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独立行政法人日本学術振興会 日独共同大学院プログラム
JSPS-DFG Japanese-German Graduate Externship

第 11 回 日独流体数学国際研究集会
The 11th Japanese-German International Workshop on Mathematical Fluid Dynamics

March 10 - 13, 2015 at Waseda University, Nishi-Waseda Campus

62W Bldg. 1st Floor High-Tech Research Conference Room 1

■ Program ■

	Tue, Mar. 10	Wed, Mar. 11	Thu, Mar. 12	Fri, Mar. 13
9:30-10:30	Grzegorz KARCH① (Univ. of Wrocław)	KARCH ③	Herbert KOCH ① (Univ. of Bonn)	KOCH ③
10:45-11:45	Herbert EGGER① (TU Darmstadt)	EGGER ②	EGGER ③	EGGER ④
12:00-12:40	Vsevolod SOLONNIKOV (Russian Acad. of Sci.)	Zhouping XIN (Chinese Univ. of Hong Kong)	Maria LUKACOVA (Univ. of Mainz)	Yoshio YAMADA (Waseda Univ.)
12:40-14:10	Lunch break			
14:10-15:10	KARCH ②	KARCH ④	KOCH ②	KOCH ④
15:40-16:20	Reinhard FARWIG (TU Darmstadt)	Itsuko HASHIMOTO (Toyama National Inst. of Tec.)	Hirofumi NOTSU (Waseda Univ.)	Martin ZIEGLER (TU Darmstadt)
16:30-16:45	Jonas SAUER (TU Darmstadt)	Naofumi MORI (Kyusyu Univ.)	Hana MIZEROVA (Univ. of Mainz)	Florian STEINBERG (TU Darmstadt)
16:45-17:00	Go TAKAHASHI (Waseda Univ.)	Katharina SCHADE (TU Darmstadt)	Bangwei SHE (Univ. of Mainz)	Yuki KANEKO (Waseda Univ.)
17:00-17:15	Zoran GRUJIC (Univ. of Virginia) *Special talk	Miho MURATA (Waseda Univ.)	Michael FISCHER (TU Darmstadt)	Dai NOBORIGUCHI (Waseda Univ.)
17:15-17:30		Hirokazu SAITO (Waseda Univ.)	Tobias SEITZ (TU Darmstadt)	Alexander DALINGER (TU Darmstadt)
17:30-17:45		Tomoya KATO (Nagoya Univ.)	Martin BOLKART (TU Darmstadt)	
17:45-18:00				18:00～ Reception

●→Main-course ●→40min talk ●→12min talk

Discussion room → Conference Room 2

Main-Course

Herbert EGGER

Technical University of Darmstadt, Darmstadt

AG Numerical Analysis and Scientific Computing, TU Darmstadt

Title:

Finite Element Methods for Saddlepoint Problems with Application to Darcy and Stokes Flow

(Short course, Waseda University, March 2015)

Abstract:

The goal of this short course is to review the basic ingredients of the analysis and discretization of saddlepoint systems that arise in the weak formulation of fluid flow problems. As one model problem, we consider the slow and steady flow of a viscous incompressible fluid in some bounded domain governed by the Stokes equations

$$\begin{aligned} -\nu\Delta\mathbf{u} + \nabla p &= \mathbf{f}, \qquad \text{in } \Omega,\\ \mathrm{div}\mathbf{u} &= 0, \qquad \text{in } \Omega. \end{aligned}$$

The system is complemented by appropriate boundary conditions. As a second model problem, we study the steady flow in a porous medium described by the Darcy equations

$$\begin{aligned} \alpha\mathbf{u} + \nabla p &= \mathbf{0}, \qquad \text{in } \Omega,\\ \mathrm{div}\mathbf{u} &= f, \qquad \text{in } \Omega. \end{aligned}$$

Again, appropriate boundary conditions are assumed to be given. Both systems can be seen as special cases of the generalized Stokes problem

$$\begin{aligned} -\mathrm{div}(\nu\nabla\mathbf{u}) + \alpha\mathbf{u} + \nabla p &= \mathbf{f} \qquad \text{in } \Omega,\\ \mathrm{div}\mathbf{u} &= f \qquad \text{in } \Omega. \end{aligned}$$

For a proper choice of the parameters ν and α, one directly arrives back at the Darcy and Stokes equations, respectively. The generalized Stokes system may however also be used to model a coupled Darcy–Stokes flow, e.g., a river flowing in a sand bed.

The weak formulation of either of the above systems leads to a saddlepoint problem of the following abstract form: Find $u \in V$ and $p \in Q$ such that

$$\begin{aligned} a(u,v) + b(v,p) &= l(v) \qquad \text{for all } v \in V,\\ b(u,q) &= g(q) \qquad \text{for all } q \in Q, \end{aligned}$$

where a, b, l, g are sutiable bilinear and linear forms defined on Hilbert spaces V and Q. Since two different spaces are involved, problems of this type are called mixed.

Under certain stability conditions on the bilinear forms a and b, the well-posedness of the above system is guaranteed by Brezzi's theorem. The validity of these conditions, which can be shown to be necessary and sufficient for the well-posedness statement, will be discussed in some detail for the model problems under consideration. Using the

representation of bilinear forms by linear operators, the mixed variational problem can also be written equivalently as linear operator equation

$$\begin{aligned} Au + B^*p &= l \qquad \text{in } V', \\ Bu &= g \qquad \text{in } Q'. \end{aligned}$$

As usual, V' and Q' denote the dual spaces of V and Q. The conditions of Brezzi's theorem thus exactly characterize when the operator $L : (u,p) \to (l,g)$ defines an isomorphism between the solution space $V \times Q$ and its dual $V' \times Q'$, the space of admissible data.

As a general strategy for the numerical approximation of variational problems, we intend to investigate the Galerkin approach. Let $V_h \subset V$ and $Q_h \subset Q$ be finite dimensional subspaces, and consider the discrete problem: Find $u_h \in V_h$ and $p_h \in Q_h$ such that

$$\begin{aligned} a(u_h, v_h) + b(v_h, p_h) &= l(v_h) \qquad \text{for all } v_h \in V_h, \\ b(u_h, q_h) &= g(q_h) \qquad \text{for all } q_h \in Q_h. \end{aligned}$$

Well-posedness of this system, which ensures that the discretization method is well-defined, follows with the same arguments as on the continuous level. The stability requirements on the discrete level however imply certain compatibility conditions of the approximation spaces V_h and Q_h that are not always satisfied, in particular, by spaces that may seem reasonable at first sight. Consequences of such a violation will be discussed.

If a stable pair of spaces V_h and Q_h has been selected and the discrete stability conditions hold, then the Galerkin approximation satisfies the following quasi-optimality condition

$$\|u - u_h\|_V + \|p - p_h\|_Q \le C\big(\inf_{v_h \in V_h} \|u - v_h\|_V + \inf_{q_h \in Q_h} \|p - q_h\|_Q\big),$$

i.e., the error of the numerical method is in the order of the best-approximation error. The right hand side of the estimate can then be further analyzed by approximation theory.

For the Galerkin discretization of differential equations, spaces V_h and Q_h consisting of piecewise polynomials have been used with great success. The resulting numerical schemes are called finite element methods. We discuss some choices of such piecewise polynomial spaces for our model problems that formally fit into the Galerkin approach, but violate the compatibility conditions. The consequences will be illustrated and possible remedies will be discussed. Some explicit error estimates for finite element methods using stable pairs of approximation spaces will finally be given.

Contents of the short course

(1) Introduction
presentation of the model problems; weak formulation;

(2) Saddlepoint theory
regular operator equations; Brezzi's theorem; Galerkin methods for mixed variational problems; the role of the discrete inf-sup condition; quasi-best-approximation;

(3) Finite-Elemen-Methods for the Stokes problem
analysis of the Stokes problem; construction of stable finite element approximations;

(4) Finite-Element-Methods for the Darcy problem
analysis of the weak formulation; construction of finite element approximations;

(5) A non-conforming Galerkoin method for the generalized Stokes problem
coupled Darcy-Stokes flow; non-conforming Galerkin methods; construction of a discontinuous Galerkin type finite element method; error estimates;

The plan is to present the contents of sections (1) and (2), and, depending on interest of the audience, either sections (3)-(4) or sections (4)-(5). Lecture notes will be provided.

Further reading

- F. Brezzi and M. Fortin: Mixed and Hybrid Finite Element Methods, Springer, 1991.
- V. Girault and P. A. Raviart: Finite Element Methods for Navier-Stokes Equations, 1986.
- G. Kanschat and B. Riviére: A Strongly Conservative Finite Element Method for the Coupling of Stokes and Darcy Flow. J. Comput. Phys., 229:5933?5943, 2010.
- H. Egger and C. Waluga: A Hybrid Discontinuous Galerkin Method for Darcy-Stokes Problems. Domain Decomposition Methods in Science and Engineering XX, 663-670, 2013.

Date:

① Tuesday, Mar. 10 10:45-11:45 ② Wednesday, Mar. 11 10:45-11:45
③ Thursday, Mar. 12 10:45-11:45 ④ Friday, Mar. 13 10:45-11:45

Grzegorz KARCH

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Title:

Large time behavior of solutions to the Navier-Stokes system in unbounded domains

Abstract:

LECTURE 1: Scaling method in simplest equations from the fluid dynamic

In this lecture, I shall explain an idea of a self-similar solution of an evolution equation using simplest linear and nonlinear equations which appear in fluid dynamics.

A function $u(x,t)$ is called a *self-similar* solution of an evolution equation if its value at a given time t_0 (*e.g.* at $t_0 = 1$) it sufficient to calculate this function for all other values of time $t > 0$ via so-called self-similar transformation. Self-similar solutions have always played in important role in the study of properties of other solutions to linear and nonlinear evolution equations. Very often, one can find explicit self-similar solutions which describe typical properties of other solutions. For example, the Gauss-Weierstrass kernel

$$G(x,t) = (4\pi t)^{-n/2} e^{-|x|^2/(4t)}$$

is the most famous solution of the heat equation $u_t = \Delta u$. This explicit solution appears in the asymptotic expansions as $t \to \infty$ of other solutions of the initial value problem for the heat equation supplemented with integrable initial conditions.

As another example, I shall speak about the large time behavior of solutions to the Cauchy problem for the convection-diffusion equation:

$$u_t - u_{xx} + (|u|^q)_x = 0, \qquad x \in \mathbb{R}\,,\ t > 0\,, \tag{1}$$

$$u(x,0) = u_0(x), \qquad x \in \mathbb{R}\,, \tag{2}$$

where $q > 1$. For integrable initial data with non-zero mass, i.e.,

$$u_0 \in L^1(\mathbb{R}) \qquad \text{and} \qquad \int_{\mathbb{R}} u_0(x)\ dx = M \neq 0,$$

this question has already been investigated and the outcome may be summarized as follows.

- If $q > 2$, the large time dynamics of solutions to (10)-(12) is dominated by the diffusion and u behaves as the self-similar solution $M\,G$ to the linear heat equation with $G(x,t) = (4\pi t)^{-1/2} \exp(-x^2/(4t))$ for $(x,t) \in \mathbb{R} \times (0,\infty)$.
- If $q \in (1,2)$ the convection term (10) is preponderant for large times and u behaves as the N*-wave self-similar function* which is a solution to the nonlinear conservation law $N_t + (|N|^q)_x = 0$. This self-similar asymptotic profile is given explicitly

by the formula

$$N_{\alpha,\beta}(x,t) = \begin{cases} \operatorname{sign} x \cdot \left(\dfrac{|x|}{qt}\right)^{1/(q-1)}, & -q\left(\dfrac{\alpha}{q-1}\right)^{(q-1)/q} \leq \dfrac{x}{t^{1/q}} \leq q\left(\dfrac{\beta}{q-1}\right)^{1/(q-1)}, \\ 0 & \text{otherwise}, \end{cases}$$

with certain constants $\alpha \geq 0$ and $\beta \geq 0$.

- In the critical case $q = 2$, equation (10) has an explicit self-similar solution which describes the large time behavior of other solutions.

I shall present and discuss so-called *scaling method* which allows to prove such results. This is rather universal technique which can be applied to several other models.

References

1. S. Benachour, G. Karch, Ph. Laurencot, *Asymptotic profiles of solutions to convection-diffusion equations*, C. R. Math. Acad. Sci. Paris, 338 (2004), 369–374.
2. P. Biler, G. Karch, and W. A. Woyczyński, *Critical nonlinearity exponent and self-similar asymptotics for Lévy conservation laws*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 18 (2001), 613–637.
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5. M.-H. Giga, Y. Giga, and J. Saal, *Nonlinear partial differential equations*, Progress in Nonlinear Differential Equations and their Applications, 79, Birkhäuser Boston Inc., Boston, MA, 2010. Asymptotic behavior of solutions and self-similar solutions.
6. G. Karch and K. Suzuki, *Spikes and diffusion waves in a one-dimensional model of chemotaxis*, Nonlinearity, 23 (2010), 3119–3137.
7. J. L. Vázquez, *Asymptotic beahviour for the porous medium equation posed in the whole space*, J. Evol. Equ., 3 (2003), 67–118. Dedicated to Philippe Bénilan.

LECTURE 2: Self-similar solutions to the Navier-Stokes system

In this lecture, we describe a method which allows to construct global-in-time solutions to certain evolution equations supplemented with sufficiently small initial conditions. In this approach, self-similar solutions and stationary solutions can be treated in the same way.

As a particular example, I consider the Navier–Stokes equations, describing the evolution of the velocity field u and pressure p of a three-dimensional incompressible viscous fluid at time t and the position $x \in \mathbb{R}^3$. These equations are given by

$$u_t - \Delta u + (u \cdot \nabla)u + \nabla p = F, \tag{3}$$

$$\operatorname{div} u = 0, \tag{4}$$

$$u(0) = u_0. \tag{5}$$

where the external force F and initial velocity u_0 are assigned.

Assume, for a moment, that $F \equiv 0$. Homogeneity properties of the system (3)–(4) imply that if u solves it, then the rescaled function

$$u_\lambda(x,t) = \lambda u(\lambda x, \lambda^2 t)$$

is also a solution for each $\lambda > 0$. Thus, it is natural to consider solutions which satisfy the scaling invariance property $U_\lambda \equiv U$ for all $\lambda > 0$, i.e. *forward self-similar* solutions. Choosing $\lambda = t^{1/2}$ we obtain, the self-similar transformation

$$U(x,t) = t^{-1/2} U(x/t^{1/2}, 1) \tag{6}$$

with the self-similar profile $U(y) \equiv U(y,1)$. Notice that if $u_\lambda \equiv u$ for all $\lambda > 0$, then from the self-similar form (6), the initial condition (5) $\lim_{t \searrow 0} u(x,t)$ is a distribution homogeneous of degree -1.

In the lecture, I explain how to construct solutions with the scaling property (6) as solution to problem (3)-(5) with sufficiently small and homogeneous initial conditions. The same reasoning can be applied to the case when non-zero external forces in equation (3) are present. Indeed, if the initial datum u_0 is homogeneous of degree -1 and if the external force $F(x,t)$ satisfies

$$\lambda^3 F(\lambda x, \lambda^2 t) = F(x,t) \quad \text{for all} \quad \lambda > 0, \tag{7}$$

the both sufficiently small, then problem (3)-(5) has a self-similar solution. Note that, in particular, we can take

$$F(x,t) = F(x) = (b_1\delta_0, b_2\delta_0, b_3\delta_0)$$

(the multiples of the Dirac delta) for sufficiently small $|b|$ with leads to the following *explicit, stationary, and self-similar* solution of the Navier-Stokes system

$$v_c^1(x) = 2\frac{c|x|^2 - 2x_1|x| + cx_1^2}{|x|(c|x| - x_1)^2}, \qquad v_c^2(x) = 2\frac{x_2(cx_1 - |x|)}{|x|(c|x| - x_1)^2}, \tag{8}$$

$$v_c^3(x) = 2\frac{x_3(cx_1 - |x|)}{|x|(c|x| - x_1)^2}, \qquad p_c(x) = 4\frac{cx_1 - |x|}{|x|(c|x| - x_1)^2},$$

where $|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$ and c is an arbitrary constant such that $|c| > 1$.

We conclude the lecture by describing and discussing recent result by Jia and Šverák (2014) on the existence of *large* self-similar solutions to problem (3)-(5) with $F \equiv 0$ and with large homogeneous initial conditions.

References

1. M. Cannone, *Harmonic analysis tools for solving the incompressible Navier-Stokes equations*, in Handbook of mathematical fluid dynamics. Vol. III, North-Holland, Amsterdam, 2004, 161–244.
2. M. Cannone and G. Karch, *Smooth or singular solutions to the Navier-Stokes system?*, J. Differential Equations, 197 (2004), 247–274.
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4. T. Kato, *Strong L^p-solutions of the Navier-Stokes equation in $\mathbf{R}^m$, with applications to weak solutions*, Math. Z., 187 (1984), 471–480.
5. P. G. Lemarié-Rieusset, *Recent developments in the Navier-Stokes problem*, vol. 431 of Chapman & Hall/CRC Research Notes in Mathematics, Chapman & Hall/CRC, Boca Raton, FL, 2002.
6. M. Yamazaki, *The Navier-Stokes equations in the weak-L^n space with time-dependent external force*, Math. Ann., 317 (2000), 635–675.

LECTURE 3: Self-similar large time behavior of solutions to the Navier-Stokes system

In Lecture 2, I showed that if we are able to construct local-in-time solutions to a certain evolution equation in a Banach space E and if the equation is invariant under some scaling transformations of the independent and dependent variables, then these two assumptions combined with a scaling property of the space E allow us to obtain global-in-time solutions for suitably small initial data. In this lecture, I show that such an abstract setting allows us also to study large-time behavior of constructed solutions.

As a particular example, I consider again the Navier–Stokes equations, however, such ideas may be directly applied to other parabolic equations with nonlinearities having a certain scaling degree.

Homogeneity properties of the system (3)–(4) imply that this problem has self-similar solutions (6). Such solutions describe the large time behavior of other solutions of problem (3)-(5). Indeed, if

$$\lim_{\lambda\to\infty} \lambda u(\lambda x, \lambda^2 t) = U(x,t)$$

in an appropriate sense, then

$$tu(xt^{1/2}, t) \to U(x,1) \qquad \text{as} \quad t\to\infty$$

(take $t = 1$, $\lambda = t^{1/2}$). During the lecture, I shall present methods which allows to prove both limit relations.

Next, I shall discuss the asymptotic behavior of solutions of the Navier-Stokes system in a two dimensional exterior domain. First, however, we recall that the Navier-Stokes system in the whole space $\mathbb{R}^2$ has an explicit self-similar solution called the Lamb-Oseen vortex

$$\Theta(t,x) = \frac{x^\perp}{2\pi|x|^2}\Big(1 - e^{-\frac{|x|^2}{4t}}\Big), \qquad \text{with} \quad x^\perp = (x_2, -x_1), \tag{9}$$

which appears in the large time expansions of other infinite energy solutions of this system in $\mathbb{R}^2$.

In this part of my lecture, my aim is to show that the scaling method allows us to prove an analogous result on the large time behavior of solutions of the 2D Navier-Stokes equations in an exterior domain $\Omega \subset \mathbb{R}^2$ with the Dirichlet boundary condition.

Let us be more precise. Assume that $\Omega \subset \mathbb{R}^2$ is an exterior domain, whose complement is a bounded, open, connected and simply connected set, with a smooth boundary Γ. We consider the incompressible Navier-Stokes equations in Ω with the Dirichlet boundary condition

$$\partial_t u - \Delta u + u\cdot\nabla u + \nabla p = 0, \qquad \operatorname{div} u = 0 \qquad \text{for} \quad t>0, \quad x\in\Omega, \tag{10}$$

$$u(t,x) = 0 \qquad \text{for} \quad t>0, \quad x\in\Gamma, \tag{11}$$

$$u(0,x) = u_0(x) \qquad \text{for} \quad x\in\Omega. \tag{12}$$

Above, u_0 must be divergence free and tangent to the boundary. In the following, we assume that the initial condition is of the following particular form

$$u_0 = \widetilde{u}_0 + \alpha H_\Omega \tag{13}$$

where $\widetilde{u}_0 \in L^2_\sigma(\Omega)$ is an arbitrary square integrable, divergence free, and tangent to the boundary vector field, and H_Ω the unique harmonic vector field in Ω (*i.e.* the unique vector field on Ω which is divergence free, curl free, vanishing at infinity, tangent to the boundary, and with circulation equal to 1 on the boundary Γ).

I will sketch the proof of the following theorem.

Theorem 1 *For every $\widetilde{u}_0 \in L^2_\sigma(\Omega)$ there exists a constant $\alpha_0 = \alpha_0(\widetilde{u}_0, \Omega) > 0$ such that for all $|\alpha| \le \alpha_0$ the solution of problem* (10)-(13) *satisfies*

$$\lim_{t\to\infty} t^{\frac{1}{2}-\frac{1}{p}} \|u(t) - \alpha\Theta(t)\|_{L^p(\Omega)} = 0 \tag{14}$$

for each $p \in (2, \infty)$.

In other words, Theorem 1 says that the large time behavior of solutions to the Navier-Stokes system in an exterior domain, supplemented with the Dirichlet boundary condition and particular initial condition (13) is described by the explicit self-similar solution (9) of the Navier-Stokes system.

References

1. M. Cannone and G. Karch, *Smooth or singular solutions to the Navier-Stokes system?*, J. Differential Equations, 197 (2004), 247–274.
2. M.-H. Giga, Y. Giga, and J. Saal, *Nonlinear partial differential equations*, Progress in Nonlinear Differential Equations and their Applications, 79, Birkhäuser Boston Inc., Boston, MA, 2010. Asymptotic behavior of solutions and self-similar solutions.
3. D. Iftimie, G. Karch, and Ch. Lacave, *Self-similar asymptotics of solutions to the Navier-Stokes system in two dimensional exterior domain*, (2011) 1-13. arXiv.org/abs/1107.2054
   This paper will be never published and it exists in the preprint version, only. The result from this paper is a particular case of our more general work published in J. London Math. Soc. (2014).
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LECTURE 4: Global asymptotic stability of solutions to the Navier-Stokes equations

During this lecture, I shall continue a discussion of properties of solutions to the initial-value problem for the Navier-Stokes equations

$$u_t - \Delta u + (u \cdot \nabla)u + \nabla p = F, \quad (x,t) \in \mathbb{R}^3 \times (0,\infty), \tag{15}$$

$$\operatorname{div} u = 0, \tag{16}$$

$$u(x,0) = u_0(x), \tag{17}$$

where $u = \big(u_1(x,t), u_2(x,t), u_3(x,t)\big)$ is the velocity of the fluid and $p = p(x,t)$ the scalar pressure.

There are two main approaches for the construction of solutions to the initial value problem (15)–(17).

In the pioneering paper by Leray, weak solutions to (15)–(17) are obtained for all divergence free initial data $u_0 \in L^2(\mathbb{R}^3)^3$ and $F = 0$. These solutions satisfy equations (15)–(16) in the distributional sense and fulfill a suitable energy inequality. Fundamental questions on regularity and uniqueness of the weak solutions to the 3D Navier-Stokes equations remain open.

The second approach leads to mild solutions which were discussed in previous lectures. These solutions are given by an integral formulation using the Duhamel principle and are obtained by means of the Banach contraction principle. Specifically, mild solutions are known to exist for large initial conditions on a finite time interval. For sufficiently small data, in appropriate scale-invariant spaces, the corresponding mild solutions are global-in-time and their dependence on data is regular.

During this lecture, I describe a certain link between these two approaches, which can be summarized as follows.

Assume that $V = V(x,t)$ is a global-in-time mild solution of (15)–(17), small in some scale-invariant space, which usually is not imbedded in $L^2(\mathbb{R}^3)$. One may show that problem (15)–(16) has a global-in-time weak solution in the sense of Leray corresponding to the initial datum $V(x,0)$ perturbed by an arbitrarily large divergence free L^2-vector field, Moreover, this weak solution converges in the energy norm as $t \to \infty$ to the mild solution $V = V(x,t)$. In other words, we show that a sufficiently small mild solution $V = V(x,t)$ of problem (15)–(16) is, in some sense, an asymptotically stable weak solution of this problem under all divergence free initial perturbations from $L^2(\mathbb{R}^3)^3$.

As a particular case, we obtain an asymptotic stability, under arbitrary large initial L^2-perturbations, of the singular stationary self-similar solutions given by formula (8).

References

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Date:

① Tuesday, Mar. 10 9:30-10:30 ② Tuesday, Mar. 10 14:10-15:10

③ Wednesday, Mar. 11 9:30-10:30 ④ Wednesday, Mar. 11 14:10-15:10

Herbert KOCH

The University of Bonn, Bonn

Title:

Dispersive equations: Wellposedness and solitary waves

Abstract:

1. The spaces U^p and V^p

The scalar differential equation

$$\dot{u} = f(t), \qquad x(0) = u_0$$

has the unique solution

$$u(t) = u_0 + \int_0^t f(s)ds.$$

Important function spaces for u are the space of bounded continuous functions, and the space of functions with derivatives in L^1, related to $f \in L^1$. In different areas including dispersive equations, stochastic differential equations and harmonic analysis one wants to 'interpolate' between this extreme spaces.

We define the p variation of $u : \mathbb{R} \to \mathbb{R}$ for $1 \le p < \infty$ by

$$\omega_p(u) = \sup_{t_1 < t_2 \cdots < t_N} \left(\sum |u(t_{i+1}) - u(t_i)|^p \right)^{1/p}$$

and

$$\|u\|_{V^p} = \max\{\omega_p(u), \|u\|_{sup}\}.$$

These spaces roughly interpolate between bounded functions and functions of bounded variation.

We define scales of function spaces U^p and $V^p_{rc} \subset V^p$ so that

(1) There are the embeddings

$$\dot{B}^{1/p}_{p,1} \subset U^p \subset V^p_{rc} \subset \dot{B}^{1/p}_{p,\infty}.$$

(2) The spaces are invariant under reparametrization.

(3) The dual space of U^p is $V^{p'}$.

2. Strichartz and bilinear estimates

Dispersion is the phenomenon that waves spread out for large time. On the analytic side this property leads to decay of the fundamental solution, often proven by stationary phase, and Strichartz estimates expressing higher space time integrability.

Waves with different velocity interact only in small space time regions. The analytic consequence are bilinear L^2 estimates, which on the Fourier side are convolution estimates for measures supported on transversal hypersurfaces.

In this lecture I will explain these estimates, and how they imply embeddings and estimates for function spaces based on the U^p and V^p spaces of the first lecture.

3. Wellposedness for dispersive equation

The techiques of the previous lectures allow to prove global wellposedness for small data for many dispersive equations. I will discuss this for the Kadomtsev-Petviashvili II equation, for generalized KdV equations and Klein-Gordon equations with quadratic nonlinearities.

4. KdV, mKdV and NLS with rough data

It is an amazing fact that many of the most interesting dispersive equations are integrable in the sense that there are many explicit formulas. In this talk I will focus on the Miura map related to the factorization of Schrödinger operators

$$(-\partial_{xx}^2 + u)\Psi = (\partial_x + v)(-\partial_x + v)\Psi$$

with

$$u = v_x + v^2.$$

The Miura map allows to relate a neighborhood of a soliton to KdV to a neighborhood of 0. I will explain some of its properties and implications for solutions to the KdV equation with a PDE based approach.

Date:

① Thursday, Mar. 12 9:30-10:30 ② Thursday, Mar. 12 14:10-15:10

③ Friday, Mar. 13 9:30-10:30 ④ Friday, Mar. 13 14:10-15:10

40 minutes talks

Reinhard FARWIG

Technical University of Darmstadt, Darmstadt

Title:

Quasi-optimal initial value conditions for the Navier-Stokes equations

Abstract:

Consider weak solutions of the instationary Navier-Stokes system in a three-dimensional domain Ω. It is well-known that an initial value $u_0 \in H^1_{0,\sigma}(\Omega)$ or even $u_0 \in \mathcal{D}(A^{1/4}) \subset L^3_\sigma(\Omega)$, where $A = -P\Delta$ denotes the Stokes operator, admits a unique regular solution in Serrin's class $L^s(0,T;L^q(\Omega))$, $\frac{2}{s}+\frac{3}{q}=1$, $2<s<\infty$, for some $T=T(u_0)$. The optimal class of initial values $u_0 \in L^2_\sigma(\Omega)$ with this property was determined by H. Sohr, W. Varnhorn and R. Farwig (Ann. Univ. Ferrara 55, 89-110 (2009)) and is given by the Besov space $\mathbb{B}^{-2/s}_{q,s} = B^{-1+3/q}_{q,s}$ of solenoidal vector fields u_0 satisfying the condition

$$\int_0^\infty \left(\|e^{-\tau A}u_0\|_q\right)^s d\tau < \infty.$$

This condition is used at (almost) all $t_0 > 0$ along a given weak solution u to find various new regularity and uniqueness conditions.

The above optimal condition can be weakened to $u_0 \in L^2_\sigma(\Omega)$ with finite integral

$$\int_0^\infty \left(\tau^\alpha\|e^{-\tau A}u_0\|_q\right)^s d\tau < \infty$$

where $\frac{2}{s}+\frac{3}{q} = 1-2\alpha$, $0<\alpha<\frac{1}{2}$, see *R. Farwig, Y. Giga, P.-Y. Hsu: Initial values for the Navier-Stokes equations in spaces with weights in time. Dept. Math, Hokkaido Univ. EPrints Server, Preprint Series no. 1060.* This condition can be described by the Besov space $\mathbb{B}^{-1+3/q}_{q,s}$, $q>3$. A weak solution with such a quasi-optimal initial value still satisfies the energy equality on some interval $[0,T)$ and Serrin's condition $u \in L^s(\varepsilon,T;L^q(\Omega))$, $\varepsilon>0$, but the classical Serrin weak-strong uniqueness theorem holds only under additional assumptions to be described in this talk.

Date:

Tuesday, Mar. 10 15:40-16:20

Zoran GRUJIC

University of Virginia, Charlottesville

Title:

Vortex stretching and anisotropic diffusion in three-dimensional viscous incompressible flows

Abstract:

The goal of this lecture is to present a mathematical framework for identifying vortex stretching–anisotropic diffusion as a primary physical mechanism behind the phenomenon of turbulent dissipation. This work is motivated by G.I. Taylor's fundamental paper "Production and dissipation of vorticity in a turbulent fluid" from 1937, as well as by numerical simulations of turbulent flows.

The numerical simulations reveal that the primary geometric signature of the regions of intense vorticity is the one of vortex filaments. In particular, the lengths of the filaments seem to be comparable to the macro-scale of the simulation in view. This, paired with the decrease of the volume of a suitably defined region of intense vorticity, yields an indirect bound on the transversal scale of the filaments – a natural micro-scale of the flow.

It turns out that this scale coincides with the scale of anisotropic diffusion, i.e., the scale of local, anisotropic sparseness of the regions of intense vorticity needed for the anisotropic diffusion to engage and control the sup-norm of the vorticity. In other words – thinking in terms of vortex filaments – we arrive at criticality.

The last step is to show that a very mild, purely geometric condition suffices to produce an extra-log decay of the volume of the region of intense vorticity, breaking the criticality. It is worth noting that within this condition, the vorticity direction is allowed to develop a singularity; consequently, 'crossing of the vortex lines' is not necessarily an obstruction to the program.

This is based on several recent papers, most of them joint work with Z. Bradshaw and R. Dascaliuc.

Date:

Tuesday, Mar. 10 17:00-18:00 *Special Talk

Itsuko HASHIMOTO

Toyama National Institute of Technology, Toyama

Title:

Asymptotic behavior of radially symmetric solutions for Burgers equation in several space dimensions

Abstract:

In this talk, we present the recent results on large-time behavior of radially symmetric solutions for Burgers equation on the exterior of a ball in multi-dimensional space, where boundary data at the far field are prescribed. For this Burgers equation, we assume that the corresponding Riemann problem for the hyperbolic part admits the rarefaction wave as an asymptotic state. In the present problem, because of the Dirichlet boundary condition, the asymptotic states are divided into four cases dependent on the signs of the boundary data and the far field data. In all cases both global existence of the solution and the asymptotic behavior are shown

without smallness conditions. We prove that in case where boundary data equal to 0 or negative, the asymptotic stability are same as one of viscous conservation law on the half line. On the other hand, if boundary data is positive, the asymptotic state is superposition of stationary wave and rarefaction wave, which is new wave phenomena. The proof is given by standard L^2 energy method and method of characteristic curve. Finally we add the small comment on the radially symmetric solutions for Burgers equation where the corresponding Riemann problem for the hyperbolic part admits the viscous shock wave as an asymptotic state.

Date:

Wednesday, Mar. 11 15:40-16:20

Maria LUKACOVA

Johannes Gutenberg University Mainz, Mainz

Title:

Error estimates for some viscoelastic fluids

Abstract:

We present the error analysis of a particular Oldroyd-B type model with the limiting Weissenberg number going to infinity. Assuming a suitable regularity of the exact solution we study the error estimates of a standard finite element method and of a combined finite element/finite volume method. Our theoretical result shows first order convergence of the finite element method and the error of the order O(h3/4) for the finite element/finite volume method. These error estimates are compared and confirmed by the numerical experiments. For the combined finite difference/finite volume scheme the second order convergence can be shown experimentaly.

This work has been done in collaboration with Jan Setbel (Academy of Sciences, Prague), Hana Mizerova and Bangwei She.

Date:

Thursday, Mar. 12 12:00-12:40

Hirofumi NOTSU

Waseda University, Tokyo

Waseda Institute for Advanced Study, Waseda University, Japan

h.notsu@aoni.waseda.jp

Title:

Error estimates of Lagrange-Galerkin finite element schemes for natural convection problems

Abstract:

Two Lagrange-Galerkin finite element (LG-FE) schemes for natural convection problems are mathematically and numerically studied. The one is a conventional LG-FE scheme with a stable element, e.g., P2/P1, and the other is a stabilized LG-FE scheme with the P1/P1 element. The schemes maintain well known two advantages of the Lagrange-Galerkin method, i.e., robustness for convection-dominated problems and symmetry of the resulting matrix.

Many LG-FE schemes for natural convection problems have been proposed and employed in scientific computation, there are, however, no results of error estimates for them to the best of our knowledge. Recently we have established stability and convergence with optimal error estimates of the two LG-FE schemes for natural convection problems. The results are proved based on the idea developed for a corresponding LG-FE scheme for the Navier-Stokes equations. In this talk, we show the core part of the proofs and two- and three-dimensional numerical results.

This is a joint work with Prof. M. Tabata at Waseda University.

Date:

Thursday, Mar. 12 15:40-16:20

Vsevolod SOLONNIKOV

St.Petersburg Department of V.A.Steklov Institute of Mathematics of the Russian Academy of Sciences, St.Petersburg

Title:

Estimates of the solution of a free boundary problem for viscous compressible fluids

Abstract:

We are concerned with the free boundary problem

$$\begin{cases} \rho(x,t)(\boldsymbol{v}_t + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v})) - \nabla\cdot T(\boldsymbol{v}) + \nabla p(\rho) = 0, \\ \rho_t + \nabla\cdot(\rho\boldsymbol{v}) = 0, \quad x \in \Omega_t^+ \cup \Omega_t^-, \quad t > 0, \\ [\boldsymbol{v}] = 0, \quad [-p(\rho)\boldsymbol{n} + T(\boldsymbol{v})\boldsymbol{n}] = 0, \quad V_n = \boldsymbol{v}\cdot\boldsymbol{n}, \quad x \in \Gamma_t, \\ \boldsymbol{v}^-(x,t) = 0, \quad x \in S, \\ \boldsymbol{v}(x,0) = \boldsymbol{v}_0(x), \quad \rho(x,0) = \rho_0(x), \quad x \in \Omega_0^+ \cup \Omega_0^-, \end{cases} \tag{0.1}$$

where $\boldsymbol{v}(x,t) = \boldsymbol{v}^\pm(x,t)$, $\rho(x,t) = \rho^\pm(x,t)$ for $x \in \Omega_t^\pm$, Ω_t^+ and Ω_t^- are bounded domains separated by a free interface $\Gamma_t = \partial\Omega_t^+$ that is given for $t = 0$ and should be found for $t > 0$. The domain $\Omega = \Omega_t^+ \cup \Gamma_t \cup \Omega_t^-$ is fixed; the surface $S = \partial\Omega$ is bounded away from Γ_t. By $T(\boldsymbol{v}) \equiv T^\pm(\boldsymbol{v})$ we mean the viscous part of the stress tensor:

$$T^\pm(\boldsymbol{v}) = \mu^\pm S(\boldsymbol{v}^\pm) + \mu_1^\pm I\nabla\cdot\boldsymbol{v}^\pm, \quad x \in \Omega^\pm,$$

$\boldsymbol{n}$ is the normal to Γ_t exterior with respect to Ω^+ and V_n is the velocity of evolution of Γ_t in the direction $\boldsymbol{n}$, $\mu^\pm, \mu_1^\pm = const > 0$, $[\boldsymbol{u}] = u^+ - u^-$ is the jump of $\boldsymbol{u}$ on Γ_t.

The pressure functions $p^\pm(\rho^\pm)$ are positive strictly increasing functions of a positive argument possessing Lipshitz continuous derivatives.

In the Lagrangean coordinates, the problem (0.1) takes the form

$$\begin{cases} r(\xi,t)\boldsymbol{u}_t - \nabla_u \cdot T_u(\boldsymbol{u}) + \nabla_u p(r) = 0, \\ r_t + r\nabla_u \cdot \boldsymbol{u} = 0, \quad \xi \in \Omega_0^+ \cup \Omega_0^-, \quad t > 0, \\ [\boldsymbol{u}] = 0, \quad [-p(r)\boldsymbol{n} + T_u(\boldsymbol{u})\boldsymbol{n}] = 0, \quad \xi \in \Gamma_t, \\ \boldsymbol{u}^- = 0, \quad \xi \in S, \\ \boldsymbol{u}(\xi,0) = \boldsymbol{v}_0(\xi), \quad r(\xi,0) = \rho_0(\xi), \quad \xi \in \Omega_0^+ \cup \Omega_0^-, \end{cases} \tag{0.2}$$

where $r = \rho(X(\xi,t),t)$, $\boldsymbol{u}(\xi,t) = \boldsymbol{v}(X,t)$,

$$X(\xi,t) = \xi + \int_0^t \boldsymbol{u}(\xi,\tau)d\tau, \quad \xi \in \Omega_0^- \cup \Omega_0^+.$$

We set $\nabla_u = J_u^{-1}A\nabla$, $S_u(\boldsymbol{u}) = (\nabla_u\boldsymbol{u}) + (\nabla_u\boldsymbol{u})^T$, $T_u(\boldsymbol{u}) = \mu S_u(\boldsymbol{u}) + \mu_1 I\nabla_u \cdot \boldsymbol{u}$, $J_u = det\mathcal{L}$, $\mathcal{L} = \left(\frac{\partial X}{\partial \xi}\right)$. The elements of the matrix A are co-factors of the elements $a_{ij} = \delta_{ij} + \int_0^t \frac{\partial u_i}{\partial \xi_j} d\tau$ of the matrix $\mathcal{L}$. The normal $\boldsymbol{n}(X)$ to Γ_t is connected with the normal $\boldsymbol{n}_0(\xi)$ to Γ_0 by

$$\boldsymbol{n}(X) = \frac{A\boldsymbol{n}_0}{|A\boldsymbol{n}_0|}.$$

We set $r = \overline{\rho} + \theta$, where $\overline{\rho} = M/|\Omega|$ is the mean density and $M = \int_\Omega \rho(x,t)dx = \int_\Omega r(\xi,t)J_u(\xi,t)d\xi = const$ is the total mass of the fluid. Clearly, $\int_\Omega \theta(\xi,t)J_u(\xi,t)d\xi = 0$. The system (0.2) can be written in the form

$$\begin{cases} \overline{\rho}\boldsymbol{u}_t - \nabla \cdot T(\boldsymbol{u}) + p'(\overline{\rho})\nabla\theta = \boldsymbol{l}_1(\boldsymbol{u},\theta), \\ \theta_t + \overline{\rho}\nabla \cdot \boldsymbol{u} = l_2(\boldsymbol{u},\theta), \quad \xi \in \Omega, \\ [\boldsymbol{u}] = 0, \quad [-p'(\overline{\rho})\theta\boldsymbol{n}_0 + T(\boldsymbol{u})\boldsymbol{n}_0] = \boldsymbol{l}_3(\boldsymbol{u},\theta), \quad \xi \in \Gamma_0, \\ \boldsymbol{u}(\xi,t) = 0, \quad \xi \in S, \\ \boldsymbol{u}(\xi,0) = \boldsymbol{v}_0(\xi), \quad \theta(\xi,0) = \theta_0(\xi) = \rho_0(\xi) - \overline{\rho}, \end{cases} \tag{0.3}$$

where $\boldsymbol{l}_1(\boldsymbol{u},\theta)$, $l_2(\boldsymbol{u},\theta)$, $\boldsymbol{l}_3(\boldsymbol{u},\theta)$ are some nonlinear functions.

We obtain existence theorems and estimates of solutions of (0.3) in the Sobolev spaces $W_2^{l,l/2}$.

Theorem 1. *Let $S, \Gamma_0 \in W_2^{3/2+l}$, $1/2 < l < 1$, $p \in C^2(\overline{\rho}/2, 3\overline{\rho}/2)$, $\overline{\rho}, p'(\overline{\rho}) > 0$. Then for arbitrary $\boldsymbol{v}_0^\pm \in W_2^{l+1}(\Omega_0^\pm)$, $\rho_0^\pm \in W_2^{l+1}(\Omega_0^\pm)$ such that $\int_\Omega \theta_0(\xi)d\xi = 0$,*

$$\begin{aligned} &\boldsymbol{v}_0(x,0) = 0, \quad x \in S, \quad [\boldsymbol{v}_0] = 0, \quad [-p(\rho_0)\boldsymbol{n}_0 + T(\boldsymbol{u}_0)\boldsymbol{n}_0] = 0, \quad x \in \Gamma_0, \\ &|\rho_0| \geq c_0 > 0, \end{aligned} \tag{0.4}$$

problem (0.3) has a unique solution defined for $t \le T$, and

$$\begin{aligned}&\sum_{\pm}(\|\boldsymbol{v}^{\pm}\|_{W_2^{l+2,l/2+1}(Q_T^{\pm})}+\|\theta^{\pm}\|_{W_2^{l+1,0}(Q_T^{\pm})}+\|\theta_t^{\pm}\|_{W_2^{l+1,0}(Q_T^{\pm})})\\&\le c(T)\sum_{\pm}(\|\boldsymbol{v}_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}+\|\theta_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}),\quad Q_T^{\pm}=\Omega^{\pm}\times(0,T).\end{aligned}\tag{0.5}$$

Theorem 2. *If, in addition, $p^+(\overline{\rho})=p^-(\overline{\rho})$ and*

$$\sum_{\pm}\|\boldsymbol{v}_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}+\sum_{\pm}\|\theta_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}\le\epsilon\ll 1,\tag{0.6}$$

then the solution is defined for all $t>0$, $\int_{\Omega}\theta(\xi,t)J_u(\xi,t)d\xi=0$ and

$$\begin{aligned}&\sum_{\pm}(\|e^{\beta t}\boldsymbol{v}^{\pm}\|_{W_2^{l+2,l/2+1}(Q_\infty^{\pm})}+\|e^{\beta t}\theta^{\pm}\|_{W_2^{l+1,0}(Q_\infty^{\pm})}+\|e^{\beta t}\theta_t^{\pm}\|_{W_2^{l+1,0}(Q_\infty^{\pm})})\\&\le c\sum_{\pm}(\|\boldsymbol{v}_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}+\|\theta_0^{\pm}\|_{W_2^{l+1}(\Omega^{\pm})}),\quad \beta>0,\quad Q_\infty^{\pm}=\Omega^{\pm}\times(0,\infty).\end{aligned}\tag{0.7}$$

Similar results are obtained in the spaces $W_p^{2,1}(Q_T^{\pm})$ with $p>3$.

Date:

Tuesday, Mar. 10 12:00-12:40

Zhouping XIN

The Chinese University of Hong Kong, Hong Kong

Title:

Uniform Estimates on Incompressible Surface Waves

Abstract:

We consider the motion of an incompressible viscous fluid, subject to the influence of gravity and surface tension forces, in a moving domain

$$\Omega(t)=\{x\in\mathbb{R}^3|\quad -b<x_3<h(t,x_1,x_2)\}.$$

The lower boundary of $\Omega(t)$ is assumed to be rigid and given satisfying a Navier-slip condition, but the upper boundary, the graph of the unknown function $h(t,x_1,x_2)$, is advected with the fluid and satisfies the stress balance dynamical boundary condition. Thus the velocity $u(t,\cdot)$, the pressure $p(t,\cdot)$, and $h(t,x_1,x_2)$ solve the following boundary value problem

$$\begin{array}{lll}\partial_t u+u\cdot\nabla u+\nabla p-\varepsilon\Delta u=0 & \text{in} & \Omega(t)\\ \nabla\cdot u=0 & \text{in} & \Omega(t)\\ p\vec{n}-2\varepsilon S(u)\vec{n}=gh\vec{n}-\sigma H\vec{n} & \text{on} & \{x_3=h(t,x_1,x_2)\}\\ \partial_t h=u\cdot\vec{N} & \text{on} & \{x_3=h(t,x_1,x_2)\}\\ u_3=0,\ (S(u)(-e_3))_i=-ku_i,\quad i=1,2, & \text{on} & \{x_3=-b\}\end{array}$$

for $S(u) = \frac{1}{2}(\nabla u + \nabla u^t)$, $\vec{n} = \frac{\vec{N}}{|\vec{N}|}$ with $\vec{N} = (-\partial_1 h, -\partial_2 h, 1)$, $\varepsilon > 0$: the viscosity, $g > 0$: gravity, $\sigma > 0$: the surface tension coefficients, k: the friction coefficient, and H is the mean curvature of the free surface. We prove that these exists a uniform time interval on which one can derive uniform estimates independent of both viscosity and surface tension coefficients. These allow one to justify the vanishing viscosity and surface tension limits by the strong compactness argument. As a byproduct, we can get the unified local well-posendess of the free-surface impressible Euler equations with or without surface tension by the inviscid limits. This is a joint work with Dr. Yanjing Wang.

Date:

Wednesday, Mar. 11 12:00-12:40

Yoshio YAMADA

Waseda University, Tokyo

Title:

Multiple spreading phenomena for a certain class of free boundary problems of reaction-diffusion equations

Abstract:

This talk is concerned with a free boundary problem for reaction-diffusion equations of the form

$$u_t = du_{xx} + uf(u), \quad t > 0,\ 0 < x < s(t),$$

with boundary conditions $u_x(t,0) = u(t,s(t)) = 0$. Here $x = s(t)$ is a fee boundary whose dynamics is determined by Stefan condition $s'(t) = -\mu u_x(t,s(t))$ with $\mu > 0$. For a certain class of reaction functions $f(u)$, it can be shown that the free boundary problem with initial data $(u(0),s(0)) = (u_0,s_0)$ has a unique global solution $(u(t),s(t))$. Our main concern is to study asymptotic behavior of solutions $(u(t),s(t))$ as $t \to \infty$. We will first prove the dichotomy result that every solution satisfies one of the following properties, (i) vanishing of solutions: $s(t)$ remains bounded for all $t \geq 0$ and $\lim_{t\to\infty} u(t) = 0$ uniformly in $[0,\infty)$, (ii) spreading of solutions: $\lim_{t\to\infty} s(t) = \infty$ and $\liminf_{t\to\infty} u(t) \geq c_0$ with $c_0 > 0$. Our next purpose is to derive transient profiles of solutions. In the spreading case, we will prove that solutions exhibit a prescribed number of spreading phenomena with different speeds of $s'(t)$ as $t \to \infty$.

Date:

Friday, Mar. 13 12:00-12:40

Martin ZIEGLER

Technical University of Darmstadt, Darmstadt

Title:

Computability and Complexity Theory of PDEs

Abstract:

Efficient numerical methods and recipes have become customary tools to Engineers, 'solving' various partial differential equations with given initial/boundary conditions on digital computers. However the quality of the approximations they produce generally remains to be assessed by human experts and experience: Only few implementations actually yield guaranteed enclosures of prescribable accurracy, and on a case-by-case basis.

In fact, Pour-El and Richards (1981) had constructed a computable C^1 initial condition $u(0)$ to the 3D wave equation with solution $u(1)$ provably *non*-computable, namely encoding the Halting problem [4]. This challenge to the Church-Turing Hypothesis (which the field of Computational Physics implicitly builds on) led to an intensive philosophical controversy and was resolved only 20 years later: when Zhong and Weihrauch (2002) established uniform computability of the wave propagator — in the Sobolev space sense [6].

After a recap of Pour-El and Richards' construction, we present own research on the algorithmic foundation of real computing and more specifically on questions of efficiency [2, 3, 1, 5]: We refine qualitative computability to quantitative complexity theory, devising provably correct algorithms with axiomatized semantics over (possibly multivalued and enriched) continuous data types yielding canonical interface declarations in contemporary object-oriented imperative programming languages, accompanied by rigorous parameterized running-time bounds and complemented by optimality proofs relative to common hypotheses from Logic/Theoretical Computer Science such as (strengthenings and variations of) the $\mathcal{P}$-vs-$\mathcal{NP}$ Millennium Prize Problem.

References:

[1] A. Kawamura, N. Müller, C. Rösnick, M. Ziegler: "Computational Benefit of Smoothness: Parameterized Bit-Complexity of Numerical Operators on Analytic Functions and Gevrey's Hierarchy", submitted.

[2] A. Kawamura, H. Ota, C. Rösnick, M. Ziegler: "Computational Complexity of Smooth Differential Equations", pp.578–589 in *Proc. 37th Int. Symp. Mathematical Foundations of Computer Science* (MFCS'2012), Springer LNCS vol.**7464**; full version in *Logical Methods in Computer Science* vol.**10:1** (2014).

[3] A. Kawamura, F. Steinberg, M. Ziegler: "Complexity of Laplace's and Poisson's Equation", p.231 in *Bulletin of Symbolic Logic* vol.**20:2** (2014); full version submitted.

[4] M.B. Pour-El, J.I. Richards: "The wave equation with computable inital data such that its unique solution is not computable", pp.215–239 in *Advances in Math.* vol.**39** (1981).

[5] S.-M. SUN, N. ZHONG, M. ZIEGLER: "On the Computability of the Navier-Stokes Equation", submitted.

[6] K. WEIHRAUCH, N. ZHONG: "Is Wave Propagation Computable or Can Wave Computers Beat the Turing Machine?", pp.312–332 in *Proc. London Mathematical Society* vol.**85:2** (2002).

Date:

Friday, Mar. 13 15:40-16:20

12 minutes talks

Martin BOLKART

Technical University of Darmstadt, Darmstadt

Title:

The Stokes equations in spaces of BMO-type

Abstract:

We consider the Stokes equations in a large class of domains including bounded domains, half space and exterior domains. We introduce a norm BMO_b measuring the BMO-seminorm in Ω and the mean value near the boundary. We will derive $L^\infty - BMO_b$-estimates for the derivatives of the solution. Finally we can prove that the solution operator is analytic in a space with BMO_b-norm.

Date:

Thursday, Mar. 12 17:30-17:45

Alexander DALINGER

Technical University of Darmstadt, Darmstadt

Technische Universität Darmstadt, 64289 Darmstadt, Germany

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Title:

On the hydrodynamic behavior of a 1D system with next neighbor interactions

Abstract:

We consider a one-dimensional system of particles interacting with each other over the next neighbors. Starting with an arbitrary configuration of particles, the aim is to understand the time evolution of the particle density. We proved that the particle density converges in the hydrodynamic limit to a solution of a non-linear heat equation. This result will be discussed in the talk.

Date:

Friday, Mar. 13 17:15-17:30

Michael FISCHER

Technical University of Darmstadt, Darmstadt

Technische Universitat Darmstadt, 64293 Darmstadt, Germany

mfischer@mathematik.tu-darmstadt.de

Title:

Shape optimization and the stabilized characteristics finite element method

Abstract:

In this talk we consider an application of the perturbation of identity method for shape optimization [1] to the discretization of the Boussinesq equations via a

stabilized characteristics finite element method [2]. After a brief introduction and problem setting we investigate the sensitivity of the solution of the stabilized characteristics finite element method with respect to domain variations in a suitable function space setting. The talk is concluded by introducing a computationally more efficient approximation of the scheme and discussing it's pros and cons.

References:

[1] F. Murat and J. Simon. Etude de problemes d'optimal design. In J. Cea, editor, *Optimization Techniques Modeling and Optimization in the Service of Man Part 2*, volume 41 of *Lecture Notes in Computer Science*, pages 54–62. Springer Berlin Heidelberg, 1976.

[2] H. Notsu. Error estimates of a pressure-stabilized characteristics finite element scheme for the navier-stokes equations. *Preprint- WIAS Discussion Paper*, 2:1–21, 2013.

Date:

Thursday, Mar. 12 17:00-17:15

Yuki KANEKO

Waseda University, Tokyo

Title:

Criteria of spreading and vanishing for a free boundary problem in mathematical ecology

Abstract:

We discuss a free boundary problem for a nonlinear diffusion equation modeling the spreading of non-native species in a radially symmetric setting, where unknown functions are population density $u(t,|x|)$ and spreading front $|x| = h(t)$ (free boundary) of the species. We focus on the asymptotic behavior of solutions, which is understood as Spreading:

$$\lim_{t\to\infty} h(t) = \infty, \quad \liminf_{t\to\infty} \|u(t,\cdot)\|_{C(0,h(t))} > 0$$

and Vanishing:

$$\lim_{t\to\infty} \|u(t,\cdot)\|_{C(0,h(t))} = 0.$$

In this talk, we will give some criteria for spreading and vanishing and show some related results.

Date:

Friday, Mar. 13 16:45-17:00

Tomoya KATO

Nagoya University, Nagoya

Title:

Solutions to nonlinear higher order Schrödinger equations on modulation spaces

Abstract:

We consider the Cauchy problem for the nonlinear higher order Schrödinger equation with small initial data on modulation spaces. Using integrability of time decay factors of time decay estimates, we show the existence of a unique global solution. As a result, we are able to deal with wider classes of a nonlinearity and a solution space.

Date:

Wednesday, Mar. 11 17:30-17:45

Hana MIZEROVÁ

Johannes Gutenberg University Mainz, Mainz

Title:

Characteristics finite element scheme preserving positive-definiteness of the conformation tensor

Abstract:

We consider a dumbbell based model for dilute polymer solutions, the so called diffusive Peterlin model. The system of equations for velocity and for the positive-definite conformation tensor describes the unsteady behaviour of some incompressible polymeric fluids. The existence and for more regular data the uniqueness of global weak solution to this problem have been proven in [1]. For the diffusive Oseen-type Peterlin model we have proposed a pressure-stabilized characteristics finite element scheme, cf. [2]. The main aim of this talk is to prove that, for sufficiently small time step, this scheme preserves the positive-definiteness of the conformation tensor. This is an important feature needed to show the stability and error estimates for the given scheme. The proof is done by the implicit function theorem following the technique from [3].

This work has been supported by the German Science Foundation (DFG) under IRTG 1529 "Mathematical Fluid Dynamics" and realized in collaboration with M. Lukáčová, M. Tabata and H. Notsu.

References:

[1] M. Lukáčová-Medvid'ová, H. Mizerová, Š. Nečasová: *Global existence and uniqueness result for the diffusive Peterlin viscoelastic model*, submitted.

[2] M. Lukáčová-Medvid'ová, H. Mizerová, H. Notsu, M. Tabata: *Error estimates of a pressure-stabilized characteristics finite element scheme for the diffusive Oseen-Peterlin model*, in preparation.

[3] S. Boyaval, T. Lelièvre, C. Mangoubi: *Free-energy-dissipative schemes for the Oldroyd-B model*, ESAIM: Mathematical Modelling and Numerical Analysis 43.3 (2009): 523-561.

Date:

Thursday, Mar. 12 16:30-16:45

Naofumi MORI

Kyusyu University, Fukuoka

Title:

Decay property of the Timoshenko system with memory-type dissipation

Abstract:

We consider the initial value problem of the Timoshenko system with a memory term. It is known that the decay property of the system is of the regularity-loss type and is weaker than that of the Timoshenko system with a frictional dissipation. In this talk, we consider the linearized system in the one-dimensional whole space, introduce the dissipative structure of the system, and, applying the energy method in the Fourier space, derive the optimal pointwise estimate of solutions, which gives a sharp decay estimate of solutions. If there is some time left, we obtain the global existence of solutions to the nonlinear problem under smallness and minimal regularity assumptions on the initial data, where we employ the energy method.

This talk is based on a joint work with Shuichi Kawashima (Kyushu University).

Date:

Wednesday, Mar. 11 16:30-16:45

Miho MURATA

Waseda University, Tokyo

Title:

Local well-posedness of the fluid-rigid body interaction problem for compressible fluids

Abstract:

We consider the system of equations describing the motion of a rigid body immersed in a compressible viscous fluid within the barotropic regime. It is shown that this system admits a unique, local strong solution in the L_p in time and L_q in space setting with $2 < p < \infty$ and $3 < q < \infty$. For the purpose, we use the contraction mapping principle based on the maximal L_p-L_q regularity. Concerning the same coupled system, Boulakia and Guerrero [1] proved an existence result for strong solution within the L_2 framework. One of the merits of our approach is less compatibility condition and regularity on initial data compared with the ones given in [1].

References:

[1] M. Boulakia and S. Guerrero, A regularity result for a solid-fluid system associated to the compressible Navier-Stokes equations, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **26** (2009), 777–813.

Date:

Wednesday, Mar. 11 17:00-17:15

Dai NOBORIGUCHI

Waseda University, Tokyo

Title:

The initial-boundary value problems for scalar conservation laws with stochastic forcing

Abstract:

In this talk we consider the first order stochastic scalar conservation laws of the following type

$$du + \operatorname{div}(A(u))dt = \Phi(u)dW(t) \quad \text{in } (0,T) \times D,$$

with the initial condition

$$u(0,\cdot) = u_0(\cdot) \quad \text{on } D,$$

and the formal boundary condtion

$$\text{“}u = u_b\text{”} \quad \text{in } (0,T) \times \partial D.$$

Here $D \subset \mathbb{R}^d$ is a bounded domain with a Lipschitz boundary ∂D, $T > 0$, A is a function from $\mathbb{R}$ to $\mathbb{R}^d$, Φ is a function from $\mathbb{R}$ to $\mathbb{R}$ and W is a one-dimensional Brownian motion defined on a stochastic basis $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}, \mathbb{P})$.

We introduce definition of kinetic solutions characterized by kinetic formulations with the boundary defect measures. In such a solution, we obtain results of uniqueness and existence.

Date:

Friday, Mar. 13 17:00-17:15

Hirokazu SAITO

Waseda University, Tokyo

Title:

Free boundary problems for the incompressible Navier-Stokes equations in some unbounded domain

Abstract:

In this talk, we would like to consider the global well-posedness of free boundary problems for the incompressible Navier-Stokes equations in some unbounded domain. More precisely, our situation is that the surface tension and the gravity force do not work on the free boundary, and the domain is bounded below by a solid surface and bounded above by a free surface. To show the global well-posedness, the L_p-L_q framework will be used in our approach.

Date:

Wednesday, Mar. 11 17:15-17:30

Jonas SAUER

Technical University of Darmstadt, Darmstadt

Fachbereich Mathematik, Technische Universität Darmstadt, 64283 Darmstadt,
jsauer@mathematik.tu-darmstadt.de

Title:

The Stokes Operator in Periodical Domains with Boundary

Abstract:

We investigate maximal regularity in L^q-spaces of the abstract Stokes operator $\mathcal{A}_q$ in spatially periodic domains with boundary, that is, in cylindrical domains. The main step is to show corresponding resolvent estimates in the periodic half space $G_+ = \{x \in G : x_1 > 0\}$, where G is the locally compact abelian group $\mathbb{R}^{n-1} \times T$ and T is the one-dimensional torus. These estimates can be obtained by using a sophisticated mirroring technique and *a priori* estimates on the periodic whole space G, which have been obtained in [2]. If one wants to read off $\mathcal{R}$-boundedness of the resolvent operator directly from the obtained resolvent estimates, one has to use an extrapolation theorem in the style of García-Cuerva and Rubio de Francia [1]. In particular, the resolvent estimates have to hold in weighted L^q_ω-spaces, where ω is a Muckenhoupt weight. Unfortunately, $\omega(x_1, \cdot)$ is not an $(n-1)$-dimensional Muckenhoupt weight in general. Therefore, classical mirroring techniques that rely on an $(n-1)$-dimensional estimates for fixed $x_1 > 0$ cannot be applied. The main aspect of my talk will be to show how to overcome these difficulties arising in the periodic half space case.

Once the periodic half space is understood, results for cylindrical domains follow easily by well-known perturbation and localization methods.

References:

[1] *García-Cuerva, J. and Rubio de Francia, J. L.*: Weighted norm inequalities and related topics. *North-Holland Publishing Co., Amsterdam*, Volume 116, 1985

[2] *Sauer, J.*: Weighted Resolvent Estimates for the Spatially Periodic Stokes Equations. *Ann. Univ. Ferrara*, DOI 10.1007/s11565-014-0221-4, 2014

Date:

Tuesday, Mar. 10 16:30-16:45

Katharina SCHADE

Technical University of Darmstadt, Darmstadt

Title:

On the Stokes resolvent estimates for cylindrical domains

Abstract:

We prove an L^∞-resolvent estimate of the Stokes problem in infinite cylinders and some cylinder-like domains. For this we introduce a new admissibility criterion of Neumann admissible domains and then derive the desired resolvent estimate for Neumann admissible domains by a contradiction argument.

Date:

Wednesday, Mar. 11 16:45-17:00

Tobias SEITZ

Technical University of Darmstadt, Darmstadt

Title:

Inverse problems for incompressible flow

Abstract:

Measurements of flow velocities are necessary for the study of fluid mechanical problems. Velocity fields obtained by such measurements naturally contain errors and are thus not suitable for direct usage. By simple filtering we can smooth these errors, but the result will in general not satisfy the governing equations. We consider the filtering process as an inverse problem with partial differential equations and can thus take into account the physical model, which is described by stationary Navier Stokes equations. In our approach we directly use measurements stemming from Magnetic Resonance Velocimetry.

Date:

Thursday, Mar. 12 17:15-17:30

Bangwei SHE

Johannes Gutenberg University Mainz, Mainz

Title:

Energy dissipative scheme for a diffusive Oldroyd-B model

Abstract:

It is a well-known fact that numerical simulation of viscoelastic fluids poses many challenging questions. In particular, when the so-called Weissenberg number,

which expresses the fraction of elastic and viscous effects is larger (e.g. $We > 1$), then unbounded numerical solutions are approached exponentially in time. We should also mention that the question of global weak existence for the Oldroyd-B model is still open. Due to these drawbacks we propose a new diffusive model for the Oldroyd-B viscoelastic fluid. In order to approximate the new model numerically a characteristic finite difference scheme has been implemented. The aim of this talk is to show that our scheme satisfies the entropy inequality on the discrete level.

Date:

Thursday, Mar. 12 16:45-17:00

Florian STEINBERG

Technical University of Darmstadt, Darmstadt

Title:

Second order representations of L_p Spaces

Abstract:

Real Complexity Theory provides a realistic and rigorous notion of how functions on the real numbers can be handled by computers. However, any computable function is necessarily continuous. In many applications, spaces that contain discontinuous and merely integrable functions play an important role. These spaces can usually be made computable metric spaces, but due to their 'breadth' the standard representations do not lead to a well behaved notion of complexity. We recall the notion of a second order representation and review the standard second order representation for continuous functions. Then we investigate different generalizations of this notion to integrable functions.

Date:

Friday, Mar. 13 16:30-16:45

Go TAKAHASHI

Waseda University, Tokyo

Title:

Extension criterion on time local solutions to the Navier-Stokes equations

Abstract:

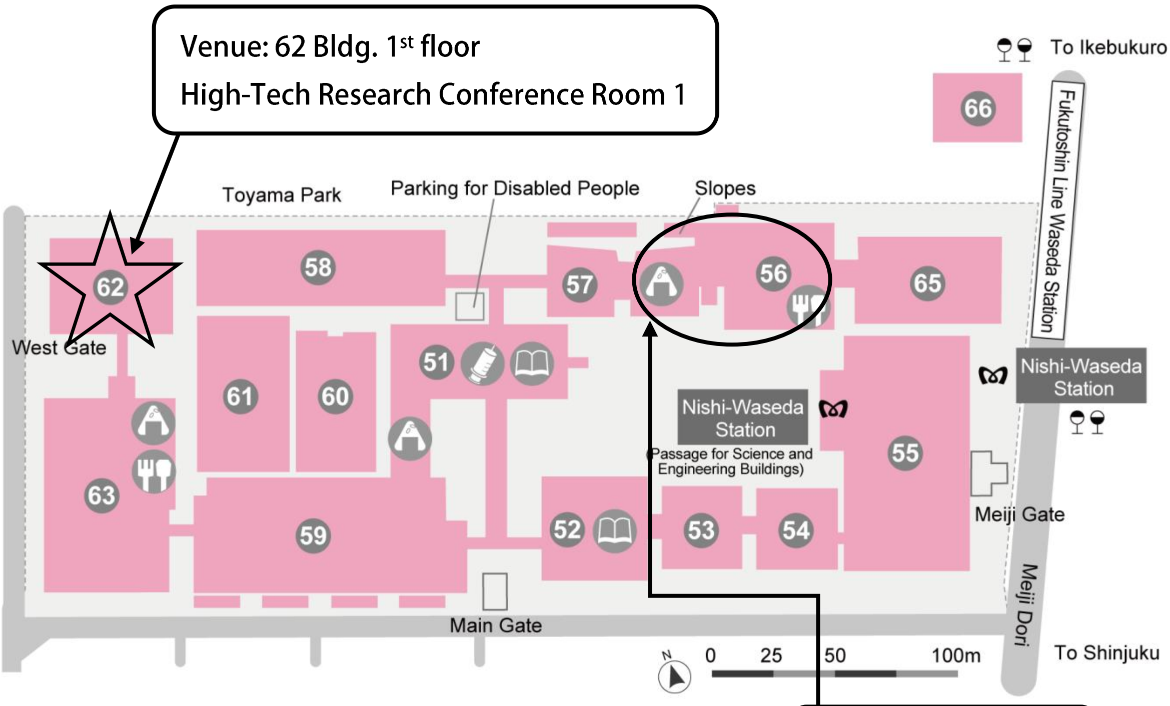
In this talk, we work on the time local classical solution to the incompressible Navier-Stokes equations, and consider the extension of time. It is widely known that the Cauchy problem for the Navier-Stokes equations is locally well-posed, and in order to hit on a clue to solve so-called “ Millennium problem ” the time extension argument of a time local solution has been discussed. We propose a time

extension criterion by estimating the Morrey type functional of the solution near the final time. To prove this theorem, we use the theory of suitable weak solutions and prove so-called ϵ regularity criterion.

Date:

Tuesday, Mar. 10 16:45-17:00

Nishi-Waseda Campus Map



Basement floor:
Shop and Cafeteria

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