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独立行政法人日本学術振興会 日独共同大学院プログラム
JSPS-DFG Japanese-German Graduate Externship

第 12 回 日独流体数学国際研究集会
The 12th Japanese-German International Workshop on Mathematical Fluid Dynamics

March 1 - 4, 2016 at Waseda University, Nishi-Waseda Campus

63 Bldg. 2nd Floor 05 Conference Room

■ Program ■

	Tue, Mar. 1	Wed, Mar. 2	Thu, Mar. 3
9:20 -9:30	Opening		
9:30 -10:30	Stefan ULBRICH① (TU Darmstadt)	ULBRICH ③	Ryo TAKADA ① (Tohoku Univ.)
10:45 -11:45	ULBRICH ②	ULBRICH ④	TAKADA ②
12:00 -12:40	Lunch Break	Jonas SAUER (TU Darmstadt)	Jun KATO (Nagoya Univ.)
12:40 -14:10	Lunch Break	Lunch Break	Lunch Break
14:10 -15:10	Robert DENK ① (Univ. of Konstanz)	DENK ②	DENK ③
15:40 -16:20	Amru HUSSEIN (TU Darmstadt)	Matthias KOTSCHOTE (Univ. of Konstanz)	Yohei TSUTSUI (Shinshu Uni.)
16:30 -16:45	Mathis GRIES (TU Darmstadt)	Andreas KLAIBER (Univ. of Konstanz)	Martin BOLKART (TU Darmstadt)
16:45 -17:00	Sebastian ZAIGLER (TU Darmstadt)	Michael FISCHER (TU Darmstadt)	Go TAKAHASHI (Waseda Univ.)
17:00 -17:15	Florian STEINBERG (TU Darmstadt)	Shinya UCHIUMI (Waseda Univ.)	David WEGMANN (TU Darmstadt)

	Fri, Mar. 4
9:30 -10:30	TAKADA ③
10:45 -11:45	TAKADA ④
12:00 -12:40	Tomasz PIASECKI (Polish Acad. of Sci.)
12:40 -14:10	Lunch Break
14:10 -15:10	DENK ④
15:40 -15:55	Tobias SEITZ (TU Darmstadt)
15:55 -16:10	Alexander DALINGER (TU Darmstadt)
16:10 -16:25	Dai NOBORIGUCHI (Waseda Univ.)
16:45 -17:00	Anton SEYFERT (TU Darmstadt)
17:00 -17:15	Shota ENOMOTO (Kyushu Univ.)
17:15 -17:30	Naofumi MORI (Kyushu Univ.)
-17:40	Closing
18:00-	Closing Reception at Cafeteria

● →Main-course ● →40min talk ● →15min talk

Main-Course

Robert DENK

University of Konstanz, Konstanz, Germany

Title:

Maximal regularity for parabolic evolution equations

Abstract:

Lecture 1: Maximal regularity and $\mathcal{R}$-boundedness

Maximal regularity is one of the standard approaches to investigate semilinear and quasilinear parabolic evolution equations. A prototype example is the graphical mean curvature flow given by

$$\partial_t u - \Delta u = -\sum_{i,j=1}^{n} \frac{\partial_i u \partial_j u}{1+|\nabla u|^2}\,\partial_i\partial_j u \quad (t\in(0,T)), \quad u(0)=u_0.$$

The main idea of maximal regularity is show that the linearized equation induces an isomorphism between suitably chosen function spaces and to reduce the nonlinear equation to a fixed-point equation in these function spaces.

In our lectures, we will consider maximal regularity in L^p-Sobolev spaces in time and space with $p\in(1,\infty)$. For the initial value $u|_{t=0}$ or for boundary values, one has to describe the trace spaces of L^p-Sobolev spaces. It turns out that these are Besov spaces of non-integer order. To deal with the nonlinear terms in the equation, one can apply Sobolev embedding results for sufficiently large p. If the linearized operator has maximal regularity, the nonlinear equation has a unique solution at least for small time intervals or for small data.

A closed operator $A\colon X\supset D(A)\to X$ acting in a Banach space has maximal L^p-regularity if the Cauchy problem

$$\partial_t u - Au = f \quad (t\in(0,T)), \quad u(0)=u_0$$

has a unique solution $u\in H_p^1((0,T);X)\cap L^p((0,T);D(A))$ depending continuously on the data f and u_0. Taking Fourier (or Laplace) transform F_t in time, we see that this is equivalent to the condition that

$$F_t^{-1} i\tau(i\tau - A)^{-1} F_t$$

defines a continuous operator in $L^p(;X)$. In this case, the (operator-valued) symbol $\tau\mapsto i\tau(i\tau-A)^{-1}$ is said to be an L^p-multiplier. A sufficient condition for symbols to be an L^p-multiplier is given by a vector-valued version of Michlin's theorem which is based on the so-called $\mathcal{R}$-boundedness of the symbol.

Lecture 2: Maximal regularity for boundary value problems

For (linearized) equations in the whole space ^{n}, $\mathcal{R}$-boundedness of the resolvent and therefore maximal L^p-regularity can be shown by vector-valued Michlin theorems. For

boundary value problems in domains, however, one has to study model problems in the half space ${}^n_+ := \{x \in^n : x_n > 0\}$ of the form

$$\begin{aligned} \lambda u - Au &= f \quad \text{in } {}^n_+, \\ \gamma_0 u &= g \quad \text{on } {}^{n-1}, \end{aligned}$$

where $\gamma_0 u := u|_{^{n-1}}$ stands for the trace of the function u on the boundary $\partial^n_+ =^{n-1}$. There is a standard method to solve such boundary value problems, using partial Fourier transform F' in the tangential variables. This reduces the problem to an ordinary differential equation in the half-line $(0, \infty)$. Taking inverse Fourier transform, we can describe the solution operator as a sum of the whole space resolvent and an operator of the form

$$(K\phi)(x', x_n) = \int_0^\infty (F')^{-1} w(\xi', x_n + y_n, \lambda)(F'\phi)(\xi', y_n) dy_n.$$

Such operators are called singular Green operators. By results on the $\mathcal{R}$-boundedness of integral operators, it can be shown that parabolic boundary value problems have maximal L^p-regularity. For general boundary conditions, the Shapiro-Lopatinskii condition has to be satisfied. This condition can equivalently be written as the invertibility of the so-called Lopatinskii matrix.

Lecture 3: Parabolic mixed-order systems

The analysis of the Stefan problem with Gibbs-Thomson correction leads to the Lopatinskii matrix

$$L(\xi', \lambda) = \begin{pmatrix} \frac{1}{\sqrt{|\xi'|^2 + \lambda}} & -|\xi'|^2 \\ & \lambda \end{pmatrix} \quad (\xi' \in^{n-1},\ \Re\lambda \geq 0).$$

This is an example of a mixed-order system (Douglis-Nirenberg system). For maximal regularity results, one has to determine Sobolev type spaces in which the corresponding operator $L(D', \partial_t)$ induces an isomorphism.

A general approach to this is given by the Newton polygon method. Here one first considers the determinant of the matrix and describes the different inhomogeneities (with respect to ξ' and λ) in a geometrical way. By this, one can define mixed-order (anisotropic) Sobolev spaces in time t and space x which are adapted to the equation and which are the appropriate function spaces for the data and for the solution.

It is possible to define a generalization of classical parabolicity (so-called N-parabolicity) which is the essential condition for maximal L^p-regularity for mixed-order systems. As applications, we mention Stefan problems and the thermoelastic plate equation.

Lecture 4: Maximal regularity for pseudodifferential operators

Let X be a Banach space. The symbol of an operator-valued symbol of class $S^\mu(^n; L(X))$ is a C^∞-function $a\colon {}^n \times^n \to L(X)$ satisfying

$$\sup \Big\{ \langle\xi\rangle^{|\alpha|-\mu} \|\partial_\xi^\alpha a(x,\xi)\|_{L(X)} : x \in^n,\ \xi \in^n \Big\} < \infty \quad (\alpha \in^n_0).$$

For such a symbol, the corresponding pseudodifferential operator $[a]$ is given by

$$([a]u)(x) = (2\pi)^{-n/2} \int_n a(x,\xi)(Fu)(\xi)d\xi.$$

If the operator norm in the definition of S^μ is replaced by the $\mathcal{R}$-bound, the resulting symbol class $S^\mu_{\mathcal{R}}({}^n; L(X))$ yields $\mathcal{R}$-bounded operator families. Therefore, parabolic pseudodifferential operators in the whole space have maximal L^p-regularity for every $p \in (1,\infty)$.

The situation for pseudodifferential (non-local) boundary value problems is more complicated. It can be shown that singular Green operators are $\mathcal{R}$-bounded, too, which implies maximal L^p-regularity. As an example, we consider the Stokes equation in an infinite cylinder with no-slip boundary conditions. Here, the elimination of the pressure leads to a non-local boundary value problem for the velocity.

Date:

① Tuesday, Mar. 1 14:10-15:10 ② Wednesday, Mar. 2 14:10-15:10

③ Thursday, Mar. 3 14:10-15:10 ④ Friday, Mar. 4 14:10-15:10

Ryo TAKADA
Tohoku University, Sendai

Mathematical Institute, Tohoku University
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Title:

Dispersive estimates for rotating fluids and stably stratified fluids

Abstract:

The most important two features in the geophysical fluid dynamics are *rotation* of the fluid and *stable stratification*, and it is known that these effects exhibit dispersive natures. In this mini-course, we shall present recent mathematical results on the dispersive estimates for the linear propagators associated with the rotation of the fluid and the stable stratification.

1. Derivation of the equations

We consider the initial value problems for the rotating Navier-Stokes equations:

$$\begin{cases} \partial_t u + (u \cdot \nabla) u = \nu \Delta u - 2\Omega e_3 \times u - \nabla p & t > 0,\ x \in \mathbb{R}^3, \\ \nabla \cdot u = 0 & t \geqslant 0,\ x \in \mathbb{R}^3, \\ u(0,x) = u_0(x) & x \in \mathbb{R}^3, \end{cases} \tag{NS$_\Omega$}$$

and the stably stratified Boussinesq equations:

$$\begin{cases} \partial_t u + (u \cdot \nabla) u = \nu \Delta u - \nabla p + \theta e_3 & t > 0,\ x \in \mathbb{R}^3, \\ \partial_t \theta + (u \cdot \nabla)\theta = \kappa \Delta \theta - N^2 u_3 & t > 0,\ x \in \mathbb{R}^3, \\ \nabla \cdot u = 0 & t \geqslant 0,\ x \in \mathbb{R}^3, \\ u(0,x) = u_0(x), \quad \theta(0,x) = \theta_0(x) & x \in \mathbb{R}^3. \end{cases} \tag{B$_N$}$$

Here, $u = (u_1(t,x), u_2(t,x), u_3(t,x))^T$, $p = p(t,x)$ and $\theta = \theta(t,x)$ are the unknown functions, representing the velocity field, the scalar pressure and the thermal disturbance about a mean state in hydrostatic balance, respectively, while $u_0 = (u_{0,1}(x), u_{0,2}(x), u_{0,3}(x))^T$ is the given initial velocity field and $\theta_0 = \theta_0(x)$ is the given initial thermal disturbance. $\Omega \in \mathbb{R} \setminus \{0\}$ is the angular frequency of the background rotation, and $N > 0$ is the Brunt-Väisälä (buoyancy) frequency for the constant stratification.

In the first lecture, we shall derive the system (NS$_\Omega$) from the original Navier-Stokes equations with the boundary condition that u behaves like $\Omega e_3 \times x + O(1)$ as $|x| \to \infty$, and the system (B$_N$) from the original Boussinesq equations by considering the perturbation about a mean state in hydrostatic balance. Moreover, we give the explicit representations of the linear semigroups generated by the linear operators in (NS$_\Omega$) and (B$_N$).

2. Dispersive estimates for linear propagators

In the second and third lectures, we shall consider the dispersive estimates (the L^1-L^∞ temporal decay estimates) for the linear propagators associated with the Coriolis force $\Omega e_3 \times u$ in (NS_Ω) and the stable stratification $-N^2 u_3$ in (B_N):

$$e^{\pm i\Omega t\frac{D_3}{|D|}} f(x) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} e^{ix\cdot\xi \pm i\Omega t\frac{\xi_3}{|\xi|}} \widehat{f}(\xi)\, d\xi,$$
$$e^{\pm iNt\frac{|D_h|}{|D|}} f(x) = \frac{1}{(2\pi)^3}\int_{\mathbb{R}^3} e^{ix\cdot\xi \pm iNt\frac{|\xi_h|}{|\xi|}} \widehat{f}(\xi)\, d\xi,$$

respectively. Here $\xi_h = (\xi_1, \xi_2)$ and $|\xi_h| = \sqrt{\xi_1^2 + \xi_2^2}$. We give the decay estimates for the above linear propagators by applying the Littman theorem to $e^{\pm i\Omega t\frac{D_3}{|D|}}$, and by making use of the stability of the stationary phase estimates under the small C^m perturbations with a large $m \in \mathbb{N}$ for $e^{\pm iNt\frac{|D_h|}{|D|}}$. Also, we shall mention the sharpness of our dispersive estimates.

3. Global existence of solutions to (NS_Ω) and (B_N)

In the fourth lecture, we apply our dispersive estimates to the study of the existence and uniqueness of global in time solutions for (NS_Ω) and (B_N). In particular, we give explicit relations between the size of given data and the rotation speed $|\Omega|$, and the buoyancy frequency N:

$$\|u_0\|_{\dot{H}^s} \leqslant C|\Omega|^{\frac{1}{2}\left(s-\frac{1}{2}\right)}, \quad \|\theta_0\|_{\dot{H}^s} \leqslant CN^{\frac{1}{2}\left(s-\frac{1}{2}\right)+1}$$

with some $s > 1/2$, which ensure the unique existence of global solutions to (NS_Ω) and (B_N), respectively. Consequently, it is shown that the size of the initial velocity u_0 and the thermal disturbance θ_0 can be taken large in proportion to the speed of rotation $|\Omega|$ and the strength of stratification N, respectively.

Date:

① Thursday, Mar. 3 9:30-10:30 ② Thursday, Mar. 3 10:45-11:45

③ Friday, Mar. 4 9:30-10:30 ④ Friday, Mar. 4 10:45-11:45

Stefan ULBRICH

Technical University of Darmstadt, Darmstadt

Title:

Theory and methods for optimization problems governed by the Navier-Stokes equations

Abstract:

The modelling and numerical simulation of flows plays an important role in physics, engineering, chemistry, medicine, and in other disciplines. In most applications, the ultimate goal is not only the mathematical modelling and numerical simulation of the flow, but rather the optimization or optimal control of the considered process. Typical examples are the optimal control of flows by actuators to reduce skin friction or the optimal shape design of an aircraft. The resulting optimization problems are very complex and a thorough mathematical analysis is necessary to design efficient solution methods.

In this short course, we will focus on the optimal control of the unsteady incompressible Navier-Stokes equations. We will consider optimization problems of the form [5, 6, 7, 9, 11]

$$\begin{aligned} &\min J(v,u) \quad \text{subject to} \\ &v_t - \nu\Delta v + (v\cdot\nabla)v + \nabla p - F(u) = 0 \quad \text{on } \Omega_T := (0,T)\times\Omega, \\ &\operatorname{div} v = 0 \quad \text{on } \Omega_T, \\ &v(0,\cdot) - v_0 = 0 \quad \text{on } \Omega, \\ &\text{suitable boundary conditions} \quad \text{on } I\times\partial\Omega, \\ &v\in V_{ad}, \quad u\in U_{ad}, \end{aligned} \tag{1}$$

where $\Omega \subset^d$ is a bounded sufficiently smooth domain, $v \in W \subset L^2(0,T;H^1(\Omega)^d)$ is the velocity field and $p : \Omega_T \to$ is the pressure. Moreover, $u \in U$ is a (usually time-dependent) control, $F : U \to Z$ is a control operator and $J : W \times U \to$ is the objective function. Finally, $U_{ad} \subset U$ and $V_{ad} \subset W$ are closed convex admissible sets for the control and the state. We will give examples from flow control applications leading to problems of this type.

We will consider the following topics.

(1) Existence of optimal solutions,

(2) Optimality conditions,

(3) Optimization methods for the efficient solution of (1).

To allow for a complete theory, we will focus on the case $d = 2$.

The problem (1) is of the general form

$$\min J(y,u) \quad \text{subject to} \quad E(y,u) = 0, \quad y \in Y_{ad}, \quad u \in U_{ad} \tag{2}$$

with state space Y, control space U and state equation $E(y,u) = 0$, $E : Y \times U \mapsto Z$. We will see that with an appropriate choice of Y and Z the operator $E : Y \times U \mapsto Z$ induced by the Navier-Stokes equations is continuously Féchet-differentiable, admits

for each $u \in U$ a unique solution $y = y(u)$ of $E(y,u) = 0$ and the linearized operator $E_y'(y,u)$ has a bounded inverse.

After studying the existence of optimal solutions of (1), we will derive first order optimality conditions by introducing and applying optimality theory for optimization problems (2) in Banach space [2, 9, 10].

We will first consider the case without state constraints, i.e., $V_{ad} = W$. This will allow us to work with standard weak solutions of (1). The optimality conditions consist of the original constraints, an adjoint equation and a stationarity condition. The adjoint equation involves the dual operator of the linearized Navier-Stokes and determines the Lagrange multiplier (adjoint state) for the state equation $E(y,u) = 0$. We will show that the adjoint state enjoys additional regularity [6, 9, 11], which is essential for the convergence theory of efficient optimization methods for the solution of (1).

Next, we will discuss optimization methods for (1). In particular, we will show that the optimality system can for the most relevant cases of admissible sets U_{ad} be rewritten as a nonsmooth system of equations that can be solved efficiently by semismooth Newton methods [7, 9, 11]. After a theoretical justification we will show numerical results.

In a next step we will consider the case of state constraints, where V_{ad} is a closed, convex and bounded subset of W. Besides pointwise bounds on the velocity (e.g. in gas transport) the case of pointwise bounds on the stresses are of particular interest in application (e.g., to avoid damage of particles in fluids within pumps or mixers).

We will derive first order optimality conditions for pointwise constraints on the velocity and partial derivatives of the velocity, respectively. This will require to apply L^p- theory for strong solutions of the Navier-Stokes equations, since L^∞-regularity of the velocity (or its derivative) is necessary to handle pointwise state constraints (or state gradient constraints) [3, 12, 13].

To solve state constrained problems efficiently, penalty approaches by Moreau-Yosida regularization in connection with semismooth Newton methods have been proven to be efficient [3, 8]. We will introduce the underlying ideas and discuss the convergence theory of this approach. Moreover, we will show numerical results.

Finally, if time permits, we will give an introduction to selected further topics, in particular to the method of mappings for shape optimization problems [1] and to reduced order model techniques for optimal online control [4, 5] governed by the Navier-Stokes equations.

References:

[1] J. A. Bello, E. Fernández-Cara, J. Lemoine, and J. Simon. The differentiability of the drag with respect to the variations of a Lipschitz domain in a Navier-Stokes flow. *SIAM J. Control Optim.*, 35(2):626–640, 1997.

[2] J. F. Bonnans and A. Shapiro. *Perturbation analysis of optimization problems.* Springer Series in Operations Research. Springer-Verlag, New York, 2000.

[3] S. Bott. *Adaptive SQP Method with Reduced Order Models for Optimal Control Problems with Constraints on the State applied to the Navier-Stokes Equations.* Dissertation, Technische Universität Darmstadt, 2015.

[4] J. Ghiglieri and S. Ulbrich. Optimal flow control based on POD and MPC and an application to the cancellation of Tollmien-Schlichting waves. *Optim. Methods Softw.*, 29(5):1042–1074, 2014.

[5] M. D. Gunzburger. *Perspectives in flow control and optimization*, volume 5 of *Advances in Design and Control*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2003.

[6] M. D. Gunzburger and S. Manservisi. The velocity tracking problem for Navier-Stokes flows with bounded distributed controls. *SIAM J. Control Optim.*, 37(6):1913–1945, 1999.

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[8] M. Hintermüller and K. Kunisch. PDE-constrained optimization subject to pointwise constraints on the control, the state, and its derivative. *SIAM J. Optim.*, 20(3):1133–1156, 2009.

[9] M. Hinze, R. Pinnau, M. Ulbrich, and S. Ulbrich. *Optimization with PDE constraints*, volume 23 of *Mathematical Modelling: Theory and Applications*. Springer, New York, 2009.

[10] F. Tröltzsch. *Optimal control of partial differential equations*, volume 112 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2010. Theory, methods and applications, Translated from the 2005 German original by Jürgen Sprekels.

[11] M. Ulbrich. Constrained optimal control of Navier-Stokes flow by semismooth Newton methods. *Systems Control Lett.*, 48(3-4):297–311, 2003. Optimization and control of distributed systems.

[12] D. Wachsmuth. Regularity and stability of optimal controls of nonstationary Navier-Stokes equations. *Control Cybernet.*, 34(2):387–409, 2005.

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Date:

① Tuesday, Mar. 1 9:30-10:30 ② Tuesday, Mar. 1 10:45-11:45

③ Wednesday, Mar. 2 9:30-10:30 ④ Wednesday, Mar. 2 10:45-11:45

40 minutes talks

Amru HUSSEIN

Technical University of Darmstadt, Darmstadt

Title:

On the primitive Equations in L^p with Temperature: Part I

Abstract:

Primitive equations of the ocean and the atmosphere are considered to be a fundamental model for many geophysical flows. In this talk primitive equations with temperature and salinity are considered in L^p. Existence and uniqueness of strong solutions is discussed for arbitrarily large data where initial values lie in certain interpolation spaces contained in $H^{2/p,p}$. Local solutions are constructed using an iteration scheme, and proving global *a priori* estimates allow one then to extend local solutions globally.

Date:

Tuesday, Mar. 1 15:40-16:20

Jun KATO

Nagoya University, Nagoya

Title:

A priori estimate for the Navier-Stokes equations with the Coriolis force and some applications

Abstract:

In this talk, we consider the Navier-Stokes equations with the Coriolis force,

$$\begin{cases} \partial_t u - \nu\Delta u + \Omega e_3 \times u + (u,\nabla)u + \nabla p = 0, & \text{in } (0,\infty)\times\mathbb{R}^3, \\ \operatorname{div} u = 0, & \text{in } (0,\infty)\times\mathbb{R}^3, \end{cases}$$

with initial data $u|_{t=0} = u_0$, where the constant $\nu > 0$ denotes the viscosity coefficient of the fluid, and $\Omega \in \mathbb{R}$ represents the speed of rotation around the vertical unit vector $e_3 = (0,0,1)$. We prove that a priori estimate,

$$\|u(t)\|_{\chi^{-1}} + (\nu - (2\pi)^{-3}\|u_0\|_{\chi^{-1}}) \int_0^t \|u(\tau)\|_{\chi^1}\, d\tau \le \|u_0\|_{\chi^{-1}},$$

which is first derived for the Navier-Stokes equations by Lei and Lin (2011), holds under the effect of the Coriolis force, where $\|f\|_{\chi^m} = \int |\xi|^m |\widehat{f}(\xi)|\, d\xi$. As an application, existence of a unique global solution for arbitrary speed of rotation is proved, provided that the initial data satisfies $\|u_0\|_{\chi^{-1}} < (2\pi)^3\nu$.

This talk is based on a joint work with Hiroki Ito (Nagoya University).

Date:

Thursday, Mar. 3 12:00-12:40

Matthias KOTSCHOTE

University of Konstanz, Konstanz

Department Mathematics and Statistics
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Title:

Spectral analysis for steady states of compressible two-phase fluids of Navier-Stokes-Allen-Cahn type

Abstract:

This talk is concerned with the so-called "Navier-Stokes-Allen-Cahn" system (NSAC) which is a combination of the Navier-Stokes equations for compressible fluids with a phase field description of Allen-Cahn type. The NSAC system admits of describing two-phase patterns in a flowing liquid including phase transformations. This model finds its counterpart in the "Navier-Stokes-Cahn-Hilliard" system that imitates again a flow of a binary mixture being macroscopically immiscible but without permitting phase transformation.

The first purpose of this talk consists in presenting a derivation of NSAC and NSCH from first principles, in the spirit of rational mechanics, for fluids of very general constitutive laws. For NSAC, this deduction confirms and extends a proposal of Blesgen [1]. Regarding NSCH, it continues work of Lowengrub and Truskinovsky [2] and provides the apparently first justified formulation in the non-isothermal case.

The second part of the talk deals with the linear system obtained by linearizing around non-constant stationary solutions. For the associated linear operator a quite detailed description of the point spectrum can be accomplished, e.g. instable eigenvalues are characterized in terms of eigenvalues of an elliptic problem.

References:

[1] T. Blesgen: A generalisation of the Navier-Stokes equations to two-phase-flows. *J. Phys. D: Appl. Phys.* **32** (1999), 1119–1123.

[2] J. Lowengrub, L. Truskinovsky: Quasi-incompressible Cahn-Hilliard fluids and topological transitions. *Proc. R. Soc. Lond. A* **454** (1998), 2617–2654.

Date:

Wednesday, Mar. 2 15:40-16:20

Tomasz PIASECKI

Polish Academy of Sciences, Warsaw

Institute of Mathematics

Title:

Stationary compressible Navier-Stokes equations with inflow boundary conditions

Abstract:

In my talk I will present results concerning existence of regular solutions in a vicinity of given laminar flows to the Navier-Stokes system describing stationary flow of a compressible fluid:

$$\begin{aligned} &\rho v \cdot \nabla v - \operatorname{div} \mathbf{T}(\nabla v, \pi) = \rho f \\ &\operatorname{div}\,(\rho v) = 0 \\ &\operatorname{div}\,(\rho E v) = \rho f \cdot v + \operatorname{div}\,(\mathbf{T}(\nabla v) v) - \operatorname{div} q, \end{aligned} \tag{1}$$

supplied with boundary conditions which will be precised, in a bounded domain. Here v denotes the velocity of the fluid, ρ the density and θ the absolute temperature, the other quantities will be explained in the talk. The mathematical theory of system (1) is quite well developed in case of homogeneous boundary conditions understood as conditions when normal component of the velocity vanishes at the boundary. However, admission of flow accross the boundary leads to substantial mathematical difficulties related to the hyperbolicity of the continuity equation. In particular, in such case we have no large data existence results, the fact which gives additional motivation to studying regular solutions for small data.

It is well known that in framework of regular solutions the existence results are subject to certain constraints on the boundary in the neighbourhood of the points where characteristics of the continuity equation become tangent to the boundary. Such situation is avoided when we consider a problem in cylindrical domain. Therefore, in the first part of my talk I will focus on results in such class of domains. Here the regular solutions are understood as functions in Sobolev spaces with weak derivatives satisfying the equations a.e. Even though we do not have to tackle the singularity of the boundary, we still have to deal with lack of compactness in the continuity equation. I will present several ways of solving this difficulty.

In the second part of my talk I will move to problem in a domain where we have to deal with boundary singularity. Here we generalize slightly the definition of solution which leads to working in fractional Sobolev-Slobodetskii spaces. Such approach allows to relax considerably the assumptions of boundary singularity compared to previous results.

Date:

Friday, Mar. 4 12:00-12:40

Jonas SAUER

Technical University of Darmstadt, Darmstadt

Fachbereich Mathematik, Technische Universität Darmstadt, 64283 Darmstadt,
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Title:

Maximal Regularity: An Easy Approach

Abstract:

Throughout the last fifteen years or so, the concept of maximal L^p regularity has been examined and applied successfully to various evolution equations, in particular to quasilinear parabolic equations. The celebrated theorem of Weis states that in Banach spaces which fulfill the geometric UMD property, maximal regularity of an operator $-A$ generating a strongly continuous semigroup is equivalent to the $\mathcal{R}$-boundedness of the family $\{it(it+A)^{-1}|t \in \mathbb{R}\}$. This characterization has been used extensively to obtain maximal regularity for a variety of differential operators. In my talk, I will show that maximal regularity for the initial value problem is equivalent to a time-periodic variant of maximal regularity, regardless of the geometric properties of the underlying Banach space. As time-periodic L^p estimates can be obtained in a fast and straight-forward way for the Laplace and the Stokes operator on different domains such as the whole space, the half space and sufficiently regular bounded domains, this constitutes a slim and elegant proof of maximal L^p regularity of these operators which completely circumvents the somewhat complicated notion of $\mathcal{R}$-boundedness. In proving the corresponding time-periodic estimates, the key idea is to replace the time axis with a torus and reformulate the problem on a locally compact abelian group. Subsequently, the L^p estimates are obtained via Fourier multipliers in a group setting, which gives the result in the whole space. Other domains can then be treated by similar methods as are known from the corresponding elliptic problems.

The talk is based on joint works with Mads Kyed [1] and Yasunori Maekawa [2].

References:

[1] M. Kyed and J. Sauer, *A Method for Obtaining Time-Periodic L^p Estimates*, preprint.

[2] Y. Maekawa and J. Sauer, *Maximal Regularity of the Time-Periodic Stokes Operator on Unbounded and Bounded Domains*, preprint.

Date:

Wednesday, Mar. 2 12:00-12:40

Yohei TSUTSUI

Shinshu University, Nagano

Title:

Applications of weighted Hardy spaces to the incompressible Navier-Stokes equations

Abstract:

Applying the theory of weighted Hardy spaces $H^p_\alpha(\mathbb{R}^n)$, we consider time decay and asymptotic expansion of solutions.

For $\varphi \in C_0^\infty(B(0,1))$ with $\hat{\varphi}(0) = 1$, $p \in (0,\infty)$ and $\alpha \in (-n/p,\infty)$, H^p_α is denoted by a space of tempered distributions with finite semi-norm

$$\|f\|_{H^p_\alpha} := \|M_\varphi[f]\|_{L^p_\alpha} < \infty,$$

where

$$M_\varphi[f](x) := \sup_{t>0} |f * \varphi_t(x)|,\ \varphi_t(x) := t^{-n}\varphi(x/t) \text{ and } \|g\|_{L^p_\alpha} := \||x|^\alpha g\|_{L^p}.$$

Firstly, we establish the existence of global solutions with initial data $a \in L^n \cap H^p_\alpha$ having small L^n norm. L^2-energy of our solutions decays as $t^{-(n+2)/4-\varepsilon}$, $(\varepsilon > 0)$. Wiegner (‘87) showed the existence of weak solutions decaying as $t^{-(n+2)/4}$. In our approach , to remove ε in our decay, div - curl estimates with critical power weight $\alpha_p = n(1-1/p)+1$ seem to be necessary. These estimates are constructed by the real interpolation theory.

To study asymptotic expansion and rapid decay, the previous papers assume pointwise estimates on the initial data; $|a(x)| \lesssim \langle x\rangle^{-\gamma}$. We show similar results with non-decaying initial data, but belongs to weighted Hardy spaces.

This talk is based on a joint work with Takahiro Okabe in Hirosaki University.

Date:

Thursday, Mar. 3 15:40-16:20

15 minutes talks

Martin BOLKART

Technical University of Darmstadt, Darmstadt

Title:

The Stokes equations in spaces of BMO-type

Abstract:

We introduce a space BMO_b with norm measuring the BMO-seminorm in the interior of the domain and the mean value near the boundary. We will then prove that the Stokes semigroup is analytic in these BMO-type spaces. As an application we can then prove that in some domains the Stokes semigroup on L^p is analytic even if the Helmholtz projection in L^p does not exist.

Date:

Thursday, Mar. 3 16:30-16:45

Alexander DALINGER

Technical University of Darmstadt, Darmstadt

Title:

On the hydrodynamic limit and equilibrium fluctuations of a particle system with nearest neighbor interactions

Abstract:

For a one-dimensional particle system with nearest neighbor interactions we study the particle density. Starting with an arbitrary configuration of particles, we describe the time evolution of the system. First, we show that the particle density converges in the hydrodynamic limit to a solution of a nonlinear heat equation. Furthermore, we consider the random fluctuations around its average state, the so called equilibrium fluctuations.

Date:

Friday, Mar. 4 15:55-16:10

Shota ENOMOTO

Kyushu University, Fukuoka

Title:

On asymptotic behavior of solutions to the compressible Navier-Stokes equations in an infinite layer under slip boundary condition

Abstract:

We consider large time behavior of solutions to the compressible Navier-Stokes equation around the motionless state in a two-dimensional infinite layer under the

slip boundary condition. We show that the asymptotic leading part of perturbations is given by a superposition of self-similar solutions of one-dimensional viscous Burgers equations if initial perturbations are sufficiently small.

Date:

Friday, Mar. 4 17:00-17:15

Michael FISCHER

Technical University of Darmstadt, Darmstadt

Title:

Shape optimization with the Boussinesq equations

Abstract:

In this talk we consider the perturbation of identity method for shape optimization based on the works of Murat and Simon, to solve shape optimization problems governed by the Boussinesq equations. To this end we give an introduction to shape optimization in general and introduce the perturbation of identity method by Murat and Simon, see [1].

After the introduction we present a general framework to show Fréchet differentiability of the control to state operator of nonlinear optimal control problems.

Then we derive the shape optimization problem governed by the Boussinesq equations and apply the abstract Fréchet differentiability theorem to prove the shape differentiability of the solution operator. This extends the work by Lindeman et al., see [2], from the Navier-Stokes to the Boussinesq equations.

References:

[1] F. Murat and J. Simon, Etude de problemes d'optimal design, *Lecture Notes in Computer Science*, **41**(1976).

[2] F. Lindemann and M. Fischer and M. Ulbrich and S. Ulbrich, Fréchet differentiability of time-dependent incompressible Navier-Stokes flow with respect to domain variations, *Preprint*, (2015)

[3] G. Wachsmuth, Differentiability of implicit functions: Beyond the implicit function theorem, *Journal of Mathematical Analysis and Applications*, **Vol. 414, 259 - 272**, (2014)

Date:

Wednesday, Mar. 2 16:45-17:00

Mathis GRIES

Technical University of Darmstadt, Darmstadt

Title:

On the primitive equations in L^p-spaces with temperature, Part II

Abstract:

In this talk, we report on an ongoing work concerning the primitive equations in the non-isothermal setting. Our aim is to extend a result on the existence of time periodic solutions for large forces in the isothermal case to the non-isothermal setting.

Date:

Tuesday, Mar. 1 16:30-16:45

Andreas KLAIBER

University of Konstanz, Konstanz

Title:

Spectral stability of internal solitary waves in continuously stratified fluids

Abstract:

Internal solitary waves (ISWs), which travel in the interior of lakes and oceans, present a fascinating observable wave phenomenon with large amplitudes and a remarkable robustness. These waves are ecologically important since they are involved in mixing and energy transport.

From the mathematical point of view, ISWs are exact solutions of the 2D Euler equations for incompressible, inviscid fluids with non-constant density. This talk is concerned with the study of their spectral stability, which has received only little attention at a rigorous mathematical level.

After introduction of the mathematical model, the first part of the talk deals with an Evans-function approach to spectral stability of ISWs. This approach starts from the Euler eigenvalue problem and is based on finite-dimensional truncations of a formal spatial-dynamics formulation. For small-amplitude ISWs, this yields a proof of the absence of small unstable modes by comparison with solitons in the Korteweg-deVries equation.

The second part of the talk deals with the definition of a “moment of instability” for ISWs (a classical tool in the stability theory of solitary waves), which presents a complementary approach to stability of ISWs.

Date:

Wednesday, Mar. 2 16:30-16:45

Naofumi MORI

Kyushu University, Fukuoka

Title:

Global existence and decay property of the Timoshenko-Cattaneo system

Abstract:

Our main focus in this talk is to show the asymptotic behavior of a nonlinear version of the Timoshenko system with heat conduction of Cattaneo's law. As it has been already proved in Said-Houari and Kasimov (2012) and Mori and Kawashima (2015), the linear version of this system is of regularity-loss type. It is well known (Hosono and Kawashima (2006), Ide and Kawashima (2008), Kubo and Kawashima (2009)) that the regularity-loss property of the linear problem creates difficulties when dealing with the nonlinear problem. In fact, the dissipative property of the problem becomes very weak in the high frequency region and as a result the classical energy method fails. To overcome this difficulty and following Ide and Kawashima (2008) and Ikehata (2002), Racke and Said-Houari (2012) use an energy method with negative weights to create an artificial damping which allows us to control the nonlinearity. They prove that for $0 \leq k \leq [s/2] - 2$ with $s \geq 8$, the solutiuon of the problem is global in time and decays as $\|\partial_x^k U(t)\|_{L^2} \leq C(1+t)^{-1/4-k/2}$, provided that the initial datum $U_0 \in H^s(\mathbb{R}) \cap L^1(\mathbb{R})$. On the other hand, in this talk, we show the global existence result of the same problem with the critical sobolev space $s = 2$ by using just an energy method without any negative weights. Besides, we also show the optimal decay result with the minimal regularity assumptions on the initial data $U_0 \in H^2(\mathbb{R}) \cap L^1(\mathbb{R})$ by using a time decay inequarity of $L^2(\mathbb{R})$-$L^q(\mathbb{R})$-$L^r(\mathbb{R})$ type first developed in Ju, Mori and Kawashima (2015). This talk is based on a joint work with Prof. Reinhard Racke of Univ. Konstanz.

Date:

Friday, Mar. 4 17:15-17:30

Dai NOBORIGUCHI

Waseda University, Tokyo

Title:

On a time-splitting method for degenerate parabolic stochastic partial differential equations

Abstract:

In this talk we consider degenerate parabolic stochastic partial differential equations of the following type

$$du + \mathrm{div}(B(u))dt = \mathrm{div}(A(u)\nabla u) + \Phi(u)d\beta(t) \quad \text{in } (0,T) \times \mathbb{T}^d,$$

with the initial condition

$$u(0,\cdot) = u_0(\cdot) \quad \text{in } \mathbb{T}^d.$$

Here $\mathbb{T}^d$ is the d-dimensional torus, $T > 0$, and β is a one-dimensional Brownian motion defined on a stochastic basis $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}, \mathbb{P})$.

We introduce the definition of kinetic solutions characterized by kinetic formulations. To prove the existence of such solutions, we will use a time-splitting approach.

Date:

Friday, Mar. 4 16:10-16:25

Tobias SEITZ

Technical University of Darmstadt, Darmstadt

Title:

Enhancement of flow measurements using fluid-dynamic constraints

Abstract:

Novel experimental modalities acquire spatially resolved velocity measurements for steady state and transient flows which are of interest for engineering and biological applications. One of the drawbacks of such high resolution velocity data is their susceptibility to measurement errors. We propose a novel filtering strategy that allows enhancement of noisy measurements to obtain reconstruction of smooth divergence free velocity and corresponding pressure fields, which together approximately comply to a prescribed flow model. The main step in our approach consists of the appropriate use of the velocity measurements in the design of a linearized flow model which can be shown to be well-posed and consistent with the true velocity and pressure fields up to measurement and modeling errors. The reconstruction procedure is formulated as a linear quadratic optimal control problem and the resulting filter has analyzable smoothing and approximation properties.

Date:

Friday, Mar. 4 15:40-15:55

Anton SEYFERT

Technical University of Darmstadt, Darmstadt

Title:

Almost periodic solutions to incompressible fluid flow problems

Abstract:

In this talk we consider almost periodic solutions in time to various problems arising in fluid mechanics such as the Navier-Stokes flow in an aperture domain or the flow past a rotating obstacle. Our approach is based on an extension of a recent result on periodic solutions of this kind of problems.

Date:

Friday, Mar. 4 16:45-17:00

Florian STEINBERG

Technical University of Darmstadt, Darmstadt

Title:

Complexity Theory in Sobolev Spaces

Abstract:

The framework of second order representations was introduced by Kawamura and Cook in 2010 and provides a rigorous notion of computation over continuous structures. It also provides a notion of time and space restricted computation. Choosing a representation means to specify what information a program computing an element of the structure has to provide. We relate the existence of a reasonable choice of information on a compact metric space to a purely metric property of the space: The metric entropy of a compact metric space measures fast the number of balls needed to cover the space grows with the radius of the balls decreasing. We show that the optimal running time of the metric is inherently connected to the metric entropy. These results are applied to show optimality of some concrete representations of function spaces.

Date:

Tuesday, Mar. 1 17:00-17:15

Go TAKAHASHI

Waseda University, Tokyo

Title:

On partial regularity and extension of solutions to the Navier-Stokes equations

Abstract:

In this talk, we work on the local-in-time classical solution to the incompressible Navier-Stokes equations. The dimension n is 3 or 4. First, we discuss the partial regularity of suitable weak solutions, which is an essential argument to obtain the local boundedness of the solutions. Next, we will establish a time extension criterion to the local-in-time solutions as an application of ϵ regularity theorem.

Date:

Thursday, Mar. 3 16:45-17:00

Shinya UCHIUMI

Waseda University, Tokyo

Title:

Exactly computable Lagrange–Galerkin schemes and their numerical results

Abstract:

We present exactly computable Lagrange–Galerkin schemes for convection-diffusion problems and the Navier–Stokes problems, which we recently developed [1, 2].

Lagrange–Galerkin schemes are powerful numerical methods for flow problems. In conventional schemes, we have to compute integration of terms which include composite functions and it is difficult to compute the values exactly in general. Although numerical quadrature is usually employed to the terms in real computations, the error caused by the quadrature may destroy the stability. We introduce a locally linearized velocity and the backward Euler method to solve ordinary differential equations which describe the position of a fluid particle. Then, the schemes become exactly computable and theoretical stability and convergence are assured.

In this talk, we introduce these exactly computable schemes and show error estimates. We also present some numerical results, which reflect these estimates and also show robustness for high Péclet number and high Reynolds number problems.

This talk is a joint work with Prof. M. Tabata at Waseda University.

References:

[1] M. Tabata and S. Uchiumi. A genuinely stable Lagrange–Galerkin scheme for convection-diffusion problems. *Japan Journal of Industrial and Applied Mathematics*, 2015. doi:10.1007/s13160-015-0196-2

[2] M. Tabata and S. Uchiumi. A Lagrange–Galerkin scheme with a locally linearized velocity for the Navier–Stokes equations. 2015. arXiv:1505.06681 [math.NA]

Date:

Wednesday, Mar. 2 17:00-17:15

David WEGMANN

Technical University of Darmstadt, Darmstadt

Title:

Decay of Non-stationary Navier-Stokes Flow with Nonzero Dirichlet Boundary Data

Abstract:

We consider the Navier-Stokes equations in a domain with compact boundary and nonzero Dirichlet boundary data β. Recently, in the case of an exterior domain the existence of a global in time weak solution which satisfies the strong energy inequality was shown in [1]. This solution is constructed as a sum of a very weak solution b to the instationary Stokes equation with nonzero boundary data and a weak solution v to a system of Navier-Stokes type with zero boundary data.

Assuming that $\beta(t) \to 0$ as $t \to \infty$, we prove in [2] that the corresponding solution v fulfills $\|v(t)\|_2 \to 0$ as $t \to \infty$. Furthermore, in a bounded domain the solution v

tends exponentially to 0 if the corresponding data are exponentially decreasing.

As a last result, we calculate a lower polynomial bound for the decay rate if Ω is unbounded. Therefore, we use a suitable spectral decomposition of the Stokes operator as introduced in [3] and Duhamel's formula.

This is a result of a collaboration with Prof. Dr. Reinhard Farwig and Prof. Dr. Hideo Kozono.

References:

[1] Farwig, R., Kozono, H., "Weak solutions of the Navier-Stokes equations with non-zero boundary values in an exterior domain satisfying the strong energy inequality," J. Differential Equations, **7**, No. 10, 2633–2658 (2014).

[2] Farwig, R., Kozono, H., Wegmann, D., "Decay of non-stationary Navier-Stokes flow with nonzero Dirichlet boundary data," Indiana Univ. Math. J., accepted for publication (2015)

[3] Borchers, W., Miyakawa, T., "Algebraic L^2 decay for Navier-Stokes flows in exterior domains. II," Hiroshima Math. J., **3**, 621–640 (1991).

Date:

Thursday, Mar. 3 17:00-17:15

Sebastian ZAIGLER

Technical University of Darmstadt, Darmstadt

Title:

Regularity Structures

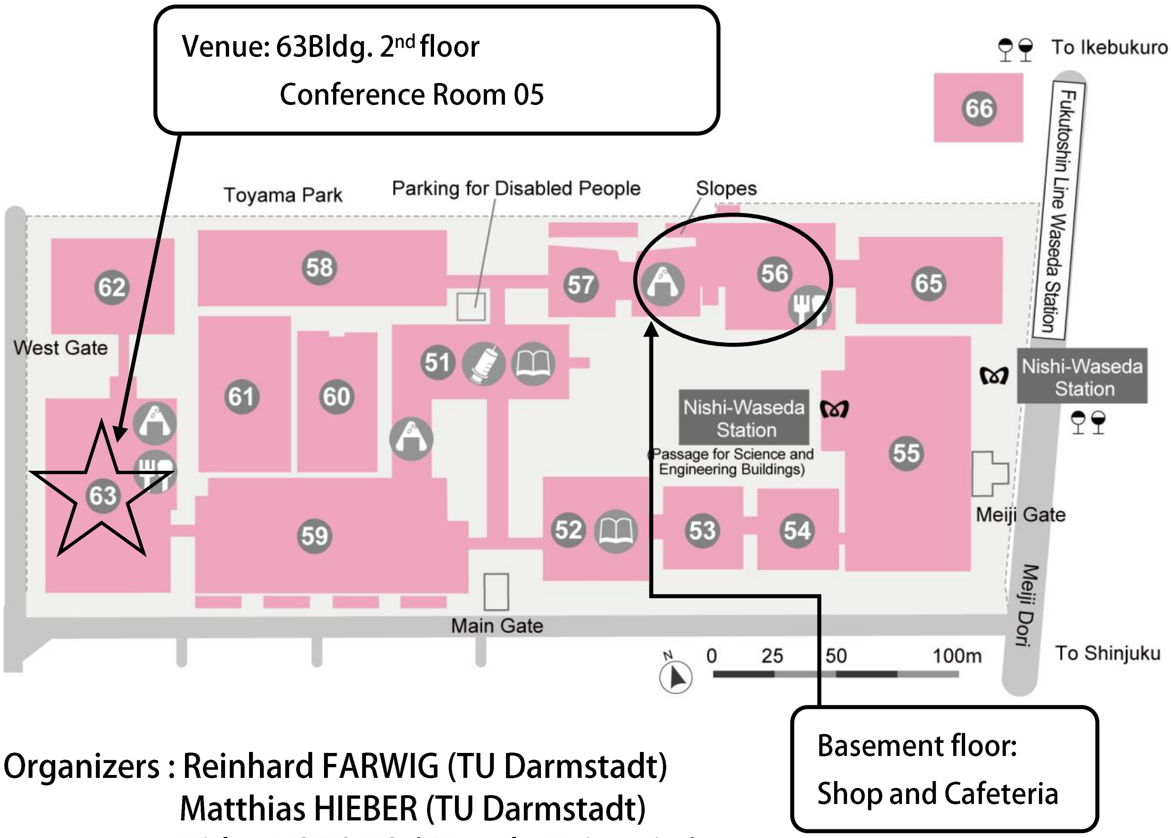
Abstract:

Regularity structures are a new method introduced by Martin Hairer to solve subcritical SPDEs with rough data, which were formerly inaccessible. They build vector spaces of abstract "Taylor expansions" of functions and distributions leading to a fixed point equation that solves a SPDE in these abstract spaces. Then it is possible to find a sequence of renormalised solutions of the SPDE so that it converges to the solution of the original problem. We will give a short introduction to this technique and outline our goal to use them on the primitive equations with white noise.

Date:

Tuesday, Mar. 1 16:45-17:00

Nishi-Waseda Campus Map



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