On the Stokes semigroup in some non-Helmholtz domains

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June 2014

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Contents

1. Introduction

Stokes system:

$$v_t - \Delta v + \nabla q = 0, \ \operatorname{div} v = 0 \ \text{ in } \ \Omega \times (0, T)$$

B.C. $\quad v = 0 \ \text{ on } \ \partial\Omega$

I.C. $\quad v|_{t=0} = v_0 \ \text{ in } \ \Omega$

Here Ω is a uniformly C^3-domain in $\mathbf{R}^n (n \geq 2)$

v : unknown velocity field

q : unknown pressure field

v_0 : a given initial velocity

Problem 1. Is the solution operator (called the Stokes semigroup) $S(t): v_0 \mapsto v(\cdot, t)$ an analytic semigroup in L^∞-type spaces?

In other words, is there $C > 0$ s.t.

$$\left\| \frac{d}{dt} S(t) f \right\|_X \leq \frac{C}{t} \|f\|_X, t \in (0,1), f \in X$$

where X is an L^∞-type Banach space?

Analyticity is a notion of regularizing effect appeared in parabolic problems in an abstract level.

Definition of analyticity

Definition 1 (semigroup). Let $S = \{S(t)\}_{t>0}$ be a family of bounded linear operators in a Banach space X. In other words, $\{S(t)\}_{t>0} \in L(X)$. We say that S is a **semigroup** in X if

(i) (semigroup property) $S(t)S(\tau) = S(t+\tau)$ for $t, \tau > 0$

(ii) (strong continuity) $S(t)f \to S(t_0)f$ in X as $t \to t_0$ for all $t_0 > 0$, $f \in X$

(iii) (non degeneracy) $S(t)f = 0$ for all $t > 0$ implies $f = 0$.

(iv) (boundedness) $\|S(t)\|_{op} \leq {}^{\exists}C$ for $t \in (0,1)$

Definition 2 (non C_0 analytic semigroup)**.** Let S be a semigroup in X. We say that S is **analytic** if ${}^{\exists}C > 0$ such that

$$\left\|\frac{d}{dt}S(t)\right\|_{op} \leq \frac{C}{t}, t \in (0,1).$$

See a book [ABHN] W. Arendt, Ch. Batty, M. Hieber, F. Neubrander, Vector-valued Laplace transforms and Cauchy problems, Birkhäuser (2011).

Definition 3 (C_0-semigroup). A semigroup S is called ***C_0*-semigroup** if $S(t)f \to f$ as $t \downarrow 0$ for all $f \in X$.

Remark. The name of analyticity stems from the fact that $S = \{S(t)\}_{t \geq 0}$ can be extended as a holomorphic function to a sectorial region of t i.e. $|\arg t| < \theta$ with some $\theta \in (0, \pi/2)$.

A classical problem

Problem 2. Is the solution operator $S(t)$ an analytic semigroup in L^r-type spaces?

This problem has a long history.

V. A. Solonnilov '77, Y. G. '81, ……

A simple example

Heat semigroup (Gauss – Weierstrass semigroup)

$$(H(t)f)(x) = e^{t\Delta}f = G_t * f$$

$$= \int_{\mathbf{R}^n} G_t(x-y)f(y)dy$$

$$G_t(x) = \frac{1}{(4\pi t)^{n/2}} \exp(-|x|^2/4t)$$

Proposition 1. (i) The family $H = \{H(t)\}_{t>0}$ is a non C_0-analytic semigroup in $L^\infty(\mathbf{R}^n)$

(and also in $BC(\mathbf{R}^n)$)

but a C_0-analytic semigroup in $BUC(\mathbf{R}^n)$

(and also in $C_0(\mathbf{R}^n)$)

(ii) The family H is a C_0-analytic semigroup in $L^r(\mathbf{R}^n)$ for all $1 \leq r < \infty$.

$$BC(\mathbf{R}^n) = C(\mathbf{R}^n) \cap L^\infty(\mathbf{R}^n)$$

$$BUC(\mathbf{R}^n) = \{ f \in BC(\mathbf{R}^n) | f\text{: uniformly continuous}\}$$

$$C_0(\mathbf{R}^n) = L^\infty\text{-closure of } C_c^\infty(\mathbf{R}^n) = \left\{ f \in C(\mathbf{R}^n) \middle| \lim_{|x|\to\infty} f(x) = 0 \right\}$$

Spaces for divergence free vector fields

$C_{c,\sigma}^{\infty}(\Omega) = \{ f \in C_c^{\infty}(\Omega) | \operatorname{div} f = 0\}$

$=$ the space of all smooth solenoidal vector fields with compact support

$C_{0,\sigma}(\Omega) = L^{\infty}$-closure of $C_{c,\sigma}^{\infty}(\Omega)$

$= \{ f \in C(\overline{\Omega}) \mid \operatorname{div} f = 0$ in $\Omega, f = 0$ on $\partial\Omega\}$

if Ω is bounded. (Maremonti '09)

$L_{\sigma}^{r}(\Omega) = L^{r}$-closure of $C_{c,\sigma}^{\infty}(\Omega)$, $1 \leq r < \infty$

More spaces

Helmholtz decomposition (Ω bounded C^1-domain, …)

$\boldsymbol{L^r(\Omega) = L^r_\sigma(\Omega) \oplus G^r(\Omega)}$ $(1 < r < \infty)$, where

$G^r(\Omega) = \{\nabla\pi \in L^r(\Omega) \mid \pi \in L^1_{loc}(\Omega)\}$.

$L^r_\sigma(\Omega) = G^{r'}(\Omega)^\perp$

$$= \left\{ f \in L^r(\Omega) \,\middle|\, \int_\Omega f \cdot \nabla\varphi dx = 0 \text{ for all } \varphi \in G^{r'}(\Omega) \right\}$$

e.g. Fujiwara – Morimoto ’79, Galdi’s book ’11

Here $1/r + 1/r' = 1$

$$L^\infty_\sigma(\Omega) := \left\{ f \in L^\infty(\Omega) \middle| \int_\Omega f \cdot \nabla\varphi dx = 0 \text{ for all } \varphi \in \widehat{W}^{1,1}(\Omega) \right\}.$$

$$C_{0,\sigma}(\Omega) \subset BUC_\sigma(\Omega) \subset L^\infty_\sigma(\Omega)$$

Analyticity results for the Stokes semigroup $S(t) = e^{-tA}$ in L^r_σ (= L^r-closure of $C^\infty_{c,\sigma}$)

(i) L^2_σ: easy since the Stokes operator is nonnegative self-adljoint.

(ii) L^r_σ: V. A. Solonnikov '77, Y. G. '81 (bdd domain) (max regularity / resolvent estimate)

… H. Abels – Y. Terasawa '09 (variable coefficient) bdd, exterior, bent half space.

(iii) $\tilde{L}^r_\sigma$ space $= \begin{cases} L^r \cap L^2_\sigma & r \geq 2 \\ L^r + L^2_\sigma & r < 2 \end{cases}$

W. Farwig, H. Kozono and H. Sohr '05, '07, '09

General uniformity C^2-domain / All except

Solonnikov appeals to the resolvent estimate

General results in L^r setting

Theorem 1 (M. Geißert, H. Heck, M. Hieber and O. Sawada, J. Reine Angew. Math., 2012).
If a uniformly C^2 domain Ω admits the Helmholtz decomposition in L^r, then $S(t)$ is a C_0-analytic semigroup in L^r_σ.
(Moreover, the maximum L^r regularity holds.)

Applications to the Navier-Stokes equations — strong solvability

$$v_t - \Delta v + v \cdot \nabla v + \nabla q = 0, \ \operatorname{div} v = 0 \ \text{ in } \ \Omega \times (0, T)$$

B.C. $\quad v = 0 \ \text{ on } \ \partial\Omega$

I.C. $\quad v|_{t=0} = v_0 \ \text{ in } \ \Omega$

T. Kato – H. Fujita ’62: L^2 theory, Ω: bdd,
$H^{1/2}$ initial data ⇒ local existence (3-D)

Y. G. – T. Miyakawa ’85: L^p theory, Ω: bdd,
L^n initial data ⇒ local existence (n-D)

…… H. Amann ’00 Besov theory, …… R. Farwig, H. Sohr, W. Varnhorn ’09 ……

Results in L^∞ setting

Theorem 2 (K. Abe – Y. G., Acta Math., 2013)**.** Let Ω be a bounded C^3-domain in $\mathbf{R}^n (n \geq 2)$. Then the Stokes semigroup $S(t)$ is a C_0-analytic semigroup in $C_{0,\sigma}(\Omega) \, (= BUC_\sigma(\Omega))$. It can be regarded as a non C_0-analytic semigroup in $L^\infty_\sigma(\Omega)$.

Remark. Whole space case is reduced to the heat semigroup. This type of analyticity result had been only known for half space where the solution is written explicitly (Desch – Hieber – Prüss '01, Solonnikov '03)

Theorem 3 (K. Abe – Y. G., J. Evol. Eq., 2012)**.** Let Ω be an C^3-exterior domain in $\mathbf{R}^n$. Then the Stokes semigroup $\{S(t)\}_{t>0}$ is a C_0-analytic semigroup in $C_{0,\sigma}(\Omega)$ and extends to a non C_0-analytic semigroup in $L^\infty_\sigma(\Omega)$. It can be extended as a C_0-analytic semigroup in $BUC_\sigma(\Omega)$.

Note that for an unbounded domain $C_{0,\sigma}(\Omega)$ is strictly smaller than $BUC_\sigma(\Omega)$ because $f \in C_{0,\sigma}(\Omega)$ implies $|f(x)| \to 0$ as $|x| \to \infty$.

Applications to the Navier-Stokes equations — L^∞ theory

Whole space, J. Leray '34 …… Y. G. – K. Inui – S. Matsui '99

Half space, V. A. Solonnikov '03, H.-O. Bae – B. J. Jin '12

$$\|S(t)P\nabla f\|_\infty \le Ct^{-1/2}\|f\|_\infty$$

Bounded and exterior domains by Ken Abe '14:

$$\|S(t)P\nabla f\|_\infty \le Ct^{-\frac{\alpha}{2}}\|f\|_\infty^{\alpha}\|\nabla f\|_\infty^{1-\alpha},\ 0<\alpha<1$$

For strong solutions of N-S,

T : existence time $\qquad T \ge \dfrac{C}{\|v_0\|_\infty^2}$

T_*: possible blow-up time $\qquad \|v(t)\|_\infty \ge C/(T_* - t)^{1/2}$

L^∞-theory for e^{-tL} where L is an elliptic operator

(i) 2^{nd} order operator on $\mathbf{R}$ (one dim): K. Yosida '66

(ii) 2^{nd} order elliptic operator K. Masuda '71, '72 book in '75

L^r theory, cutoff procedure for resolvent and interpolation

(iii) higher order, H. B. Stewart '74, '80

Masuda – Stewart method

(iv) degenerate + mixed B. C. K. Taira, '04

See also: P. Acquistapace, B. Terrani (1987)

A. Lunardi (1995) Book.

More recent. nonsmooth coefficient / nonsmooth domain

Heck – Hieber – Stavarakidis (2010) VMO coeff., higher order

Arendt – Schaetzle (2010) 2^{nd} order, Lipschitz domain

Takuya Suzuki (2014) C^1-domain, any order.

Extensions of L^∞-theory

(i) Theorems 2 and 3 are obtained by a direct analysis of semigroup. It applies to a perturbed half space (K. Abe).

(ii) Resolvent estimate is obtained by extending a method of Masuda – Stewart (Maximum analyticity angle is obtained, K. Abe, Y. G. and M. Hieber, to appear.)

(iii) Cylindrical domains are OK. work in progress (K. Abe, Y. G., K. Schade, T. Suzuki)

(iv) Counterexample for a layer domain $\{0 < x_n < 1\}$ for $n \geq 3$. (L. von Below)

Problem

We know if L^r admits the Helmholtz decomposition, $S(t)$ is analytic in L^r_σ. Is the Helmholtz decomposition really necessary to conclude that $S(t)$ is analytic in L^r_σ? (If $r = 2$, the Helmholtz decomposition always exists and the Stokes operator is self-adjoint so that $S(t)$ is always analytic no matter what Ω is.)

Our answer

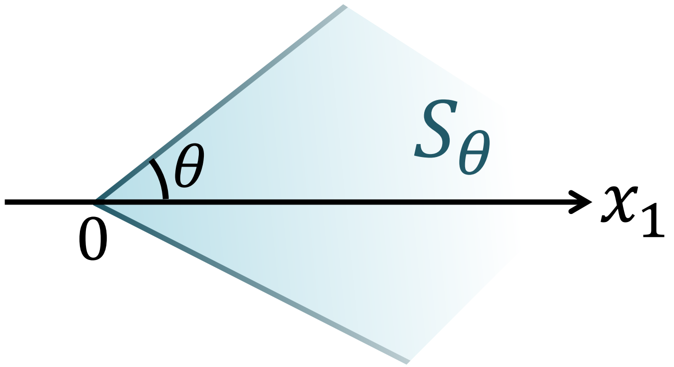
The Helmholtz decomposition in L^r is not a necessary condition so that $S(t)$ is a C_0 analytic semigroup in $L^r_\sigma(\Omega)$.

2. Bogovski's example and main results

$$S_\theta = \{x = (x_1, x_2) \mid |\arg x| < \theta/2\}$$

We say that a planar domain Ω is a **sector-like domain** with opening angle $0 < \theta < 2\pi$ if

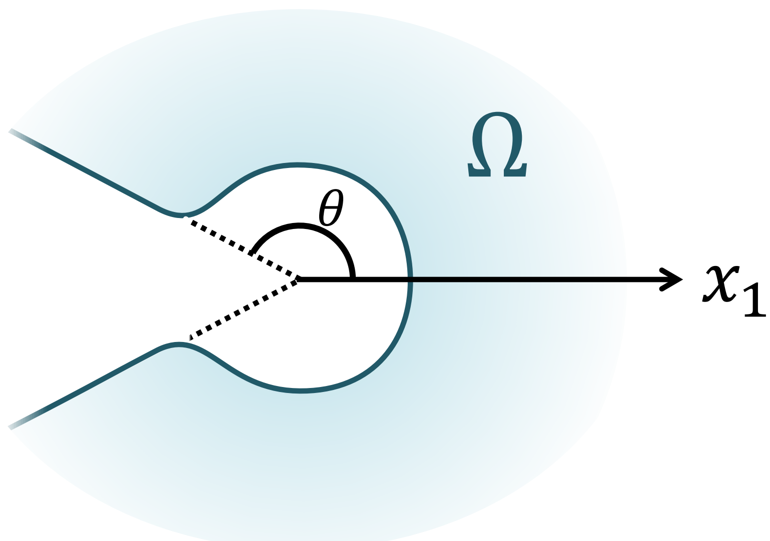
$$\Omega \setminus D_R = S_\theta \setminus D_R$$



for some $R > 0$, where D_R is an open disk of radius R centered at the origin.

Bogovski's example

Proposition 2. The L^r Helmholtz decomposition $(r > 2)$ fails for a smooth sector-like domain when $\theta > \pi \,/\, (1 - 2/r)$.



Note that for $4/3 < \mathrm{r} < 4$, L^r Helmholtz decomposition holds for all $\pi \leq \theta < 2\pi$.

Main results

Theorem 4 [AGSS]. Let Ω $(\subset \mathbf{R}^2)$ be a C^3 sector-like domain. Then the Stokes semigroup $S(t)$ is a C_0-analytic semigroup in $L^r_\sigma(\Omega)$ for any $2 \leq r < \infty$.

This follows from interpolation with L^2 result and the following L^∞-result.

Theorem 5 [AGSS]. Let Ω be a C^3 sector-like domain. Then $S(t)$ is a C_0-analytic semigroup in $C_{0,\sigma}(\Omega)$. (Moreover, $t\|\nabla^2 S(t)v_0\|_\infty \leq C_T\|v_0\|_\infty$ for $t \in (0,T)$.)

AGSS = K. Abe, Y. G., K. Schade, T. Suzuki

Idea of the proof – a blow-up argument
a key observation

(Harmonic) pressure gradient estimate by velocity gradient:

$$\sup_{x\in\Omega} d_\Omega(x)\,|\nabla q(x,t)| \leq C\|\nabla v\|_{L^\infty(\partial\Omega)}(t)$$

$$d_\Omega(x) = \mathrm{dist}\,(x, \partial\Omega)\,.$$

Pressure should be related to velocity

Bounds of ∇q is not enough to guarantee the uniqueness.

Parasitic solution: $v = g(t),\ q(x,t) = -g'(t) \cdot x$ in $\mathbf{R}^n$.

Poiseuille flow type: half space

$$v = (v^1(x_n), 0, \dots, 0), \quad q = f(t)x_1$$

$$\begin{cases} (\partial_t - \Delta)v^1 = -f(t) \\ v^1 = 0 \ \text{ on } \ \{x_n = 0\} \end{cases} \qquad (\operatorname{div} v = 0 \text{ is automatic})$$

3. Neumann problems with singular data

Equations for the pressure

Consider

$$v_t - \Delta v + \nabla q = 0 \quad \text{in} \quad \Omega.$$

Take divergence to get

$$\Delta q = 0 \quad \text{in} \quad \Omega$$

since $\operatorname{div} v = 0$. Take inner product with n_Ω (unit exterior normal) and use $v_t \cdot n_\Omega = 0$ to get

$$\partial q / \partial n_\Omega = n_\Omega \cdot \Delta v \quad \text{on} \quad \partial\Omega.$$

Lemma 1. If $\mathrm{div}\, v = 0$, then

$$n_\Omega \cdot \Delta v = \mathrm{div}_{\partial\Omega} W(v)$$

with

$$W(v) = -(\nabla v - {}^t(\nabla v)) \cdot n_\Omega.$$

In three dimensional case,

$$n_\Omega \cdot \Delta v = -\mathrm{div}_{\partial\Omega}(\omega \times n_\Omega)$$

where $\omega = \mathrm{curl}\, v$ (vorticity). In any case W is a **tangent** vector field.

Neumann problem

The pressure solves

(NP) $$\Delta q = 0 \text{ in } \Omega$$

$$\partial q / \partial n_\Omega = \mathrm{div}_{\partial\Omega} W \text{ on } \partial\Omega.$$

Enough to prove that

$$\|d_\Omega \nabla q\|_\infty \leq C\|W\|_\infty$$

for all **tangential** vector field W.

Strictly admissible domain

Definition 4 (Weak solution of (NP))**.** (Ken Abe – Y. G., ’12) Let Ω be a domain in $\mathbf{R}^n$ $(n \geq 2)$ with C^1 boundary. We call $q \in L^1_{loc}(\overline{\Omega})$ a **weak** solution of (NP) for $W \in L^\infty(\partial\Omega)$ with $W \cdot n_\Omega = 0$ if q with $d_\Omega \nabla q \in L^\infty(\Omega)$ fulfills

$$\int_\Omega q\Delta\varphi dx = \int_{\partial\Omega} W \cdot \nabla\varphi d\mathcal{H}^{n-1}$$

for all $\varphi \in C_c^2(\overline{\Omega})$ satisfying $\partial\varphi/\partial n_\Omega = 0$ on $\partial\Omega$.

Definition 5 (Strictly admissible domain). Let Ω be a uniformly C^1 domain. We say that Ω is **strictly admissible** if there is a constant C such that

$$\|d_\Omega \nabla q\|_\infty \leq C\|W\|_{L^\infty(\partial\Omega)}$$

holds for all weak solution of (NP) for tangential vector fields W. Note that strictly admissibility implies admissibility defined below.

Admissible domain

Let $P: \tilde{L}^r(\Omega) \to \tilde{L}^r_\sigma(\Omega)$ be the Helmholtz projection and $Q = I - P$. Applying Q to the Stokes equation to get

$$\nabla q = Q[\Delta v].$$

Here $\tilde{L}^r = L^r \cap L^2,$

$\tilde{L}^r_\sigma = L^r_\sigma \cap L^2$ for $r > 2.$

Admissible domain (continued)

Definition 6 (Ken Abe – Y. G., Acta Math., 2013)**.** Let Ω be a uniformly C^1-domain. We say that Ω is **admissible** if there exists $r \geq n$ and a constant $C = C_\Omega$ such that

$$\sup d_\Omega(x)\, |Q[\nabla \cdot f](x)| \leq C \|f\|_{L^\infty(\partial\Omega)}$$

hold for all matrix value $f = \left(f_{ij}\right) \in C^1(\overline{\Omega})$ satisfy $\nabla \cdot f \left(= \sum_j \partial_j f_{ij}\right) \in \tilde{L}^r(\Omega)$,

$$\operatorname{tr} f = 0 \text{ and } \partial_\ell f_{ij} = \partial_j f_{i\ell}$$

for all $i, j, \ell = \{1, \dots, n\}$.

Remark. (i) This is a property of the solution of the Neumann problem for the Laplace operator. In fact, $\nabla q = Q[\nabla \cdot f]$ is formally equivalent to

$$-\Delta q = \mathrm{div}(\nabla \cdot f) \quad \text{in} \quad \Omega$$
$$\partial q / \partial n_\Omega = n_\Omega \cdot (\nabla \cdot f) \quad \text{on} \quad \partial\Omega.$$

Under the above condition for f we see that q is harmonic in Ω since

$$\mathrm{div}(\nabla \cdot f) = \sum_{i,j} \partial_i \partial_j f_{ij} = \sum \partial_j \partial_j f_{ii} = 0.$$

(ii) The constant C_Ω depends on Ω but independent of dilation, translation and rotation.

(iii) If Ω is admissible, we easily obtain the pressure gradient estimate by taking $f_{ij} = \partial_j v^i$.

(iv) It turns out that $\partial q/\partial n_\Omega = \mathrm{div}_{\partial\Omega}(n_\Omega \cdot (f - {}^t f))$.

Remark. Strictly admissibility implies admissibility.
Example of strictly admissible domains

(a) half space
(b) C^3 bounded domain
(c) C^3 exterior domain

Note that layer domain $\{a < x_n < b\}$ is not strictly admissible.
Consider $q(x_1, \ldots, x_n) = x_1$.

Theorem 6 (Ken Abe – Y. G., Acta Math., 2013)**.** If Ω is C^3 and admissible, then $S(t)$ is a C_0-analytic semigroup in $C_{0,\sigma}(\Omega)$. (The conclusion of Theorem 5 holds.)

Estimates for harmonic pressure gradient

Theorem 7 (A key step)**.** If Ω is a C^2 sector-like domain, then it is admissible (not strictly admissible).

A strictly admissibility is proved for a bounded domain (K. Abe – Y. G.), exterior domain (K. Abe – Y. G.), a perturbed half space (K. Abe). For a bounded domain there is another proof by C. Kenig, F. Lin, Z. Shen (2013).

Sector-like domain

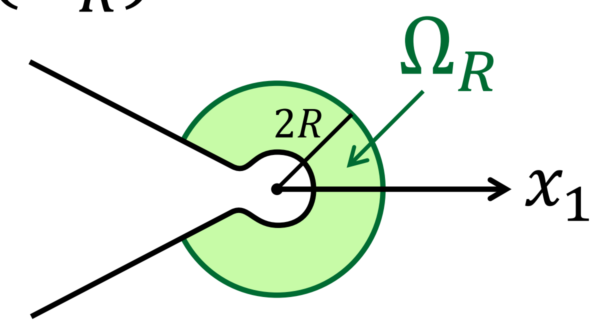
Lemma 2. Let Ω be a C^2 sector-like domain in $\mathbf{R}^2$. Then there is a constant C such that

$$\|d_\Omega(x)\nabla u\|_\infty \leq C\|g\|_\infty$$

for all weak solution $u \in L^1_{loc}(\overline{\Omega}_R)$ of

(NP) $\Delta u = 0$ in Ω_R

$$\frac{\partial u}{\partial n_\Omega} = \mathrm{div}_{\partial\Omega}\, g \quad \text{on} \quad \partial\Omega_R$$



satisfying $g \cdot n_\Omega = 0$ on $\partial\Omega \cap D_{2R}$ and $g = 0$ on $\partial D_{2R} \cap \Omega$ provided that $\|d_\Omega \nabla u\|_\infty < \infty$.

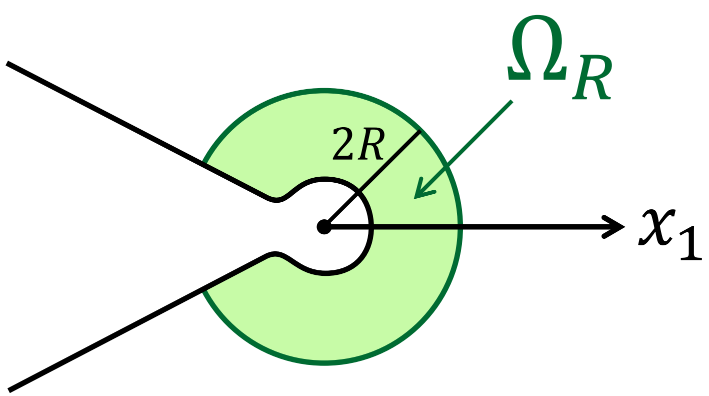
Here $\Omega_R := \Omega \cap D_{2R}$.

Sector-like domain (continued)

Note that C is independent of R. This estimate yields

Lemma 3. There exists a constant C such that all weak solutions $u \in L^1_{loc}(\overline{\Omega})$ of (NP) with $\boldsymbol{\nabla u} \in \boldsymbol{L^2(\Omega)}$ and $g \in L^\infty(\partial\Omega)$ with $g \cdot n_\Omega = 0$ fulfills $\|d_\Omega \nabla u\|_\infty \leq C\|g\|_\infty$.

This yields Theorem 7.



Idea of the proof of Lemma 2

We argue by contradiction $^\exists\{u_m, g_m, R_m\}_{m=1}^{\infty}$, such that

$$1 = \|d_\Omega \nabla u_m\|_{L^\infty(\Omega_{R_m})} > m\|g_m\|_{L^\infty(\partial\Omega \cap D_{2R_m})}$$

Case A $\quad R_m \to \infty$ as $m \to \infty$

Case B $\quad \overline{\lim}_{m\to\infty} R_m < \infty$

We only discuss case A. We take $x_m \in \Omega_{R_m}$ such that

$$|d_\Omega(x_m)\nabla u_m(x_m)| > 1/2\,.$$

We may assume $u_m(x_m) = 0$ by subtracting a constant.

Compactness

Case 1 $\quad {}^{\exists}\text{subsequence}\ \{x_{m_k}\} \to \hat{x}$

Case 2 $\quad |x_m| \to \infty$

Case 1 (a) $\hat{x} \in \Omega$, (**Case 1 (b)** $\hat{x} \in \partial\Omega$)

u_m converges to u locally uniformly with its derivatives in Ω and $u(\hat{x}) = 0$. We now apply the uniqueness to conclude $u \equiv 0$ which contradicts $|d(\hat{x})\nabla u(\hat{x})| \geq 1/2$.

Uniqueness

Lemma 4. Let Ω be a C^2 sector-like domain. Let $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ be a solution of (NP) in Ω with $g = 0$. Assume that $\|d_\Omega \nabla u\|_\infty < \infty$. Then u is a constant function.

Other cases

Case 1 (b) can be handled by uniqueness in a half space with blow-up argument

$$v_m(x) = u_m(x_m + d_m x),\ \ d_m = d_\Omega\,(x_m).$$

Case 2 can be treated by scaling-down argument

$$w_m(x) = u_m(x/|x_m|)$$

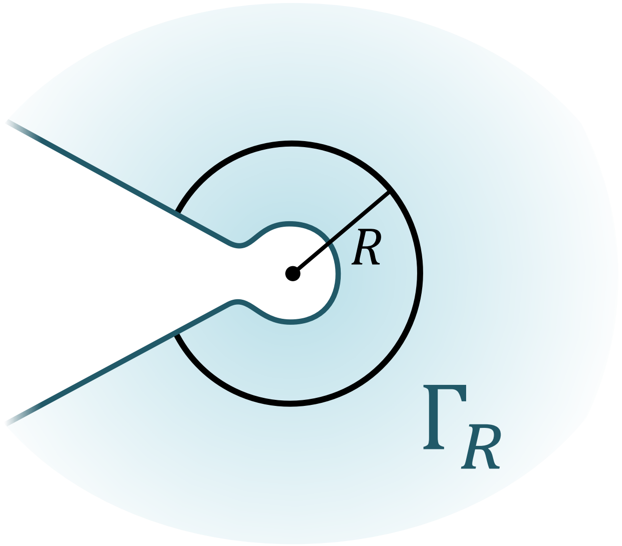
and uniqueness (with zero flux condition) in $S_\theta \cap D_S$ with some $S > 0$ or S_θ.

A key observation for uniqueness

(1) Assumption implies zero flux condition

$$\int_{\Gamma_R \cap \Omega} \frac{\partial u}{\partial r} \, d\mathcal{H}^1 = 0, \qquad R \gg 1,$$

where $\Gamma_R = \partial D_R$. This implies that u is bounded in Ω.



(2) We may assume $\int_{\Gamma_R \cap \Omega} u \, d\mathcal{H}^1 = 0$ for $R \gg 1$. If u attains its maximum, the strong maximum principle implies $u = \mathrm{const.}$

(3) In the case max is not attained, we consider sequence x_m such that $u(x_m) \to \sup u$, $|x_m| \to \infty$. We scale down and obtain a contradiction to the uniqueness result in a sector under zero flux condition.

Summary

- We prove that a C^2 sector-like domain is admissible.
- A similar idea works for a domain with finitely many cylindrical outlets.

Theorem 4. Let $\Omega\ (\subset \mathbf{R}^2)$ be a C^3 sector-like domain. Then the Stokes semigroup $S(t)$ is a C_0-analytic semigroup in $L^r_\sigma(\Omega)$ for any $2 \le r < \infty$.

Theorem 5. Let Ω be a C^3 sector-like domain. Then $S(t)$ is a C_0-analytic semigroup in $C_{0,\sigma}(\Omega)$.
(Moreover, $t\|\nabla^2 S(t) v_0\|_\infty \le C_T \|v_0\|_\infty$ for $t \in (0,T)$.)