Last update: February 28, 2017

独立行政法人日本学術振興会　日独共同大学院プログラム

JSPS-DFG Japanese-German Graduate Externship

第 14 回　日独流体数学国際研究集会

The 14th Japanese-German International Workshop on Mathematical Fluid Dynamics

March 8 - 10, 2017 at Waseda University, Nishi-Waseda Campus

63 Bldg. 2nd Floor, 05 Meeting Room

■ Program ■

	Wed, Mar. 08		Thu, Mar. 09		Fri, Mar. 10
9:20-9:30	Opening				
9:30-10:40	Reinhard RACKE ① (Univ. of Konstanz)	9:30-10:40	RACKE ②	9:30-10:40	KAGEI ③
10:50-11:30	Mitsuo HIGAKI (Kyoto University)	10:50-12:00	KAGEI ②	10:50-12:00	RACKE ③
11:40-12:20	Yutaka TERASAWA (Nagoya Univ.)	12:15-12:55	Anupam PAL CHOUDHURY (TU Darmstadt)	12:15-12:55	Hideyuki MIURA (Tokyo Inst. Tech.)
	Lunch break		Lunch Break		Lunch Break
14:20-15:00	Patrick TOLKSDORF (TU Darmstadt)	14:40-15:20	Amru HUSSEIN (TU Darmstadt)	14:50-15:30	Giorgio METAFUNE (Univ. of Salento)
15:30-16:40	Yoshiyuki KAGEI ① (Kyushu Univ.)	15:50-16:30	Nobu KISHIMOTO (Kyoto University)	16:00-17:10 70 min.	Taiga KUMAGAI (Waseda Univ.)
17:00-18:00 60 min.	Shota ENOMOTO (Kyushu Univ.)	16:50-18:10 80 min.	Marc WRONA (TU Darmstadt)		Anton SEYFERT (TU Darmstadt)
	Go TAKAHASHI (Waseda Univ.)		Sebastian ZAIGLER (TU Darmstadt)		Suma'inna (Waseda Univ.)
	David WEGMANN (TU Darmstadt)		Mathis GRIES (TU Darmstadt)		Klaus KRESS (TU Darmstadt)
	Andreas SCHMIDT (TU Darmstadt)		Shinya UCHIUMI (Waseda Univ.)		Naoto KAJIWARA (Univ. of Tokyo)
			Tobias TOLLE (TU Darmstadt)	17:15-17:20	Closing
			Tomoya KEMMOCHI (Univ. of Tokyo)	18:00-	Closing Recaption @Takeuchi Lounge

● →Main-course　● →40min talk　● →12min talk

Main-Course

Yoshiyuki KAGEI
Kyushu University, Fukuoka, Japan

Title:

Stability and bifurcation analysis of the compressible Navier-Stokes equations

Abstract:

In this mini-course I will talk about the instability of the plane Poiseuille flow and the bifurcation of traveling wave solutions.

1 The perturbation equations

We consider the compressible Navier-Stokes equations in an two-dimensional infinite layer Ω

$$\partial_t \rho + \mathrm{div}\,(\rho v) = 0, \tag{1}$$

$$\rho(\partial_t v + v \cdot \nabla v) - \nu \Delta v - (\nu + \nu')\nabla \mathrm{div}\, v + \nabla P(\rho) = \nu \rho \boldsymbol{e}_1. \tag{2}$$

Here the equations are written in a non-dimensional form;

$$\Omega = \{x = (x_1, x_2) \,:\, x_1 \in \mathbf{R},\, 0 < x_2 < 1\};$$

$\rho = \rho(x,t)$ and $v = {}^\top(v^1(x,t), v^2(x,t))$ are the density and the velocity, respectively; $P = P(\rho)$ is the pressure which is a smooth function of ρ satisfying

$$P'(1) > 0;$$

ν and ν' are the non-dimensional parameters that satisfy

$$\nu > 0 \text{ and } \nu + \nu' \geq 0;$$

and $\boldsymbol{e}_1 = {}^\top(1,0) \in \mathbf{R}^2$.

The system (1)–(2) is considered under the boundary condition

$$v|_{x_2=0,1} = 0. \tag{3}$$

One can see that (1)–(3) has a stationary solution $u_s = {}^\top(\phi_s, v_s)$, called the Poiseuille flow:

$$\phi_s = 1, \quad v_s = \frac{1}{2} x_2(\ell - x_2)\boldsymbol{e}_1.$$

We consider the stability of the Poiseuille flow u_s under the perturbation $u = (\phi, w) = (\gamma^2(\rho - 1), v - v_s)$ which is periodic in x_1. The perturbation u is governed by the system of equations

$$\partial_t \phi + v_s^1 \partial_{x_1} \phi + \gamma^2 \mathrm{div}\, w = f^0(\phi, w), \tag{4}$$

$$\partial_t w - \nu \Delta w - \tilde{\nu} \nabla \mathrm{div}\, w + \nabla \phi - \frac{\nu}{\gamma^2} \phi \boldsymbol{e}_1 + v_s^1 \partial_{x_1} w + (\partial_{x_2} v_s^1) w^2 \boldsymbol{e}_1 = \boldsymbol{f}(\phi, w), \tag{5}$$

where $f^0(\phi, w)$, $\boldsymbol{f}(\phi, w)$ are the nonlinearities; $\gamma = \sqrt{P'(1)}$; and $\tilde{\nu} = \nu + \nu'$. This system is considered under the boundary conditions:

$$w|_{x_2=0,1} = 0, \quad \phi,\ w\text{: } \tfrac{2\pi}{\alpha}\text{-periodic in } x_1, \tag{6}$$

Here α is a given positive number (wave number). The Reynolds number R and the Mach number M are given by $\mathrm{R} = 1/(16\nu)$ and $\mathrm{M} = 8/\gamma$, respectively.

2 Instability of the Poiseuille flow

We set $\Omega_\alpha = [-\frac{\pi}{\alpha}, \frac{\pi}{\alpha}) \times (0,1)$. We define the linearized operator L on $X = L^2_*(\Omega_\alpha) \times L^2(\Omega_\alpha)$ defined by

$$D(L) = \left\{ u = {}^\top(\phi, w) \in X : w \in H^1_{0,per}(\Omega_\alpha),\, Lu \in L^2(\Omega_\alpha) \right\},$$

$$L = \begin{pmatrix} v_s^1 \partial_{x_1} & \gamma^2 \mathrm{div} \\ \nabla & -\nu\Delta - \tilde{\nu}\nabla\mathrm{div} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\frac{\nu}{\gamma^2}\boldsymbol{e}_1 & v_s^1\partial_{x_1} + (\partial_{x_2} v_s^1)\boldsymbol{e}_1 {}^\top\boldsymbol{e}_2 \end{pmatrix}.$$

Here $L^2_*(\Omega_\alpha) = \{\phi \in L^2(\Omega_\alpha) : \int_{\Omega_\alpha} \phi(x)\,dx = 0\}$ and $\boldsymbol{e}_2 = {}^\top(0,1) \in \mathbf{R}^2$.

We have the following instability result.

Theorem 1 ([2]) *There are positive numbers r_0 and η_0 such that if $\alpha \le r_0$, then*

$$\sigma(-L) \cap \left\{ \lambda \in \mathbf{C} : |\lambda| \le \eta_0 \right\} = \{\lambda_{m\alpha}(\nu) : |m| = 1, \cdots, k\}$$

for some $k \in \mathbf{N}$, where $\lambda_{m\alpha}(\nu)$ is a simple eigenvalue of $-L$ and satisfies

$$\lambda_{m\alpha}(\nu) = -\frac{i}{6}(\alpha m) + \kappa_0(\alpha m)^2 + O(|\alpha m|^3)$$

as $|\alpha m| \to 0$. Here κ_0 is the constant given by

$$\kappa_0 = \frac{1}{12\nu}\left(\frac{1}{280} - \gamma^2 - \frac{\nu^2}{15\gamma^2} - \frac{\nu\tilde{\nu}}{30\gamma^2} \right).$$

Therefore, if $\gamma^2 < \frac{1}{280}$ and $2\nu^2 + \nu\tilde{\nu} < 30\gamma^2\left(\frac{1}{280} - \gamma^2\right)$, then $\kappa_0 > 0$, and so the Poiseuille flow $u_s = {}^\top(\phi_s, v_s)$ is unstable.

3 Bifurcating traveling waves

We next consider the bifurcation of the Poiseuille flow. We fix γ in such a way that $\frac{1}{280} - \gamma^2 > 0$, and regard ν as a bifurcation parameter. We denote the linearized operator L by L_ν. If ν is suitably large, then κ_0 satisfies $\kappa_0 < 0$. When ν decreases, κ_0 satisfies $\kappa_0 = 0$ at a certain value of ν, and below such a value of ν, κ_0 satisfies $\kappa_0 > 0$. It then follows that for each $0 < \alpha \ll 1$ there exists a positive number ν_0 such that

$$\mathrm{Re}\,\lambda_{\pm\alpha}(\nu_0) = 0\,; \quad \mathrm{Re}\,\lambda_{\pm\alpha}(\nu) < 0 \Leftrightarrow \nu > \nu_0\,; \quad \mathrm{Re}\,\lambda_{\pm\alpha}(\nu) > 0 \Leftrightarrow \nu < \nu_0.$$

We assume that the spectrum of $-L_{\nu_0}$ satisfies the following condition.

$$\sigma(-L_{\nu_0}) \cap \{\lambda; \mathrm{Re}\,\lambda = 0\} = \{\lambda_\alpha(\nu_0), \lambda_{-\alpha}(\nu_0)\}. \tag{7}$$

Under this assumption we will prove the existence of bifurcating traveling wave solutions.

Theorem 2 ([3]) *Assume that* (7) *holds true. Then there exists a solution branch* $\{\nu, u\} = \{\nu_\varepsilon, u_\varepsilon\}$ $(|\varepsilon| \ll 1)$ *satisfying*

$$\begin{aligned}
&\nu_\varepsilon = \nu_0 + O(\varepsilon),\\
&u_\varepsilon = u_\varepsilon(x_1 - c_\varepsilon t, x_2),\\
&u_\varepsilon(x_1 + \tfrac{2\pi}{\alpha}, x_2) = u_\varepsilon(x_1, x_2),\\
&u_\varepsilon(x_1, x_2) = \varepsilon \begin{pmatrix} 1 \\ \frac{1}{2\gamma^2}(-x_2^2 + x_2) \\ 0 \end{pmatrix} \tfrac{\sqrt{2}}{2} \cos \alpha x_1 (1 + O(\alpha)) + O(\varepsilon^2),\\
&c_\varepsilon = \tfrac{1}{6} + O(\varepsilon).
\end{aligned}$$

Remark 3 Iooss–Padula ([1]) showed that for each ν there exists a positive number Λ such that the set $\sigma(-L_\nu) \cap \{\lambda; \mathrm{Re}\,\lambda \geq -\Lambda\}$ consists of finite number of eigenvalues with finite multiplicities. Therefore, it seems very unlikely that (7) does not hold for all $0 < \alpha \ll 1$.

References:

[1] G. Iooss and M. Padula, Structure of the linearized problem for compressible parallel fluid flows, Ann. Univ. Ferrara, Sez. VII, **43** (1998), pp. 157–171.

[2] Y. Kagei and T. Nishida, Instability of plane Poiseuille flow in viscous compressible gas, J. Math. Fluid Mech., **17** (2015), pp. 129–143.

[3] Y. Kagei and T. Nishida, T raveling waves bifurcating from plane Poiseuille flow of the compressible Navier-Stokes equation, preprint, 2015, MI Preprint Series 2015-9, Kyushu University.

Date:

① Wednesday, Mar. 08 15:30-16:40 ② Thursday, Mar. 09 10:50-12:00

③ Friday, Mar. 10 09:30-10:40

Reinhard RACKE

University of Konstanz, Konstanz

Title:

Hyperbolic Navier-Stokes Equations

Abstract:

The classical *incompressible* Navier-Stokes equations in the whole space $\mathbb{R}^n$, $n = 2, 3$,

$$\begin{aligned} u_t - \mu\Delta u + (u\cdot\nabla)u + \nabla p &= 0 && \text{in } (0,\infty)\times\mathbb{R}^n, && (0.1)\\ \operatorname{div} u &= 0 && \text{in } (0,\infty)\times\mathbb{R}^n, && (0.2)\\ u(0,\cdot) &= u_0 && \text{in } \mathbb{R}^n, && (0.3) \end{aligned}$$

with $\mu > 0$ being the viscosity, for the velocity vector $u = u(t,x) : (0,\infty)\times\mathbb{R}^n \to \mathbb{R}^n$ of a fluid, and $p = p(t,x) : (0,\infty)\times\mathbb{R}^n \to \mathbb{R}$ the related pressure, arise from the transport law

$$u_t + (u\cdot\nabla)u + \nabla p = \operatorname{div} S \tag{0.4}$$

and the constitutive law for the tensor S,

$$S = \frac{\mu}{2}(\nabla u + (\nabla u)'), \tag{0.5}$$

together with the incompressibility (zero divergence) condition (0.2) and initial conditions (0.3). We replace the Fourier type relation (0.5) by the Cattaneo type relation

$$\tau S_t + S = \frac{\mu}{2}(\nabla u + (\nabla u)'). \tag{0.6}$$

The Fourier type constitutive assumption (0.5) — in addition to the pressure contribution — leads to the well-known parabolic-type classical Navier-Stokes system (0.1)–(0.3); there, in particular, we have the effect of an infinite propagation speed of signals, as it is well-known as modeling problem/paradox for heat equations, or, more generally, for flux type equations (diffusion problems, ...) where the flux relation is given by the Fourier type. There are applications, however, where it is more reasonable to work with hyperbolic models.

Differentiating the transport equation (0.4) with respect to t, and using the new relation (0.6), we obtain the new hyperbolicly perturbed Navier-Stokes system

$$\begin{aligned} \tau u_{tt} - \mu\Delta u + u_t + \nabla p + \tau\nabla p_t &= -(u\cdot\nabla)u - (\tau u_t\cdot\nabla)u - (\tau u\cdot\nabla)u_t && (0.7)\\ & \qquad \text{in } (0,\infty)\times\mathbb{R}^n, &&\\ \operatorname{div} u &= 0 \qquad \text{in } (0,\infty)\times\mathbb{R}^n, && (0.8)\\ u(0,\cdot) = u_0, \qquad u_t(0,\cdot) &= u_1 \qquad \text{in } \mathbb{R}^n. && (0.9) \end{aligned}$$

It will turn out that, in this system, at least the vorticity $\nabla \times u$ has finite propagation speed.

There is not only the hyperbolic character of a wave equation for u, that complicates things by less regularization properties, but we notice the nonlinearities which are — in contrast to the classical case — of highest order, see the term $(\tau u \cdot \nabla)u_t$. The equation (0.7) can be regarded as a *damped* wave equation only for *small* values of u, since the term $(\tau u_t \cdot \nabla)u$ might disturb the positive damping term u_t for large u. — This is a hint to think about a possible blow-up situation for large data.

Therefore, we will combine and apply techniques known for nonlinear heat equations, where additional difficulties will arise through the Helmholtz projections. The result will be first a – more general – local existence theorem, and then a global existence theorem for small data. This can be extended to an almost global existence theorem for large data relating the admissible size of the data to the size of the relaxation parameter τ.

The *compressible* Navier-Stokes equations with heat conduction in $[0,\infty) \times \mathbb{R}^n$, for $n \geq 1$, can be written in the following form

$$\begin{aligned}
\partial_t\rho + \mathrm{div}\,(\rho u) &= 0, && (0.10)\\
\partial_t(\rho u) + \mathrm{div}\,(\rho u \otimes u) + \nabla p &= \mathrm{div}\, S, && (0.11)\\
\partial_t(\rho(e + \frac{1}{2}u^2)) + \mathrm{div}\,(\rho u(e + \frac{1}{2}u^2) + up) + \mathrm{div}\, q &= \mathrm{div}\,(uS), && (0.12)
\end{aligned}$$

where ρ, $u = (u_1, u_2, \cdots, u_n)$, p, S, e and q represent fluid density, velocity, pressure, stress tensor, specific internal energy per unit mass and heat flux, respectively. The equations (0.10), (0.11) and (0.12) are the consequence of conservation of mass, momentum and energy, respectively.

To complete the system, we need to impose constitutive assumptions on p, S, e and q. First, we assume the fluid to be a Newtonian fluid, that is,

$$S = \mu(\nabla u + (\nabla u)^T) + \mu'\nabla \mathrm{div}\, u, \qquad (0.13)$$

where μ and μ' are the coefficient of viscosity and the second coefficient of viscosity, respectively, satisfying

$$\mu > 0,\ \mu' + \frac{2}{n}\mu \geq 0.$$

The heat flux q is assumed to satisfy

$$\tau\partial_t q + q + \kappa\nabla\theta = 0, \qquad (0.14)$$

which represents Cattaneo's law (Maxwell's law, ...) and gives rise to heat waves with finite propagation speed. Here, $\tau > 0$ is the constant relaxation time and $\kappa > 0$ is the constant heat conductivity. For small relaxation parameter τ a global well-posedness result is obtained for small data.

Instead of relaxing in the heat equation, one may think of relaxing the constitutive law for the stress tensor. Maxwell's relaxation replaces (0.13) by the differential equation

$$\tau\partial_t S + S = \mu(\nabla u + \nabla u^T - \frac{2}{n}\mathrm{div} u\, I_n) + \lambda \mathrm{div} u\, I_n, \tag{0.15}$$

where $\lambda := \mu' + \frac{2}{n}\mu$. A splitting of the tensor in the form

$$S = S_1 + S_2 I_n \tag{0.16}$$

and studying

$$\begin{aligned} \partial_t\rho + \mathrm{div}(\rho u) &= 0, && (0.17)\\ \partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) + \nabla p &= \mathrm{div}(S_1) + \nabla S_2, && (0.18)\\ \partial_t(\rho(e + \frac{1}{2}u^2)) + \mathrm{div}(\rho u(e + \frac{1}{2}u^2) + up) - \kappa\triangle\theta &= \mathrm{div}(u(S_1 + S_2 I_n)), && (0.19) \end{aligned}$$

$$\begin{aligned} \tau_1\partial_t S_1 + S_1 &= \mu(\nabla u + \nabla u^T - \frac{2}{n}\mathrm{div} u I_n), && (0.20)\\ \tau_2\partial_t S_2 + S_2 &= \lambda \mathrm{div} u, && (0.21) \end{aligned}$$

is successful in leading to a global well-posedness result for small data.

The plan of the lectures is as follows:

Lecture I:

Survey and motivation for relaxation. Local well-posedness for hyperbolic *incompressible* Navier-Stokes equations [3].

Lecture II:

Proof of local existence. Global well-posedness for small data [3, 4].

Lecture III:

Almost global existence for large data [5]. *Compressible* Navier-Stokes equations with hyperbolic heat or with revised Maxwell's law [1, 2].

References:

[1] Hu, Y., Racke, R.: Compressible Navier-Stokes equations with hyperbolic heat conduction. *J. Hyperbolic Differential Equations.* **13** (2016). 233–247.

[2] Hu, Y., Racke, R.: Compressible Navier-Stokes equations with revised Maxwell's law. *J. Math. Fluid Mech.* (2016) (*accepted*).

[3] Racke, R., Saal, J.: Hyperbolic Navier-Stokes equations I: local well-posedness. *Evolution Equations Control Theory* **1** (2012), 195–215.

[4] Racke, R., Saal, J.: Hyperbolic Navier-Stokes equations II: global existence of small solutions. *Evolution Equations Control Theory* **1** (2012), 217–234.

[5] Schöwe, A.: Global strong solution for large data to the hyperbolic Navier-Stokes equation. arXiv:1409.7797 (2014).

Date:

① Wednesday, Mar. 08 9:30-10:40 ② Thursday, Mar. 09 09:30-10:40

③ Friday, Mar. 10 10:50-12:00

40 minutes talks

Mitsuo HIGAKI

Kyoto University, Kyoto

Department of Mathematics, Kyoto University, Japan.

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Title:

Navier wall law for nonstationary viscous incompressible flows

Abstract:

In fluid mechanics, it is a basic subject to understand the mathematical structure of flows near a solid wall with a rough surface. In the following we consider the initial-boundary value problem of the Navier-Stokes system in the two-dimensional rough-boundary domain $\Omega^\varepsilon = \{x = (x_1, x_2) \in \mathbb{R}^2 \mid \varepsilon\omega(\frac{x_1}{\varepsilon}) < x_2 < \infty\}$.

$$\begin{cases} \partial_t u^\varepsilon - \Delta u^\varepsilon + u^\varepsilon \cdot \nabla u^\varepsilon + \nabla p^\varepsilon = 0\,, & t > 0\,, \quad x \in \Omega^\varepsilon\,, \\ \operatorname{div} u^\varepsilon = 0\,, & t \geq 0\,, \quad x \in \Omega^\varepsilon\,, \\ u^\varepsilon(x_1, x_2) \text{ is } 2\pi\text{-periodic in } x_1\,, & t \geq 0\,, \\ u^\varepsilon|_{t=0} = u_0\,, & x \in \Omega^\varepsilon\,. \end{cases}$$

The Dirichlet (no-slip) boundary condition is imposed on the rough boundary $\partial\Omega^\varepsilon$.

$$u^\varepsilon = 0 \qquad \text{on} \quad \partial\Omega^\varepsilon\,.$$

The unknown functions $u^\varepsilon = u^\varepsilon(t,x) = (u_1^\varepsilon(t,x), u_2^\varepsilon(t,x))^\top$ and $p^\varepsilon = p^\varepsilon(t,x)$ are respectively the velocity field and the pressure field of the fluid. The initial data u_0 is assumed to be given by the zero-extension of some velocity field a on the half-plane $\mathbb{R}^2_+ = \{x \in \mathbb{R}^2 \mid x_2 > 0\}$. The boundary function $\omega : \mathbb{R} \to (-1, -\frac{1}{2})$ is assumed to be smooth and 2π-periodic. The parameter $\varepsilon = \frac{1}{N}$, $N \in \mathbb{N}$, characterizes the amplitude and the pulse width (namely the "roughness") of the rough boundary $\partial\Omega^\varepsilon$.

The Navier wall law in fluid mechanics is a typical approach to describe the effect of such a rough boundary on the flows, which consists of the next two steps: (i) replacing the original boundary by an artificially flat one, and (ii) imposing a new condition on the flat boundary, which reflects the averaged effect of the roughness.

However, the derivation of the Navier wall law often relies on formal computations, and it is therefore important to justify the wall law with a mathematical rigor. The justification of the Navier wall law is directly related to the higher order expansion of the rough domain flow u^ε in the parameter ε. In order to make such an expansion rigorous, one naturally needs the high regularity of the flow appearing in the limit as ε goes to zero. Although recently Mikelić, Nečasová, and Neuss-Radu (2013) discusses the Navier wall law for the nonstationary flow with an external force,

its argument is based on the assumptions that the initial data is zero, and that the external force is smooth and identically zero near $t = 0$. Thus the regularity problem of the ε-zero limit flow is essentially avoided by these special assumptions.

In this talk we verify the Navier wall law for the initial data in C^1 class under natural compatibility conditions. Our proof relies on the boundary layer analysis and the L^∞ theory of the Navier-Stokes system in the half plane.

Date:

Wednesday, Mar. 08 10:50-11:30

Amru HUSSEIN

Technical University of Darmstadt, Darmstadt

Title:

Strong L^p Well-Posedness of the 3D Primitive Equations

Abstract:

Primitive equations are considered to be a fundamental model for geophysical flows. Here, the L^p theory for the primitive equations is developed. In this study, the linearized Stokes type problem plays a prominent role, and it turns out that it can be treated efficiently using perturbation methods for H^∞-calculus.

Applying maximal L^p regularity and proving H^2 *a priori* bounds one proves that this set of equations is globally strongly well-posed for arbitrary large initial data lying in a trace space which is a subspaces of $H^{2/p,p}$, $1 < p < \infty$. Thus, the general L^p setting admits rougher data than the usual L^2 theory with initial data in H^1.

Date:

Thursday, Mar. 09 14:40-15:20

Nobu KISHIMOTO

Kyoto University, Kyoto

Title:

Remark on global regularity for the rotating Navier-Stokes equations in a periodic domain

Abstract:

We consider the rotating incompressible Navier-Stokes equations on a three-dimensional torus. In the periodic setting, Babin, Mahalov and Nicolaenko ('97, '99) constructed global smooth solutions for large Coriolis parameters (i.e., high speed rotation) for periodic domains of arbitrary aspect ratios. Their proof was based on analysis of the limit equation, in which nonlinear interactions are restricted onto the resonant frequencies. They pointed out that the regularity properties of solutions to the limit equation depend discontinuously on the aspect ratios. In particular, for almost all aspect ratios they obtained a priori estimate

of Sobolev norms of solutions independent of the viscosity coefficient (and they could even show long time existence for inviscid flow), while in the worst case the estimate they obtained depends exponentially on the inverse of the viscosity coefficient. Note that the regular torus (i.e., the same period in all three directions) or torus with rational aspect ratios is included in the worst case.

In this talk, we focus on the case of regular (or rational) torus and give an improved estimate on the resonant interactions. The key ingredient in our proof is to count the total number of nontrivial resonant frequencies via combinatorial argument using the divisor bound. Such an argument is a standard tool in the study of periodic nonlinear dispersive equations, while it seems less developed in the context of equations of rotating fluids in a periodic domain due to complicated dispersion relations. As an application, we show global regularity for the rotating Navier-Stokes equations with fractional Laplacian. This talk is based on a joint work with Tsuyoshi Yoneda (The University of Tokyo).

Date:

Thursday, Mar. 09 15:50-16:30

Giorgio METAFUNE

University of Salento, Salento

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Università del Salento, Lecce, Italy.
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Title:

Sharp heat kernel bounds for a class of parabolic operators with singular coefficients

Abstract:

We study parabolic problems associated to the second order elliptic operator in $\mathbb{R}^N$

$$L = \Delta + (a-1)\sum_{i,j=1}^{N} \frac{x_i x_j}{|x|^2} D_{ij} + c\frac{x}{|x|^2}\cdot\nabla - b|x|^{-2}$$

with $a > 0$ and b, c real coefficients.

Note that the second order coefficients are discontinuous when $a \neq 1$ and singularities appear in the drift and potential terms. The choice $a = 1$, $c = 0$ yields the Schrödinger operator with inverse square potential. The condition

$$D := \frac{b}{a} + \left(\frac{N-1+c-a}{2a}\right)^2 \geq 0$$

is necessary and sufficient for the existence of a realization of L generating a positive semigroup and reduces to the classical one in the case of Schrödinger operators.

The operator L becomes self-adjoint in a suitable weighted L^2-space which we use as a tool for construncting the generated semigroup. However, generation in the unweighted L^p-spaces is also characterized. Letting

$$s_1 := \frac{N-1+c-a}{2a} - \sqrt{D}, \quad s_2 := \frac{N-1+c-a}{2a} + \sqrt{D}$$

it turns out that there exists a realizazion $L_{p,int}$ between the minimal and the maximal operator that generates a semigroup in $L^p(\mathbb{R}^N)$ if and only if $s_1 < N/p < s_2 + 2$.

We describe the domain and show that the generated semigroup is bounded analytic of angle $\pi/2$ and positive for $t > 0$. As a consequence, the spectrum of $L_{p,int}$ coincides with the half-line $(-\infty, 0]$.

We prove that the semigroup is represented by a kernel $p(t, x, y)$ which satisfies the double side estimates

$$p(z,x,y) \approx Ct^{-\frac{N}{2}} \left(\frac{|x|}{|y|}\right)^{-\frac{\gamma}{2}} \left[\left(\frac{|x|}{t^{\frac{1}{2}}} \wedge 1\right)\left(\frac{|y|}{t^{\frac{1}{2}}} \wedge 1\right)\right]^{-\frac{N}{2}+1+\sqrt{D}} \exp\left(-\frac{c|x-y|^2}{t}\right)$$

where $\gamma = (N-1+c)/a - N + 1$ and the constants c, C may differ in the upper and lower bounds. Note that $\gamma = 0$ if and only if L is self-adjoint.

Integrating the above kernel estimates with respect to t we also obtain precise kernel bounds of the Green function.

Finally we remark that kernel estimates for $|x|^\alpha L$ can be obtained from the results above via a change of variablels. However, this tranformation does not relate the kernel of $|x|^\alpha \Delta$ to that of the Laplacian but rather to the kernel of a suitable L as above, where discontinuities necessarily appear.

Most of the content of these lecture is based on joint works with Chiara Spina, Luigi Negro (University of Salento) and Motohiro Sobajima (Tokyo University of Science).

Date:

Friday, Mar. 10 14:50-15:30

Hideyuki MIURA

Tokyo Institute of Technology, Tokyo

Title:

On uniqueness for the harmonic map heat flow in supercritical dimensions

Abstract:

We examine the question of uniqueness for the equivariant reduction of the harmonic map heat flow in the energy supercritical dimension. It is shown that, generically, singular data can give rise to two distinct solutions which are both stable, and satisfy the local energy inequality. We also discuss how uniqueness can be retrieved.

Date:

Friday, Mar. 10 12:15-12:55

Anupam PAL CHOUDHURY

Technical University of Darmstadt, Darmstadt

anupampcmath@gmail.com

Title:

On the nematic liquid crystal flows

Abstract:

In this talk, we shall briefly review some existing results on the nematic liquid crystal flows in a bounded domain and discuss some ongoing works in the case of Dirichlet boundary conditions for the director field. We shall be mainly focusing on the $L_p - L_q$ maximal regularity approach.

Date:

Thursday, Mar. 09 12:15-12:55

Yutaka TERASAWA

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Title:

Finite energy for Navier-Stokes equations and Liouville-type theorems in two dimensional domains

Abstract:

This talk is based on a joint work with Professor Hideo Kozono (Waseda University) and Professor Yuta Wakasugi (Ehime University).

We consider the Cauchy problem for the Navier-Stokes equations

$$\begin{cases} v_t - \Delta v + (v \cdot \nabla)v + \nabla p = 0, & (x,t) \in \mathbb{R}^n \times (0,T), \\ \operatorname{div} v = 0, & (x,t) \in \mathbb{R}^n \times (0,T), \\ v(x,0) = v_0(x), & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

Here $v = v(x,t) = (v_1(x,t), \dots, v_n(x,t))$ and $p = p(x,t)$ denote the velocity and the pressure, respectively. Also, $v_0(x) = (v_0^1(x), \dots, v_0^n(x))$ stands for the given initial velocity.

Serrin, Shinbrot and Taniuchi proved the conditions for the energy identity of weak solutions of (1). Here we give another condition which ensures the energy identity of a smooth solution and as its application, we also give a Liouville-type result. Our first main result is the following.

Theorem 1 *Let $n \geq 2$, $v_0 \in L^2_\sigma(\mathbb{R}^n)$ and let (v, p) be a smooth solution of* (1). *Assume that there exist q_1, q_2, r_1, r_2 satisfying*

$$3 \leq q_1 \leq \frac{3n}{n-1},\ 3 \leq r_1 \leq \infty \quad \textit{and} \quad (q_1, r_1) \neq \left(\frac{3n}{n-1}, \infty\right), \tag{2}$$

$$2 \leq q_2 \leq \frac{2n}{n-2},\ 2 \leq r_2 \leq \infty \quad \textit{and} \quad \begin{cases} (q_2, r_2) \neq \left(\frac{2n}{n-2}, \infty\right) & (n \geq 3), \\ q_2 \neq \infty & (n = 2) \end{cases} \tag{3}$$

such that $v \in L^3(0, T; L^{q_1,r_1}(\mathbb{R}^n)) \cap L^2(0, T; L^{q_2,r_2}(\mathbb{R}^n))$. Assume also that the pressure p satisfies $p \in L^\infty(\mathbb{R}^n \times (0, T)$. Then, we have that

$$v \in L^\infty(0, T; L^2_\sigma(\mathbb{R}^n)) \cap L^2(0, T; \dot{H}^1_\sigma(\mathbb{R}^n))$$

and that

$$\|v(t)\|^2_{L^2} + 2\int_0^t \|\nabla v(\tau)\|^2_{L^2} d\tau = \|v_0\|^2_{L^2}$$

for all $t \in (0, T)$.

An immediate consequence of this theorem is the following Liouville-type theorem.

Corollary 2 *Let $n \geq 2$, and let $v_0 \equiv 0$ in $\mathbb{R}^n$. Suppose that v is a smooth solution of (1) with its associated pressure p. If $v \in L^3(0, T; L^{q_1,r_1}(\mathbb{R}^n)) \cap L^2(0, T; L^{q_2,r_2}(\mathbb{R}^n))$ for such (q_1, r_1) and (q_2, r_2) as in (2) and (3), respectively, and if $p \in L^\infty(\mathbb{R}^n \times (0, T))$, then it holds that $v(x, t) \equiv 0$ on $\mathbb{R}^n \times (0, T)$.*

We also treat the marginal case of (2) and (3) and get the condition for finite energy of solutions and get the corresponding Liouville-type theorem.

We next handle the equation for vorticity in 2D unbounded domains. Under some asymptotic behavior at infinity and some conditions at the boundary, we prove that the vorticity are square integrable.

Date:

Wednesday, Mar. 08 11:40-12:20

Patrick TOLKSDORF

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Title:

On the Stokes and Navier-Stokes equations in bounded Lipschitz domains

Abstract:

To perform certain iteration or fixed point methods to prove the existence of solutions to the Navier-Stokes equations in L^p-spaces, properties like maximal Ł^q-regularity of the Stokes operator, L^p-L^q-estimates of the Stokes semigroup, and

gradient estimates of the Stokes semigroup are needed. In this talk, I will give an overview of how to obtain these properties if the underlying domain is a bounded Lipschitz domain and I will present the key steps of their proofs.

Date:

Wednesday, Mar. 08 14:20-15:00

12 minutes talks

Shota ENOMOTO

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Title:

Asymptotic behavior of solutions to the compressible Navier-Stokes equation around space-time periodic states

Abstract:

We consider large time behavior of solutions to the compressible Navier-Stokes equation around space-time periodic states in an infinite layer of $\mathbb{R}^n$ ($n = 2, 3$) under the action of a space-time periodic external force. If the external force is sufficiently small, then the compressible Navier-Stokes system has a space-time periodic solution. We show that the space-time periodic states is asymptotically stable if the initial perturbation is sufficiently small. Furthermore, it is shown that the asymptotic leading part is given by a product of a solution of the one-dimensional viscous Burgers equation and a space-time periodic function when $n = 2$, and by a product of a solution of the two-dimensional heat equation and a space-time periodic function when $n = 3$.

This talk is based on a joint work with Professor Yoshiyuki Kagei (Kyushu University) and Mr. Mohamad Nor Azlan.

Date:

Wednesday, Mar. 08 17:00-18:00

Mathis GRIES

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Title:

A maximum regularity approach to the free boundary value problem for the primitive equations of the ocean

Abstract:

The free boundary value problem for the primitive equations of the ocean is presented and the equations are then transformed to yield a set equations on a fixed domain. We solve the linearised problem via the method of maximal L^p—-regularity for parabolic problems and then introduce a fixed-point argument for the quasilinear problem.

Date:

Thursday, Mar. 09 16:50-18:10

Naoto KAJIWARA

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Title:

Strong time periodic solution for semilinear parabolic equation on a real interpolation space

Abstract:

We study the time periodic solution for semilinear parabolic equations. By Da Prato and Grisvard theorem, a generator of analytic semigroups on X has the maximal regularity property in a real interpolation space $D_A(\theta, p)$. In the previous talk by Kress, he constructs the linear theory for time periodic solution in this real interpolation space. We prove existence and uniqueness of the time-periodic solution in the maximal regularity class $L^q(0, T; D_A(\theta, p))$ for the semilinear problem with polynomial nonlinearities from the linear theory and the fixed-point argument under a condition on p, q, θ.

This talk is based on a joint work with Prof. Matthias Hieber, Dr. Patrick Tolksdorf and Mr. Klaus Kress (Technical University of Darmstadt). .

Date:

Friday, Mar. 10 16:00-17:10

Tomoya KEMMOCHI

The University of Tokyo, Tokyo

Title:

Error estimate for the finite element semi-discretization of the non-stationary hydrostatic Stokes equation

Abstract:

We consider the finite element method for the non-stationary hydrostatic Stokes equation. The equation is the linearized problem of the primitive equation, which describes the model for geophysical flows, such as the ocean and the atmosphere. In this talk, we present the finite element semi-discretization. That is, we discretize the space variables only. We provide an optimal error estimate for the velocity by the semigroup approach. As in the case for the Stokes problems, the inf-sup condition plays a crucial role.

Date:

Thursday, Mar. 09 16:50-18:10

Klaus KRESS

Technical University of Darmstadt, Darmstadt

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Title:

Strong time periodic solutions for linear parabolic equations in a real interpolation space

Abstract:

In this talk, we study time periodic solutions for linear parabolic equations. By the Da Prato and Grisvard theorem, a generator of an analytic semigroup on X has the maximal regularity property in a real interpolation space $D_A(\theta, p)$. We prove the existence and uniqueness of a time periodic solution in the maximal regularity class $W^{1,q}(0,T; D_A(\theta,p)) \cap L^q(0,T; D_A(\theta,p))$ for a given T-periodic right hand side $f : \mathbb{R} \to D_A(\theta,p)$ with $f_{|(0,T)} \in L^p(0,T; D_A(\theta,p)$. In the following talk, Kajiwara extends the result to the semilinear problem with polynomial nonlinear terms.

This talk is based on a joint work with Prof. Matthias Hieber, Dr. Patrick Tolksdorf (Technical University of Darmstadt) and Naoto Kajiwara (The University of Tokyo).

Date:

Friday, Mar. 10 16:00-17:10

Taiga KUMAGAI

Waseda University, Tokyo

Title:

A class of Hamilton-Jacobi equations and ode systems on graphs

Abstract:

We consider the problem

$$\begin{cases} u^\varepsilon - \dfrac{b \cdot Du^\varepsilon}{\varepsilon} + |Du^\varepsilon| = f & \text{in } \Omega, \\ u^\varepsilon = 0 & \text{on } \partial\Omega, \end{cases}$$

where ε is a positive parameter, Ω is an open subset of $\mathbb{R}^2$ determined through a Hamiltonian H, $u^\varepsilon : \overline{\Omega} \to \mathbb{R}$ denotes the unknown, $f : \overline{\Omega} \to \mathbb{R}$ is a given, continuous, and nonnegative function, and $b : \mathbb{R}^2 \to \mathbb{R}^2$ is a Hamiltonian vector field.

We study the asymptotic behavior of solutions as ε goes to zero and show that the limit of the solutions is described as solutions of a system of odes on a graph. Freidlin-Wentzel, Freidlin-Weber, Sowers, by probabilistic techniques, and Ishii-Souganidis, by pde techniques, studied stochastic perturbation problems for Hamiltonian flows. Our problem can be seen as perturbation problems for Hamiltonian flows by optimal control theoretic disturbance.

Date:

Friday, Mar. 10 16:00-17:10

Andreas SCHMIDT

Technical University of Darmstadt, Darmstadt

Title:

The Navier-Stokes equations with the Coulomb boundary condition

Abstract:

We will consider the Navier-Stokes equations on manifolds with the Coulomb friction law boundary condition. This condition means that the solution can slip at the boundary once the tangential stress exceeds a given threshold. If that is the case, the tangential component of the solution has the opposite direction of the tangential stress. We will formulate this system, that has already been studied in the Euclidean case, on manifolds and obtain the existence of weak solutions. We will also discuss the dependence of the solution on the threshold.

Date:

Wednesday, Mar. 08 17:00-18:00

Anton SEYFERT

Technical University of Darmstadt, Darmstadt

Title:

The Poisson Equation in Exterior domains with Hodge Boundary Conditions

Abstract:

We consider a system of Poisson equations with Hodge boundary conditions (also called perfectly conducting wall condition), which is a boundary condition of zeroth and first order. Using an analogous method as in the case of the Stokes equations with Dirichlet boundary conditions, we show the existence of unique solutions in exterior domains with gradients in Lorentz spaces. The analogous treatment is mainly possible due to the availability of Sobolev embeddings in Lebesgue and Lorentz spaces on exterior domains despite the lack of zero boundary conditions.

Date:

Friday, Mar. 10 16:00-17:10

Suma'inna

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Title:

The Existence of $\mathcal{R}$-bounded Solution Operators of the Thermoelastic Plate Equation with Dirichlet Boundary Conditions

Abstract:

We consider the linearized thermoelastic plate equation with the Dirichlet boundary condition in general domain Ω, given by

$$\begin{aligned} u_{tt} + \Delta^2 u + \Delta\theta &= f_1 \quad \text{in } (0,\infty)\times\Omega, \\ \theta_t - \Delta\theta - \Delta u_t &= f_2 \quad \text{in } (0,\infty)\times\Omega, \end{aligned}$$

with the initial conditions $u|_{t=0} = u_0$, $u_t|_{t=0} = u_1$, $\theta|_{t=0} = \theta_0$ in Ω and the boundary condition $u = \theta = 0$, $\partial_\nu u = g$ on $(0,\infty)\times\Gamma$. Here, $u = u(x,t)$ denotes a vertical displacement at time t and at the point $x = (x_1, \cdots, x_n) \in \Omega$, while $\theta = \theta(x,t)$ describes the temperature. This work extends the result obtained by Naito and Shibata that study the problem in the half space case. We prove the existence of $\mathcal{R}$-bounded solution operators for the corresponding resolvent problem on general domains. Then, the operator-valued Fourier multiplier theorem gives the maximal L_p-L_q regularity of the above time-dependent problem. For this matter, we use the ideas due to Denk and Shibata in the free boundary condition case.

Date:

Friday, Mar. 10 16:00-17:10

Go TAKAHASHI

Waseda University, Tokyo

Title:

Partial regularity and extension of solutions to the Navier-Stokes equations

Abstract:

In this talk, we work on local-in-time classical solutions to the incompressible Navier-Stokes equations. The objective of this talk is to establish a time-extension criterion to the local-in-time solutions. There are quite a few time-extension criteria for local-in-time strong solutions, some of which were obtained by improving upon some inequalities such as critical Sobolev inequalities. Now we take a slightly different viewpoint and focus on partial regularity of the solutions. First we discuss partial regularity of solutions, which is an essential argument to obtain the local boundedness of the solutions. Then we will establish a time-extension criterion to the local-in-time solutions as an application of an ϵ-regularity theorem.

Date:

Wednesday, Mar. 08 17:00-18:00

Tobias TOLLE

Technical University of Darmstadt, Darmstadt

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Title:

Numerical modelling of jumps in two-phase flows in the context of one-field formulated Navier-Stokes equations

Abstract:

Methods for the numerical simulation of two-phase flows can be distinguished by their formulation of the Navier-Stokes equations. The first group uses two separate sets of equations which are coupled at the interface by jump conditions. The other group employs a one-field formulation of the Navier-Stokes equations which introduces additional terms acting at the interface. While the latter is superior in handling large interfacial deformations, it is challenging to maintain the property changes at the interface as sharp as possible without deteriorating the numerical accuracy.

This talk discusses several approaches on how to deal with interfacial discontinuities in the context of a hybrid Level Set / Front Tracking method on unstructured meshes. For illustration, these approaches are tested within a hydrodynamic test case. To provide some context, the results are compared to simulations performed with a state-of-the-art volume of fluid solver.

This talk is based on joint work with Tomislav Maric and Prof. Dieter Bothe from the *Mathematical Modelling and Analysis* group, TU Darmstadt.

Date:

Thursday, Mar. 09 16:50-18:10

Shinya UCHIUMI

Waseda University, Tokyo

Title:

A Lagrange-Galerkin scheme for high-Reynolds-number flow problems

Abstract:

The Lagrange-Galerkin scheme is a powerful numerical method for flow problems. Thanks to introduction of the locally linearized velocity, it is possible to implement the scheme exactly without numerical quadrature [Tabata and Uchiumi, Math. Comp., to appear]. There, convergence of the numerical solution has been proved for widely used Taylor-Hood element. Here we present a variant of this scheme which is more robust for high-Reynolds-number problems.

Date:

Thursday, Mar. 09 16:50-18:10

David WEGMANN

Technical University of Darmstadt, Darmstadt

Title:

Existence of Strong Solutions and Decay of Turbulent Solutions of Navier-Stokes Flow with Nonzero Dirichlet Boundary Data

Abstract:

We consider the Navier-Stokes equations in a domain with compact boundary and nonzero Dirichlet boundary data β. A solution is constructed as a sum of a very weak solution b to the instationary Stokes equations with nonzero boundary data and a weak solution v to a system of Navier-Stokes type with zero boundary data.

Assuming that $\beta(t) \to 0$ as $t \to \infty$, we proved in [2] that there exists a solution v which fulfills $\|v(t)\|_2 \to 0$ as $t \to \infty$. Furthermore, in a bounded domain the solution v tends exponentially to 0 if the corresponding data is exponentially decreasing.
As a last result, we calculated a lower polynomial bound for the decay rate if Ω is unbounded. Therefore, we used a suitable spectral decomposition of the Stokes operator as introduced in [1] and Duhamel's formula.

Recently, we proved the same decay result for an arbitrary turbulent solution. The main tool for the proof is to show the existence of a strong solution after some time $T > 0$ and to identify the turbulent solution with the strong solution in the time interval $[T, \infty)$.

This is a result of a collaboration with Prof. Dr. Reinhard Farwig and Prof. Dr. Hideo Kozono.

References:

[1] Borchers, W., Miyakawa, T., "Algebraic L^2 decay for Navier-Stokes flows in exterior domains. II," Hiroshima Math. J., **3**, 621–640 (1991).

[2] Farwig, R., Kozono, H., Wegmann, D.,"Decay of non-stationary Navier-Stokes flow with nonzero Dirichlet boundary data," Indiana Univ. Math. J., to appear (2015)

Date:

Wednesday, Mar. 08 17:00-18:00

Marc WRONA

Technical University of Darmstadt, Darmstadt

Title:

Well-posedness of the primitive equations with only horizontal viscosity for several classes of inital data

Abstract:

The focus of this talk are the primitive equations for planetary atmospheric and oceanic dynamics with only horizontal viscosity. Several classes of initial data are investigated including $W^{1,p}, p > 2$, as well as the anisotripoc spaces $H^n \cap L^2_{xy}H^k_z, k \geq n$. Global well-posedness for these classes is shown via a uniform a priori bound for systems with full viscosity. After establishing well-posedness for these regularized systems, a system version of the classical Grönwall inequality and a logarithmic Sobolev inequality are used to obtain the desired a priori bound.

Date:

Thursday, Mar. 09 16:50-18:10

Sebastian ZAIGLER

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Title:

Regularity structures for the primitive equations

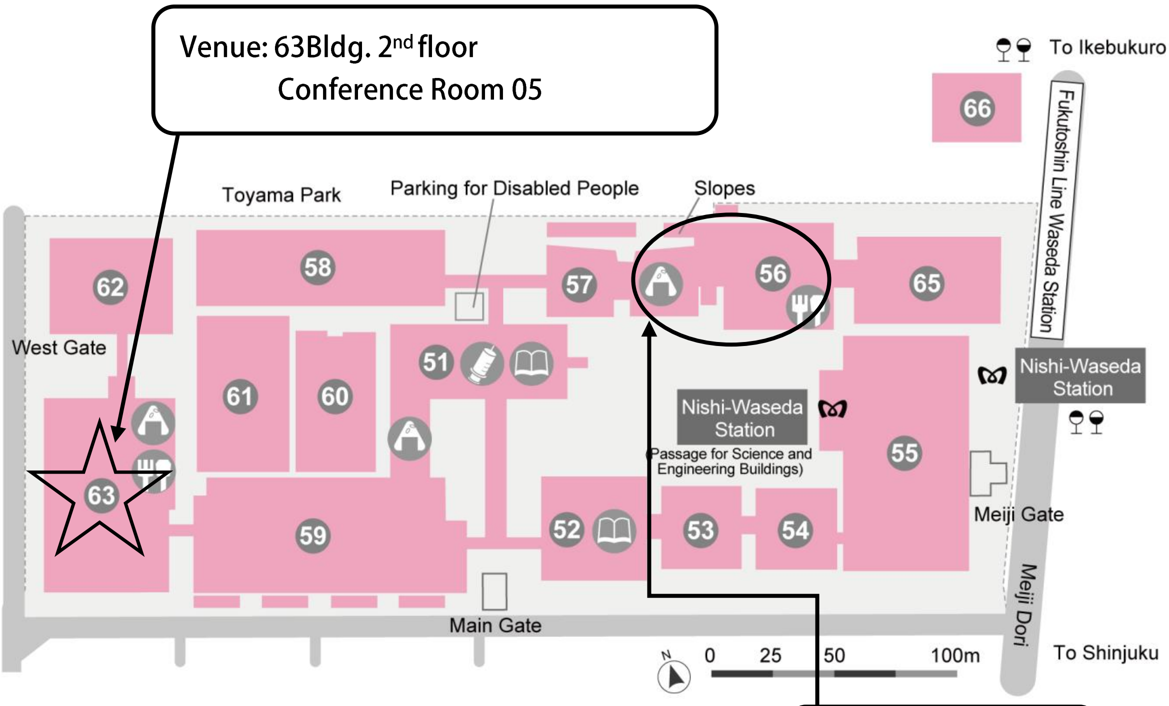
Abstract:

The new technique of regularity structures introduced by Martin Hairer is a method to find local solutions for subcritical SPDEs with rough data, which were formerly inaccessible. Regularity structures are spaces of abstract "Taylor expansions", in which a solution of a SPDE can be found by solving fixed point equations. The sequence of these fixed points, which are renormalised solution of the SPDE, converges to a solution of the SPDE. We will give a short introduction how to modify them to solve the primitive equations with white noise.

Date:

Thursday, Mar. 09 16:50-18:10

Nishi-Waseda Campus Map



Basement floor:
Shop and Cafeteria

Organizers : Reinhard FARWIG (TU Darmstadt)
Matthias HIEBER (TU Darmstadt)
Hideo KOZONO (Waseda University)
Yoshihiro SHIBATA (Waseda University)

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- JSPS Grant No. 16H06339, New development of mathematical theory of turbulence by collaboration of the nonlinear analysis and computational fluid dynamics (Hideo KOZONO)
- JSPS Grant No.24224004, Construction of mathematical theory to investigate the macro structure and the meso structure of the fluid motion (Yoshihiro SHIBATA)

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