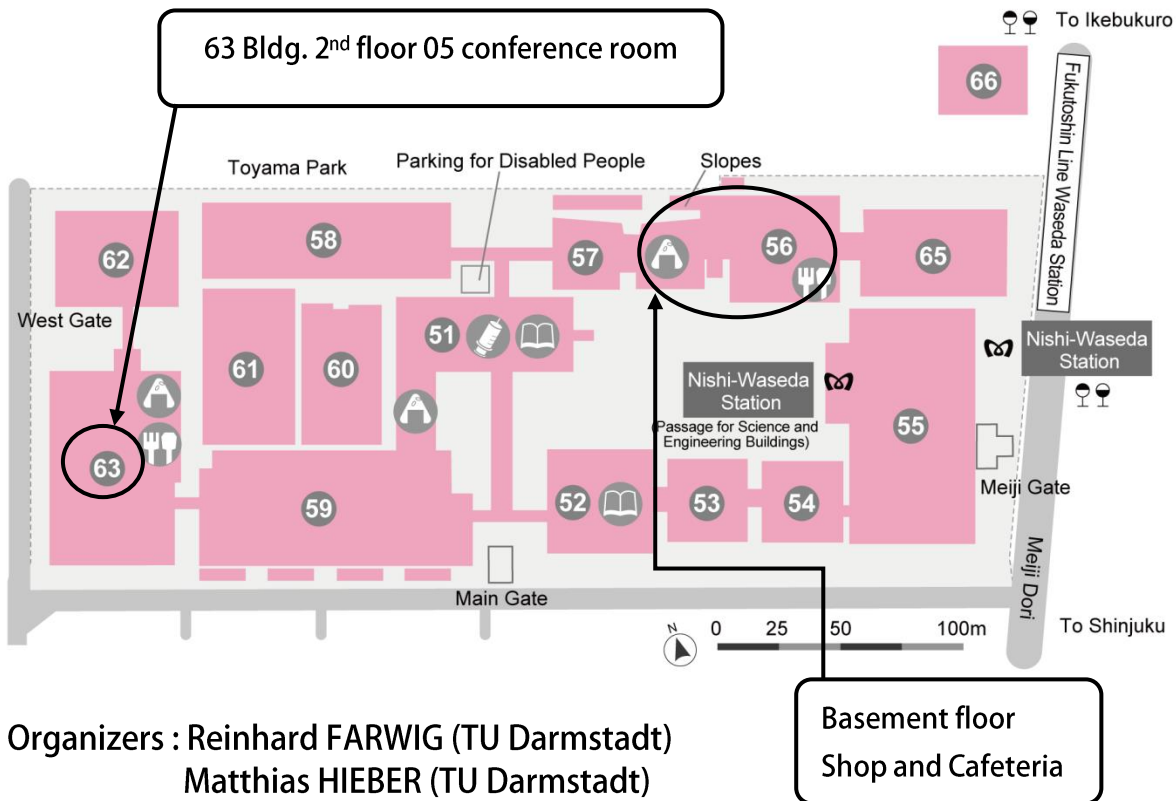
first we give an overview of some models for viscoelastic fluids and discuss existence results of weak solutions. We will also highlight a connection between open

analytical questions and numerical simulations indicating blow up. Moreover we present some stabilization techniques used in the numerical methods. In the second part, we will introduce the characteristic method and present its application for these models.

Date:

Friday, Nov. 8 16:40-16:55

## Nishi-Waseda Campus Map



Organizers : Reinhard FARWIG (TU Darmstadt)  
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 Hideo KOZONO (Waseda University)  
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- Institute of Mathematical Fluid Dynamics, Waseda University
- JSPS Grant No. 24224003, Mathematical theory of turbulence  
 by the method of modern analysis and computational science (Hideo KOZONO)
- JSPS Grant No.24224004, Construction of to investigate the fluid structure  
 from the macroscopic view point and the mesoscopic view point (Yoshihiro SHIBATA)

